EOQ model with one time price discount involves reorder points

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Abstract

To encourage customers to make larger purchases, suppliers frequently offer price discounts. This study presents the construction of EOQ models with price discounts that can occur at reorder points or in between reorder points. The supplier offers the consumer a price discount all at once when they request a special order quantity. Numerical examples are used to explain the suggested models and solution methodologies.

Keywords: Inventory, Order Quantity, Reorder point, Special reorder point.
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1. INTRODUCTION

A typical tactic is to entice customers to make larger purchases by providing all-unit or increasing quantity discounts. The price discounts simply apply the reduced unit price to units purchased above a certain amount. While all-units discounts apply the lower unit price to the entire lot when a bigger quantity is purchased. As a result, while the price discount for increasing quantity can produce numerous unit prices for an item within the same lot, the all-units discount yields the same unit price for every item in the given lot.

Heuristics for procurement from several providers with different quantity discounts were created by Burke et al. in 2008. EOQ and quantity discounts under date-times supplier credit were examined by Carlson et al. in 1996. In order to lower ordering costs, Latha et al. (2021) and Lin (2008) EOQ models with backorder price discounts. When lead time demand is under control, Lee et al. (2007) explored a computational algorithmic approach for the best inventory policy that includes back-order discounts and reduced ordering costs. An inventory model with random discount offers was created by Mahdi Tajbakhsh et al. in 2011. A distribution-free strategy with adjustable setup costs, price breaks for backorders, and manageable lead times was examined by Malik and Sarkar (2018). Sai (2007) investigated a supply chain management optimization strategy that includes a quantity discount policy. In response to a brief price markdown tied to order quantity, Tirpathi and Tomar (2015) devised an ideal order policy for a degrading good with time-dependent demand. Yang (2004) thought about employing quantity discounts as a pricing strategy for deteriorating goods when demand is price-sensitive. Leopoldo Eduardo Cardenas-Barron et al. (2010), developed optimal order size inventory model with one-time discount.

In 1997, Wee and Yu developed a model of depreciating inventory with a temporary price decrease. Cardenas-Barron et al. (2011) created an arithmetic-geometric inequality to address the vendor-buyer integrated inventory system. Ravithammal et al. (2019) used an algebraic approach with inventory level constraints to analyse the EOQ inventory model. Muniappan et al. concentrated an integrated economic order quantity model with inventory level and storage capacity restrictions (2020). A deterministic production inventory model for buyer-producer relationships with quantity discounts for goods with defined life spans was created by Ravithammal et al (2015).

2. NOTATIONS AND ASSUMPTIONS

The model uses the following notations and assumptions.

2.1 Notations

D Demand rate, units per time unit

R_1 Buyer's fixed ordering cost / order

H_b Inventory carrying cost per unit per time unit $(H_b = iC)$

s_c Buyer's unit screening cost / order / unit

Q Optimum Order quantity, units

Q' Special order quantity in reorder point, units

Q'' Special order quantity between reorder points, units

q Amount of on hand inventory when special order is placed, units

C Product unit cost

i Inventory carrying cost rate

μ Fixed - percentage discount offered by supplier,

X Additional units beyond Q to purchase to take advantage of discount

2.2 Assumptions

The following characteristics apply to this model

- Consistent demand rate is taken into account
- The supplier's offer period is extremely brief
- The Planning horizon is one year
- The offer period can occur at a reorder point or in between reorder points
- The one-time fixed-percentage discount is provided
- Price discount applicable for additional units only

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3. MODEL FORMULATION

Three cases are discussed in this section. Case (i) depicts the scenario where the buyer decides to keep purchasing Q units despite not taking advantage of the reduced pricing. Case (ii) depicts the scenario where the buyer places a special order quantity at reorder point with price discount for additional units. And case (iii)

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depicts the scenario where the buyer places a special order quantity between reorder points with price discount for additional units.

