Chapter 6

Product Root Sum Mean labeling on Subdivision Graphs

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Abstract

Let G=(V, E) be a simple graph with p vertices and q edges an injective function f:V→{1,2,3,…,q+1} is said to be a Product Root Sum Mean Labeling if the induced function f* defined on edges by $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$ yields different values. In this paper we prove the Product Root Sum Mean Labeling on Subdivision Graphs such as Subdivision graph of Triangular Snake graph, Quadrilateral Snake graph, Triangular Ladder graph, Total Comb graph and Middle Comb graph.

Keywords: Subdivision Graphs, Triangular Snake graph, Quadrilateral Snake graph, Triangular Ladder graph, Total Comb graph, Middle Comb graph, Product Root Sum Mean Labeling.

1. Introduction

In 1967, Rosa came up with the concept of graph labeling [2]. Assigning integers to the vertices, edges, or both, under specific circumstances, is known as graph labeling. Many graph labeling techniques, including graceful, anti-magic, harmonious, magic, odd

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graceful, prime, bi-magic labeling, square sum labeling, difference labeling, mean labeling, even mean labeling, odd mean labeling, Root Square Mean Labeling, Root Square Even Mean labeling, Root Square Odd Mean labeling, and Product Root Sum Mean Labeling, have been examined in more than 2500 papers over the years [3]. A mean labeling in G (V, E) is an 1-1 function $f: v \rightarrow \{0, 1, \ldots, p\}$ such that $\frac{1}{2}[f(u) + f(v)]$, if $f(u) + f(v)$ is even and $\frac{1}{2}[f(u) + f(v) + 1]$ if $f(u) + f(v)$ is odd, is distinct for any edge [5]. The Root Square Odd and Even Mean Labeling, Product Root Sum Mean Labeling was introduced [9], [10]. Let G = (u,v) be a graph an 1-1 function $f: v \rightarrow \{1, \ldots, e + 1\}$ is PRSML, if the function f^* defined by $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$ yield different values. In this research paper we propose the above said labeling Subdivision Graphs such as Triangular Snake Graph, Quadrilateral Snake Graph, Triangular Ladder Graph, Total Comb Graph, and Middle Comb Graph.

2. Main Results

Theorem 2.1: Subdivision graph of Triangular Snake Graph TS_m is a Product Root Sum Mean Label Graph.

Proof:

Let V = $\{v_1, v_2, v_3 \ldots\ldots, v_{n-1}, v_n;\ u_1, u_2, u_3, \ldots u_{n+1};\ w_1, w_2, w_3 \ldots w_n;\ x_1, x_2, x_3, \ldots x_n;\ y_1, y_2, y_3 \ldots\ldots y_n\}$ be the vertex set and the edge set be E=$\{ v_iu_i,\ 1 \le i \le n\ ;\ u_{i+1}v_i,\ 1 \le i \le n\ ;\ u_iw_i,\ 1 \le i \le n;\ w_iy_i\ ,\ 1 \le i \le n;\ u_{i+1}x_i\ 1 \le i \le n \}$ defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 4i - 2;\ 1 \le i \le n + 1;\ f(v_i) = 6i - 3\ ;\ 1 \le i \le n\ ;\ f(w_i) = 6i - 1;\ 1 \le i \le n;\ f(x_i) = 6i + 1;\ 1 \le i \le n;\ f(y_i) = 4i;\ 1 \le i \le n$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.2: The subdivision graph of Triangular Ladder Graph TL_n is a Product Root Sum Mean Graph.

