VENDOR – BUYER INVENTORY MODEL UNDER BUDGET AND FLOOR SPACE CONSTRAINTS WITH QUANTITY DISCOUNT

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Abstract

This analysis suggests an inventory model for a quantity discount approach between a vendor and a buyer. The cost of an integrated system is calculated using basic analytical geometry and an algebraic method, and it complies with floor space and budgetary restrictions. The multiplier approach developed by Lagrange is used to solve this kind of problem. To illustrate the theory, a mathematical model is provided, and the optimal solution's sensitivity to key parameters is examined.

Keywords: Inventory, constraints, Order Quantity, Analytical Geometry Technique, Algebraic Technique.

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1. INTRODUCTION

The degradation of goods like natural products, vegetables, pharmaceuticals, and so forth is frequently observed in various stock structures, leading to excessive damages in both quality and quantity of goods. A variety of tactics, such as exchange credit, cash limitations, and amount limits, are used to increase market demand in an effort to increase income.

Inventory model for damaged items with deficiencies fusing expansion and time worth of money was created by Chakrabarty et al. [3]. A multi-item stock administration model in a three-level production network with different people at each level was taken into consideration by Ghourchiany and Bafrouei [4]. Ravithammal et al. [7] focused a stock model for esteem discount with need, placing in a delay and redo. Muniappan et al. [5] developed an EOQ model with stock and product house limit impediments. Ravithammal et al. [8] made an EOQ stock model using arithmetical procedure with stock level necessity. Vediappan et al. [11] focused on fused coordination stock model utilizing Lagrange multiplier procedure. Zhan et al. [12] made forces through stock control in supply chains. Taleizadeh et al. [9] encouraged a financial solicitation amount model with inadequate defer buying and continuous markdown. Muniappan et al. [6] concentrated an EPQ motivation stock model for weakening items including to some extent multiplied deficiencies. The focus of Chang et al[2] .'s research is on how inquiry errors and exchange credits affect the financial request amount model for goods with poor quality. Accordingly incorporated stock models for multiple items under combination of circulations were constructed by Uthayakumar and Kumar [10]. Settlement of the merchant buyer fused stock system with mathematical numerical dissimilarity was explored by Cardenas-Barron et al. [1].

2. NOTATIONS AND ASSUMPTIONS

The model uses the following notations and assumptions.

2.1 Notations

D Demand rate per time

H_v Vendors unit holding cost per order per unit

H_b Buyers unit holding cost per order per unit

$\mathrm{s_c}$ Sellers unit screening cost

k Buyers multiples of order with coordination

m Vendor's multiples of order with coordination

$\mathrm{R_1}$ Buyers unit ordering cost per order

$\mathrm{R_2}$ Vendor's unit setup cost per order

$d(k)$ Discount factor

p Buyer's unit purchase cost / order

$\mathrm{Q_c}$ Optimum Order quantity

F Buyer's unit purchase cost / order

X Total available storage space for buyer.

W Buyer's maximum available budget to purchase product.

2.2 Assumptions

- Demand rate is known and predictable.
- The buyer screening the damaged products. Likewise, the vendor offers quantity discount assuming that the buyer's organization amount is bigger than normal amount.
- Integrated system cost is formulated for system optimization.
- The lot size $Q_\mathrm{c}$doesn't surpassesthe available storage space and budget level. i.e., It fulfills both stockroom limit imperative and financial plan level requirement. Mathematically, the constraints will be taken as $FQ_\mathrm{c} \leq X$ and $pQ_\mathrm{c} \leq W$ where F means space involved per item, X denotes total available storage space for buyer and W denotes maximum accessible stock.

3. MODEL FORMULATION

In this section, integrated system cost is planned for system progression. The optimum order quantity determine by utilizing with and without restrictions.

Case (i): Integrated system cost with no Constraint

The total cost for purchaser and seller is communicated as follows

i.e., $\mathrm{TC_b} = \frac{\mathrm{DR_1}}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2}$ (1)

and $\mathrm{TC_v} = \frac{\mathrm{DR_2}}{\mathrm{km}Q_c} + \frac{\mathrm{H_v km}Q_c}{2} + \mathrm{pD\,d(k)}$ (2)

The integrated system cost is communicated as follows

$$\mathrm{TC_s} = \mathrm{TC_b} + \mathrm{TC_v}$$

$$\mathrm{TC_s} = \frac{\mathrm{DR_1}}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2} + \frac{\mathrm{DR_2}}{\mathrm{km}Q_c} + \frac{\mathrm{H_v km}Q_c}{2} + \mathrm{pD\,d(k)} \quad (3)$$

Equation (3) can be composed as

$$\mathrm{TC_s} = Q_c\left[\frac{H_b + s_c + \mathrm{H_v km}}{2}\right] + \frac{1}{Q_c}\left[\mathrm{DR_1} + \frac{\mathrm{DR_2}}{\mathrm{km}}\right] + \mathrm{pD\,d(k)} \quad (4)$$

Equation (4) It is of the structure $a_1 Q_j + \frac{a_2}{Q_j} + a_3$.