Case (i): EOQ with no Discount

The total cost for buyer can be written as

TC_b = Ordering cost+ Carrying cost +Screening cost +Purchasing cost

$$TC_b = \frac{DR_1}{Q} + \frac{H_b}{2} + \frac{s_c}{2} + DC \qquad (1)$$

Equation (1) can be composed as

$$TC_b = \left[\frac{H_b + s_c}{2}\right] Q + \frac{1}{Q}[DR_1] + DC \qquad (2)$$

Equation (2) is of the structure $a_1 Q + \frac{a_2}{Q} + a_3$.

Q will be taken as, $Q = \sqrt{\frac{a_2}{a_1}}$

$$Q = \sqrt{\frac{2DR_1}{H_v + s_c}} \qquad (3)$$

Case (ii): Price Discount for additional quantity in reorder point

In this case the buyer cost TC_{b1} contains

- The overall ordering cost for the year is the cost to place the unique order for the ( ) units, plus times the number of additional orders that are necessary to supply the remaining $Q - X)$ units of demand.
- The holding cost is derived as carrying additional units for entire period at $-\mu)$ + during the period the average carrying units at $-\mu)$ + for the remaining period the carrying average inventory of $\frac{(}{-})$ at units.
- The screening cost is derived as at per units for entire year + for the remaining time of $\frac{X}{-})$ at per unit.
- The purchased cost is derived as $-X)$ for usual price + the cost of the units purchased at the discount price is $\cdot \mu)X$.

Hence, buyer cost TC_{b1} will be written as

$$TC_{b1} = R_1 + \frac{R_1\,(D-Q-X)}{Q} + \frac{H_b(1-\mu)QX}{D} + H_b(1-\mu)\left(\frac{X}{2}\right)\left(\frac{X}{D}\right) + \frac{H_bQ}{2}\left(\frac{D-X}{D}\right) + \frac{s_cQ}{2} + \frac{s_cQ}{2}\left[\frac{(D-Q-X)}{D}\right] + C(1-\mu)X + C(D-X) \qquad (4)$$

$$D(X) = TC_b - TC_{b1}$$

$$D(X) = \left[\frac{H_b\,\mu}{2D} - \frac{H_b}{2D}\right]X^2 + \left[\frac{R_1}{Q} - \frac{H_bQ}{2D} + \frac{H_b\mu Q}{D} + \frac{s_cQ}{2D} + C\mu\right]X + \frac{s_cQ^2}{2D} - \frac{s_cQ}{2} \qquad (5)$$

Equation (5) it is of the structure $a_1 Q^2 + a_2 Q + a_3$.

Q will be taken as, $Q = \frac{-a_2}{2\,a_1}$

$$X = \left(\frac{\mu}{1-\mu}\right)\left[Q + \frac{DC}{H_b}\right] + \frac{1}{1-\mu}\left[\frac{2DR_1 + s_c Q^2}{2QH_b} - \frac{Q}{2}\right] \quad (6)$$

Therefore special order quantity is $Q' = Q + X$

Case (iii) Price Discount for additional quantity between reorder points

In this case the buyer cost TC_{b2} contains

- The special order for more units and the final order, along with the number of additional orders that will be required to meet the remaining $) - X$ demand, bring the total yearly ordering cost to .
- The overall cost of screening and purchasing is equal to that for case (ii).
- The holding cost is derived as carrying additional units for entire period at $-\mu)$ + during the period the average carrying units at $-\mu)$ + for the remaining period the carrying average inventory of $\frac{)-X}{)})$ at units + plus carrying an average of for the entire cycle in which is accepted the price discount at per unit.

Hence, buyer cost TC_{b2} will be written as

$$TC_{b2} = 2R_1 + \frac{R_1(D-Q-X)}{Q} + \frac{H_b(1-\mu)Xq}{D} + H_b(1-\mu)\left(\frac{X}{2}\right)\left(\frac{X}{D}\right) + \frac{H_bQ}{2}\left(\frac{D-Q-X}{D}\right) + \frac{H_bQ}{2}\left(\frac{Q}{D}\right) + \frac{s_cQ}{2} + \frac{s_cQ}{2}\left(\frac{D-Q-X}{D}\right) + C(1-\mu)X + C(D-X) \quad (7)$$

$$D(X) = TC_b - TC_{b2}$$

$$D(X) = \left[\frac{H_b\mu}{2D} - \frac{H_b}{2D}\right]X^2 + \left[\frac{R_1}{Q} - \frac{H_bQ}{2D} - \frac{H_bq}{D} + \frac{H_b\mu q}{D} + \frac{s_cQ}{2D} + C\mu\right]X + \frac{s_cQ^2}{2D} - \frac{s_cQ}{2} \quad (8)$$

Equation (8) it is of the structure $a_1Q^2 + a_2Q + a_3$.