Proof:

Let V = {u_1, u_2, u_3,…,u_{n-1}, u_n, u_{n+1};v_1, v_2, v_3……, v_{n-1}, v_n;w_1, w_2, w_3…..w_{n+1}; x_1, x_2, x_3,…..x_n; y_1, y_2, y_3…….y_n; z_1, z_2, z_3…….z_n} be the vertex set and the edge set be E={ v_iu_i, $1 \le i \le n + 1$; $u_{i+1}x_i$, $1 \le i \le n$; v_iw_i, $1 \le i \le n$; u_ix_i, $1 \le i \le n$; w_iz_i , $1 \le i \le n$; z_iw_{i+1}, $1 \le i \le n$; w_iy_i , $1 \le i \le n$; y_iu_{i+1}, $1 \le i \le n$ } defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 6i - 2; 1 \le i \le n + 1$, $f(v_i) = 6i - 4$; $1 \le i \le n$, $f(w_i) = 6i$; $1 \le i \le n$, $f(x_i) = 6i - 3$; $1 \le i \le n$, $f(y_i) = 6i - 1$; $1 \le i \le n$, $f(z_i) = 6i + 1$; $1 \le i \le n$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.3 The subdivision graph of Quadrilateral Snake Graph Q_n is a Product Root Sum Mean Labeling Graph.

Proof:

Let V = { u_1, u_2, u_3,…u_n,.u_{n+1}; v_1, v_2, v_3……, v_{n-1}, v_n; w_1, w_2, w_3…..w_n; x_1, x_2, x_3,…..x_n; y_1,y_2,y_3…….y_n; z_1, z_2, z_3…….z_n; t_1, t_2, t_3…….t_n} be the vertex set and the edge set be E={ v_iu_i, $1 \le i \le n$; $u_{i+1}v_i$, $1 \le i \le n$; u_iw_i, $1 \le i \le n$; $u_{i+1}t_i$, $1 \le i \le n$; w_ix_i, $1 \le i \le n$;t_iy_i , $1 \le i \le n$; z_ix_i $1 \le i \le n$; x_iy_i, $1 \le i \le n$ } defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 4i - 2$, $1 \le i \le n + 1$; $f(v_i) = 8i - 5$,$1 \le i \le n$; $f(w_i) = 8i - 7$, $1 \le i \le n$;$f(x_i) = 8i - 4$,$1 \le i \le n$; $f(y_i) = 8i$,$1 \le i \le n$; $f(z_i) = 8i - 3$, $1 \le i \le n$; $f(t_i) = 8i - 1$,$1 \le i \le n$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.4: The subdivision graph of Total Comb Graph $T(P_n \odot K_1)$

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is a Product Root Sum Mean Graph.

Proof:

Let V = { u_1, u_2, u_3,..., u_{n-1},.u_n; v_1, v_2, v_3......, v_{n-1}, v_n; w_1, w_2, w_3.....w_n; x_1, x_2, x_3,.....x_n; y_1,y_2,y_3.......y_n; z_1, z_2, z_3.......z_n; p_1, p_2, p_3,..., p_{n-1}, q_1, q_2, q_3,..., q_{n-2}; r_1, r_2, r_3,..., r_{n-1}; s_1, s_2, s_3,..., s_{n-1}; t_1, t_2, t_3.......t_{n-1}; j_1, j_2, j_3,..., j_{n-1}; k_1, k_2, k_3,..., k_{n-1}} be the vertex set and the edge set be E={ u_iv_i, $1 \le i \le n$; u_iz_i, $1 \le i \le n$; v_iw_i, $1 \le i \le n$; w_ix_i, $1 \le i \le n$; w_ij_i, $1 \le i \le n$; x_iy_i, $1 \le i \le n$; z_iy_i, $1 \le i \le n$; y_ip_i, $1 \le i \le n$; $y_{i+1}p_i$, $1 \le i \le n$; y_ir_i, $1 \le i \le n$; r_is_i, $1 \le i \le n$; s_it_i, $1 \le i \le n$; $y_{i+1}t_i$, $1 \le i \le n$; j_ix_i, $1 \le i \le n$; s_ik_i , $1 \le i \le n$} defined for all values of n. Define a map f: V → {1, 2, 3......q+1} such that the vertex labels are $f(u_i) = 12i$,$1 \le i \le n$;$f(v_i) = 12i - 8$,$1 \le i \le n$;$f(w_i) = 12i - 4$,$1 \le i \le n$; $f(x_i) = 12i - 2$,$1 \le i \le n$ $f(y_i) = 12i - 10$,$1 \le i \le n$;$f(z_i) = 12i - 6$,$1 \le i \le n$;$f(p_i) = 14i - 9$,$1 \le i \le n - 1$;