Q will be taken as, $Q = \frac{-a_2}{2a_1}$

$$X = \left(\frac{1}{1-\mu}\right)\left[\frac{Q}{2} + \frac{\mu DC}{H_b}\right] + \frac{1}{1-\mu}\left[\frac{2DR_1 + s_cQ^2}{2QH_b}\right] - q \quad (9)$$

Therefore special order quantity, $Q'' = Q + X$

4. NUMERICAL EXAMPLE

Example 1

Let $)0000$, 200 , $.1$, , 10 , 0.001 , 000.

The Optimal solution is,

$Q = 1400$, $TC_b = 1.1428X\ 10^5$, $X = 502$, $TC_{b1} = 1.2094X\ 10^5$, $X = 1201$, $TC_{b2} = 1.2095X\ 10^5$.

Example 2

Let $)0000$, 500 , $.05$, , $).1$, $).01$, 00.

The Optimal solution is,

$Q = 8944$, $TC_b = 6.2236X\ 10^4$, $X = 4130$, Q' = 13075, $TC_{b1} = 6.1967X\ 10^4$,

$X = 8557, Q^{''} = 17502, TC_{b2} = 6.1786X\,10^4$.

Example 3

Let 30000 150 , .08 , ,).01 , 000.

The Optimal solution is,

$Q = 1438,\ TC_b = 1.7669\ X\ 10^4,\ X = 1024,\ Q^{'} = 2463, TC_{b1} = 1.8035X\ 10^4,$

$X = 1736, Q^{''} = 3175, TC_{b2} = 1.8047X\ 10^4.$

Example 4

Let 1000 , 100 , 2 , ,).01 ,).05 , 00.

The Optimal solution is,

$Q = 572,\ TC_b = 3.3493\ X\ 10^3,\ X = 293, Q^{'} = 866, TC_{b1} = 3.2384\ X\ 10^3,$

$X = 564,\ Q^{''} = 1137, TC_{b2} = 3.3345\ X\ 10^3.$

4.1 Sensitivity Analysis

By changing each boundary in turn while leaving the remaining borders alone, the sensitivity analysis is completed. The results are shown in Table 1.

Table 1: Effects of changes in the value of system parameters

	Decision variable	Q	TC_b	X	$Q^{'}$	TC_{b1}	X	$Q^{''}$	TC_{b2}
D	40000	1252	0.9275X10^5	401	1653	0.9871 X10^5	1027	2279	0.9872 X10^5
	45000	1328	1.0355 X10^5	451	1779	1.0986 X10^5	1115	2443	1.0987 X10^5
	50000	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2095 X10^5
	55000	1468	1.2498 X10^5	552	2020	1.3198 X10^5	1285	2753	1.3198 X10^5
	60000	1533	1.3565 X10^5	602	2135	1.4296 X10^5	1368	2901	1.4297 X10^5
R_1	200	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2095 X10^5
	1000	3131	1.3194 X10^5	503	3634	1.4629 X10^5	3390	6521	1.4631 X10^5
	1800	4200	1.4285 X10^5	504	4704	1.6166 X10^5	3390	7590	1.6168 X10^5
	2600	5048	1.5150 X10^5	505	5553	1.7367 X10^5	3027	8075	1.7370 X10^5
	3400	5773	1.5889 X10^5	506	6279	1.8383 X10^5	3390	9163	1.8386 X10^5
i	0.08	1403	1.1425 X10^5	627	2030	1.2089 X10^5	1327	2730	1.2090 X10^5
	0.09	1401	1.1427 X10^5	557	1958	1.2092 X10^5	1257	2658	1.2093 X10^5
	0.01	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2095 X10^5
	0.11	1398	1.1430 X10^5	456	1854	1.2096 X10^5	1155	2553	1.2095 X10^5
	0.12	1397	1.1431 X10^5	418	1815	1.2098 X10^5	1116	2513	1.2099 X10^5
C	2	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2095 X10^5
	4	1386	2.1142 X10^5	502	1888	2.2102 X10^5	1194	2580	2.2103 X10^5
	6	1373	3.1456 X10^5	502	1875	3.2109 X10^5	1188	2561	3.2112 X10^5

	8	1360	4.1470 X10^5	502	1862	4.2117 X10^5	1181	2541	4.2120 X10^5
	10	1348	5.1483 X10^5	502	1850	5.2124 X10^5	1175	2523	5.2128 X10^5
波$_c$	6	1796	1.1114 X10^5	502	2298	1.1622 X10^5	1399	3195	1.1623 X10^5
	8	1561	1.1281 X10^5	502	2063	1.1873 X10^5	1282	2843	1.1874 X10^5
	10	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2095 X10^5
	12	1280	1.1562 X10^5	501	1781	1.2295 X10^5	1141	2421	1.2296 X10^5
	14	1186	1.1685 X10^5	501	1687	1.2479 X10^5	1094	2280	1.2480 X10^5
μ	0.0006	1400	1.1428 X10^5	301	1701	1.2100 X10^5	1000	2400	1.2101 X10^5
	0.0008	1400	1.1428 X10^5	401	1801	1.2097 X10^5	1101	2501	1.2098 X10^5
	0.0010	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2095 X10^5
	0.0012	1400	1.1428 X10^5	602	2002	1.2092 X10^5	1301	2701	1.2092 X10^5
	0.0014	1400	1.1428 X10^5	702	2102	1.2089 X10^5	1402	2802	1.2089 X10^5
q	2000	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2095 X10^5
	6000	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2097 X10^5
	10000	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2099 X10^5
	14000	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2101 X10^5
	18000	1400	1.1428 X10^5	502	1902	1.2094 X10^5	1201	2601	1.2103 X10^5