$f(q_i) = 14i - 1$,$1 \le i \le n - 2$;$f(r_i) = 14i - 11$,$1 \le i \le n - 1$;$f(s_i) = 14i - 7$,$1 \le i \le n - 1$;$f(t_i) = 14i - 3$,$1 \le i \le n - 1$;$f(j_i) = 14i - 13$,$1 \le i \le n - 1$;$f(k_i) = 14i - 5$,$1 \le i \le n - 1$;Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.5: The subdivision graph of Middle Comb Graph M($P_{n\odot}$ K_1) is a Product Root Sum Mean Graph.

Proof:

Let V = { u_1, u_2, u_3,..., u_n,.u_{n+1}; v_1, v_2, v_3......, v_n, v_{n+1}; w_1, w_2, w_3.....w_n, w_{n+1}; x_1, x_2, x_3,.....x_n, x_{n+1}; y_1,y_2,y_3.......y_n, y_{n+1}; z_1, z_2, z_3.......z_n; p_1, p_2, p_3,..., p_n, q_1, q_2, q_3,..., q_n; r_1, r_2, r_3,..., r_n; s_1, s_2, s_3,..., s_n; t_1, t_2, t_3.......t_{n-1}} be the vertex set and the edge set be E={ u_iv_i, $1 \le i \le n + 1$; v_iw_i, $1 \le i \le n+1$; w_ix_i, $1 \le i \le n+1$; x_iy_i, $1 \le i \le n+1$; w_ir_i, $1 \le i \le n$; r_ip_i, $1 \le i \le n$; s_ip_i, $1 \le i \le n$; p_iw_{i+1}, $1 \le i \le n$; y_iz_i, $1 \le i \le n$; q_ip_i, $1 \le i \le n$; $y_{i+1}q_i$, $1 \le i \le n - 1$; p_it_i, $1 \le i \le n - 1$; $t_ip_i + 1$, $1 \le i \le n - 1$;

$z_i p_i$ $1 \leq i \leq n$; } defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 10i - 8$, $1 \leq i \leq n+1$; $f(v_i) = 10i - 6$, $1 \leq i \leq n+1$; $f(w_i) = 10i - 4$, $1 \leq i \leq n+1$; $f(x_i) = 10i - 2$, $1 \leq i \leq n+1$; $f(y_i) = 10i$, $1 \leq i \leq n+1$; $f(z_i) = 12i - 9$, $1 \leq i \leq n$; $f(p_i) = 12i - 7$, $1 \leq i \leq n$; $f(q_i) = 12i - 3$, $1 \leq i \leq n$; $f(r_i) = 12i - 11$, $1 \leq i \leq n$; $f(s_i) = 12i - 5$, $1 \leq i \leq n$; $f(t_i) = 12i - 1$, $1 \leq i \leq n-1$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$ Thus, the edge labels are distinct. Hence the theorem.

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Chapter 7

The Digital Lookout: Enhancing Roadside Safety Through Explainable Deep Learning

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Abstract

Each year, about 3% of the world's GDP, and 1.3 million people, die in traffic accidents. This report analyzes the role of explainable deep learning as digital lookouts for roadside safety and roadside risk intelligent hazard detection and explainable decision making. We study the combination of cutting-edge technologies and frameworks including explainable AI methods SHAP and Grad-CAM with YOLOv8, ResNet-50, and Faster R-CNN. We show YOLOv8 as the best for our metrics with 96.2% detection accuracy for 95 FPS. Also, SHAP values reach 92.5% on interpretability. Explainable AI systems create 100ms on-the-fly detection of and collisions and explainable AI systems show 35-60% real time detections reduction of 100ms on-the-fly or explainable decision making. We create the first transparent explainability integrated deep learning solution to safe systems to real world explainable AI products systems for the first time.

Keywords: Explainable AI, Deep learning, Roadside safety, Object detection, Grad-CAM, Autonomous vehicles.

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