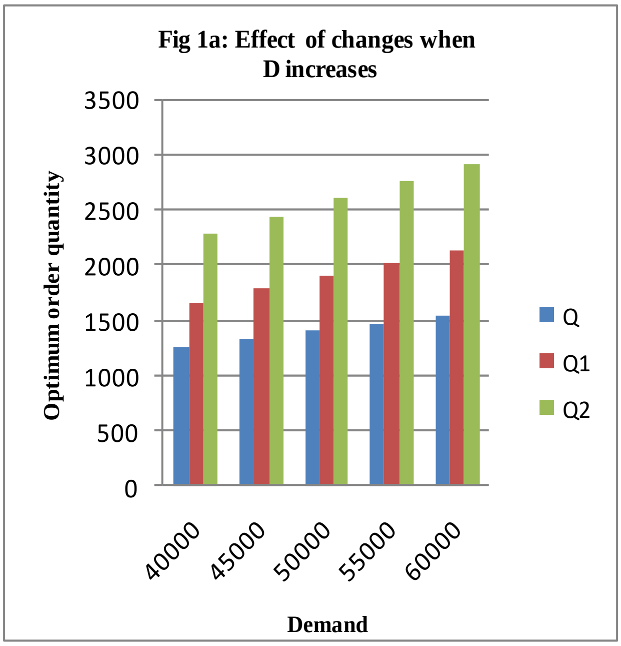


Fig 1a: Effect of changes when D increases

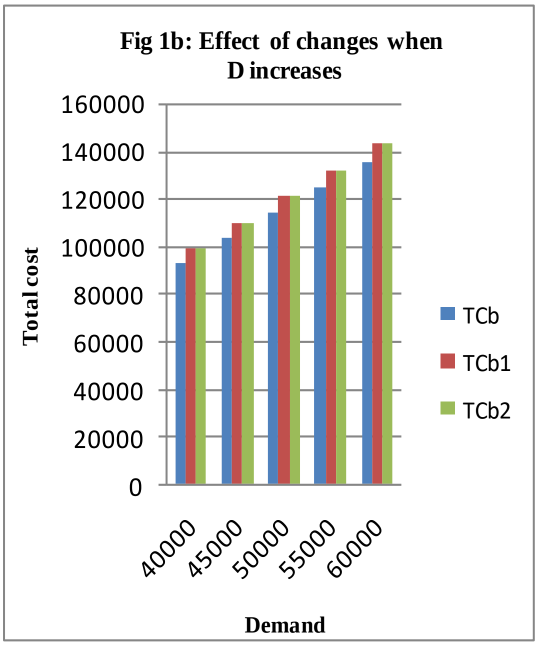


Fig 1b: Effect of changes when D increases

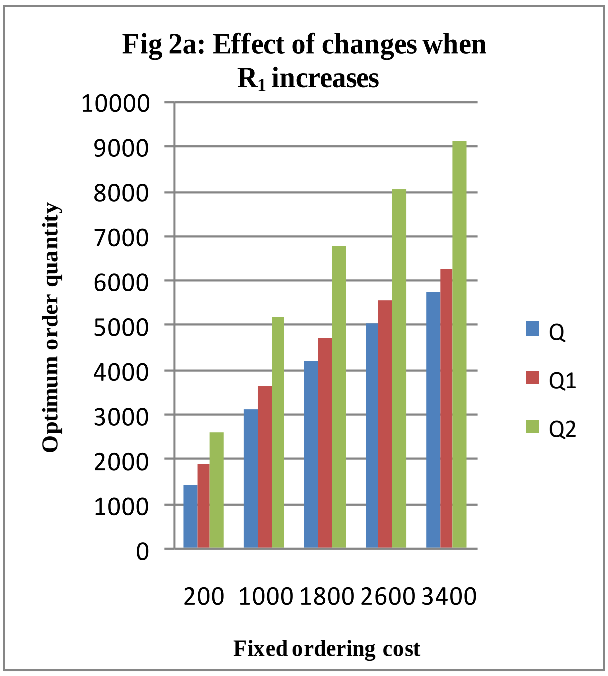


Fig 2a: Effect of changes when R_1 increases

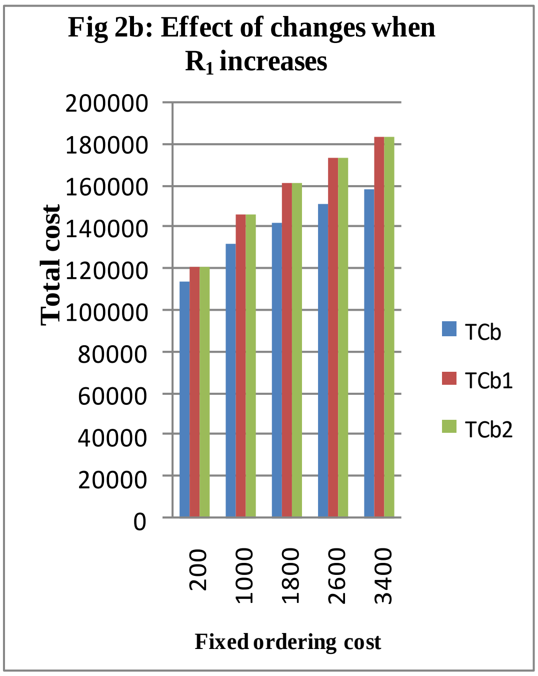


Fig 2b: Effect of changes when R_1 increases

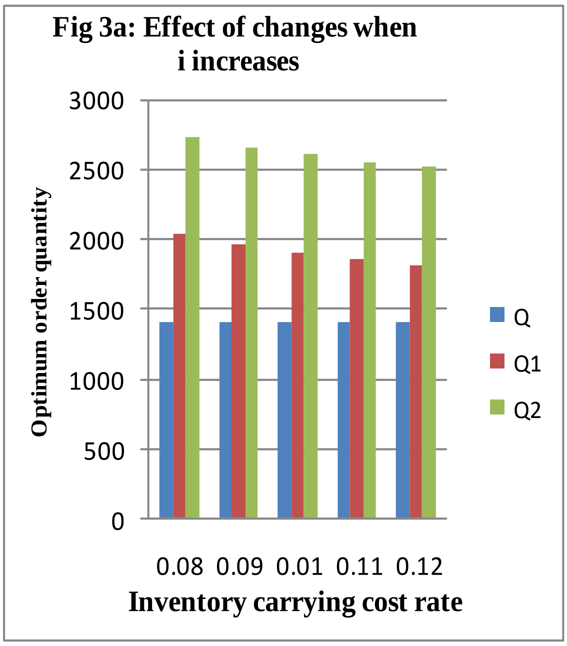


Fig 3a: Effect of changes when i increases

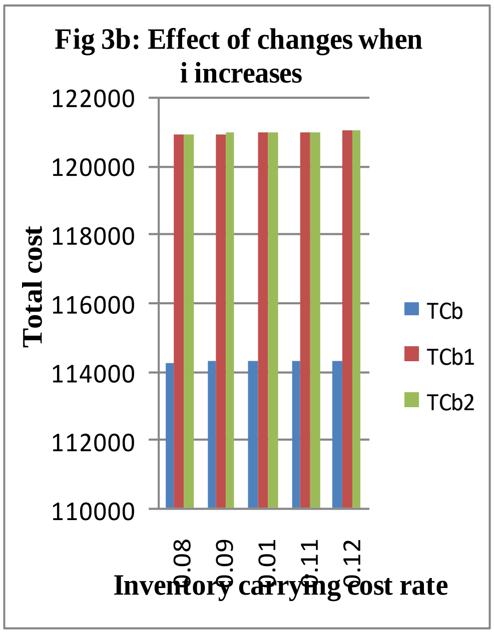


Fig 3b: Effect of changes when i increases

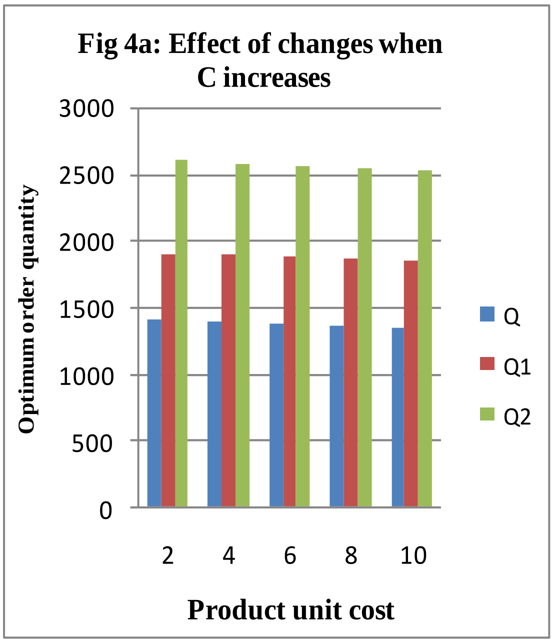


Fig 4a: Effect of changes when C increases
3000
2500
2000
1500
1000
500
0
Optimum order quantity
2
4
6
8
10
Product unit cost
Q
Q1
Q2

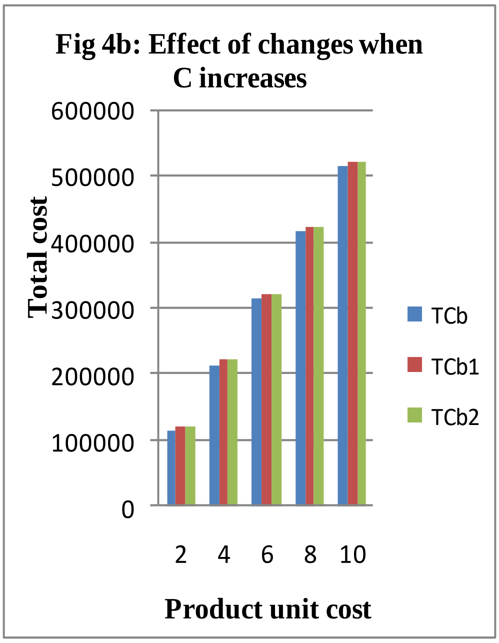


Fig 4b: Effect of changes when C increases
600000
500000
400000
300000
200000
100000
0
Total cost
2
4
6
8
10
Product unit cost
TCb
TCb1
TCb2

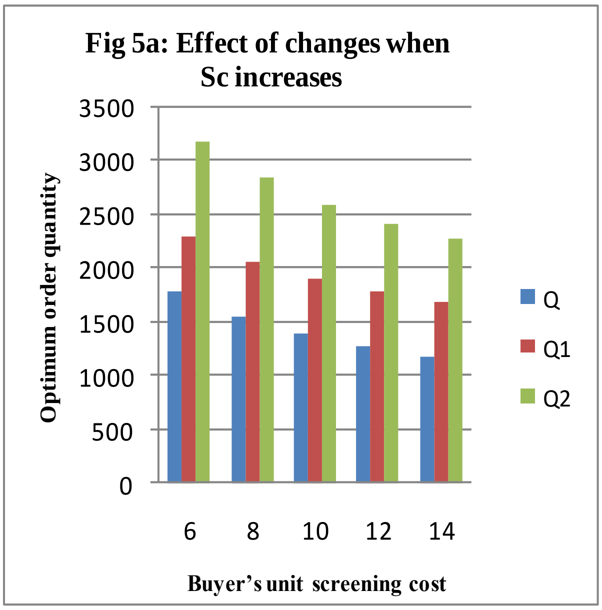


Fig 5a: Effect of changes when Sc increases
3500
3000
2500
2000
1500
1000
500
0
Optimum order quantity
6
8
10
12
14
Buyer's unit screening cost
Q
Q1
Q2

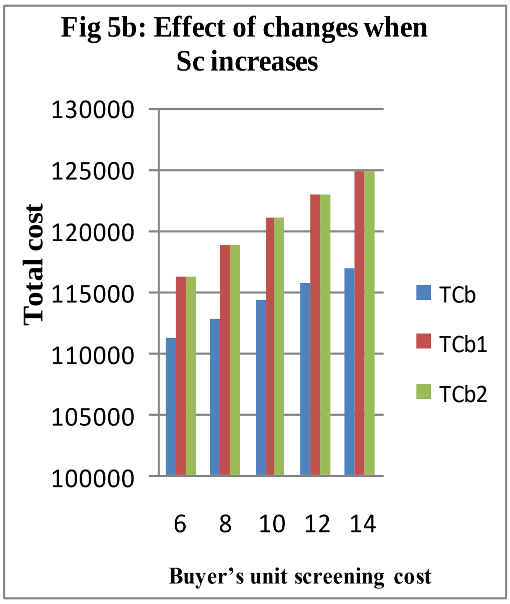
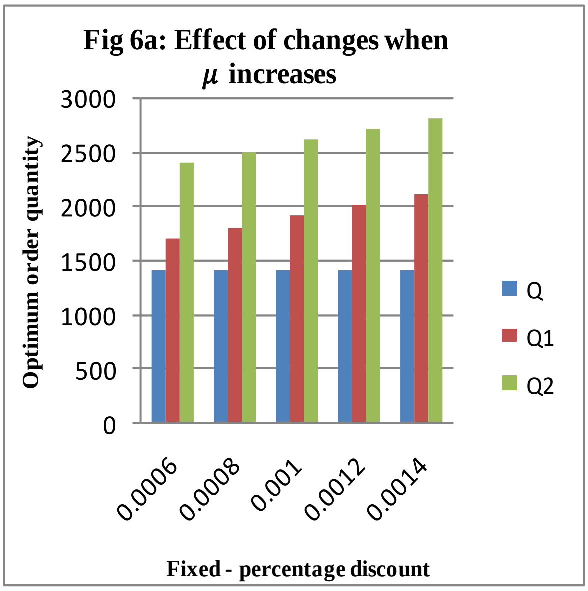
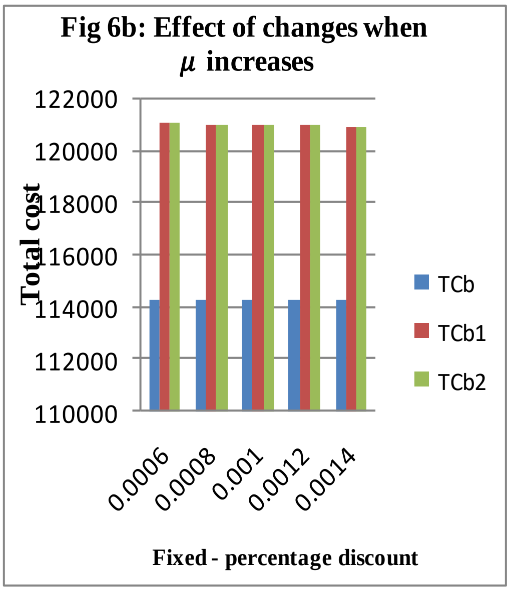
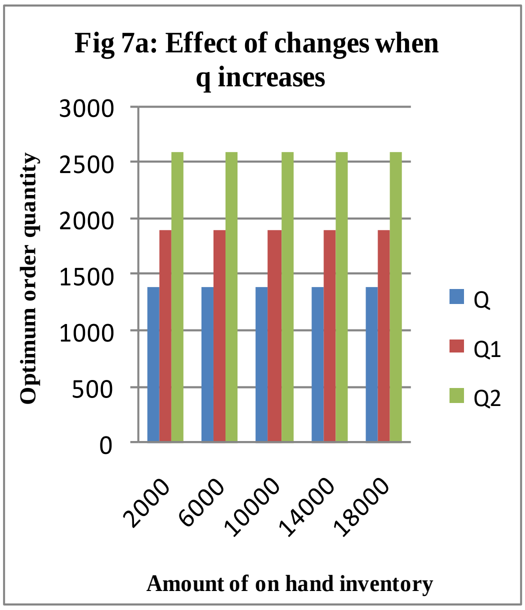
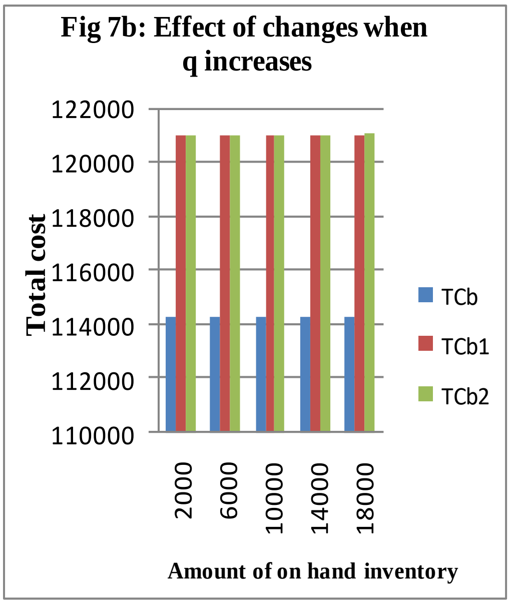


Fig 5b: Effect of changes when Sc increases
130000
125000
120000
115000
110000
105000
100000
Total cost
6
8
10
12
14
Buyer's unit screening cost
TCb
TCb1
TCb2









CONCLUSION

This study looked into the potential impacts on a retailer's replenishment strategy of a price reduction linked to the special order quantity. In order to optimize the total cost savings, between special and ordinary order policies in the reorder point and between reorder points, this study set out to identify the best order policy. The sensitivity of the model's output to changes in the various parameters was assessed using a numerical simulation. Future study should focus on expanding the proposed model to encompass varied demand patterns, discounts, shortages, and multiuser, multi-product offerings.

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