Q_j will be taken as, $\mathrm{Q_j} = \sqrt{\frac{\mathrm{a_2}}{\mathrm{a_1}}}$

$$Q_c = \sqrt{\frac{2\mathrm{D}\left[\mathrm{R_1} + \frac{\mathrm{R_2}}{\mathrm{km}}\right]}{H_b + s_c + \mathrm{H_v km}}} \quad (5)$$

Case (ii): Integrated system cost with floor space Constraint

Here, take into account the buyer's floor area restrictions while ignoring budget restrictions. The Lagrange multiplier function α is now added to the cost of the integrated system and is expressed as follows: The integrated system cost with floor space Constraint

$$\mathrm{TC_s} = \mathrm{TC_b} + \mathrm{TC_v} + \alpha\,(\mathrm{F}Q_c - \mathrm{X})$$

$$= \frac{\mathrm{DR_1}}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2} + \frac{\mathrm{DR_2}}{\mathrm{km}Q_c} + \frac{\mathrm{H_v km}Q_c}{2} + \mathrm{pD\,d(k)} + \alpha\,(\mathrm{F}Q_c - \mathrm{X}) \quad (6)$$

Equation (6) can be composed as

$$\mathrm{TC_s} = Q_c\left[\frac{\mathrm{H_b}}{2} + \frac{\mathrm{s_c}}{2} + \frac{\mathrm{H_v km}}{2} + \alpha\mathrm{F}\right] + \frac{1}{Q_c}\left[\mathrm{DR_1} + \frac{\mathrm{DR_2}}{\mathrm{km}}\right] - \alpha\mathrm{X} + \mathrm{pD\,d(k)} \quad (7)$$

Equation (6) is of the structure $a_1 Q_c + \frac{a_2}{Q_c} + a_3$.

Q_c will be taken as, $Q_c = \sqrt{\frac{\mathrm{a_2}}{\mathrm{a_1}}}$

$$Q_c = \sqrt{\frac{2\mathrm{D}\left(\mathrm{R_1} + \frac{\mathrm{R_2}}{\mathrm{km}}\right)}{(\mathrm{H_b} + \mathrm{s_c} + \mathrm{H_v km} + 2\alpha\mathrm{F})}} \quad (8)$$

Where, $\alpha = \frac{2\mathrm{DF^2}\left(\mathrm{R_1} + \frac{\mathrm{R_2}}{\mathrm{km}}\right) - \mathrm{X^2}[\mathrm{H_b} + \mathrm{s_c} + \mathrm{H_v km}]}{2\mathrm{FX^2}}$

Case (iii): Integrated system cost with Budget Constraint

Here, consider the buyer's budget constraint and ignore floor space constraint. Now, Lagrange multiplier function β is added on integrated system cost and it can be formulated as follows:

The integrated system cost is

$$\mathrm{TC_s} = \mathrm{TC_b} + \mathrm{TC_v} + \beta\,(\mathrm{p}Q_c - \mathrm{W})$$

$$\mathrm{TC_s} = \frac{\mathrm{DR_1}}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2} + \frac{\mathrm{DR_2}}{\mathrm{km}Q_c} + \frac{\mathrm{H_v km}Q_c}{2} + \mathrm{pD\,d(k)} + \beta\,(\mathrm{p}Q_c - \mathrm{W}) \quad (9)$$

Equation (9) can be composed as

$$\mathrm{TC_s} = Q_c\left[\frac{\mathrm{H_b+s_c+H_v km}+2\beta \mathrm{p}}{2}\right] + \frac{1}{Q_c}\left[\mathrm{D}\left(\mathrm{R_1} + \frac{\mathrm{R_2}}{\mathrm{km}}\right)\right] - \beta W + \mathrm{pD\,d(k)} \quad (10)$$

Equation (6) is of the structure $a_1 Q_c + \frac{a_2}{Q_c} + a_3$.

$Q_c$will be taken as, $Q_c = \sqrt{\frac{a_2}{a_1}}$

$$Q_c = \sqrt{\frac{2\mathrm{D}\left(\mathrm{R_1}+\frac{\mathrm{R_2}}{\mathrm{km}}\right)}{(\mathrm{H_b+s_c+H_v km}+2\beta \mathrm{p})}} \quad (11)$$

Where,$\beta = \frac{2\mathrm{Dp^2}\left(\mathrm{R_1}+\frac{\mathrm{R_2}}{\mathrm{km}}\right) - \mathrm{W^2[H_b+s_c+H_v km]}}{2\mathrm{pW^2}}$

Case (iv): Integrated system cost with floor space Constraint and Budget Constraint

Here, consider the buyer's budget constraint and floor space constraint. Now, Lagrange multiplier function α and β is added on integrated system cost and it can be formulated as follows:

The integrated system cost is

$$\mathrm{TC_s} = \mathrm{TC_b} + \mathrm{TC_v} + \alpha\,(\mathrm{F}Q_c - \mathrm{X}) + \beta\,(\mathrm{p}Q_c - \mathrm{W})$$

$$\mathrm{TC_s} = \frac{\mathrm{DR_1}}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2} + \frac{\mathrm{DR_2}}{\mathrm{km}Q_c} + \frac{\mathrm{H_v km}Q_c}{2} + \mathrm{pD\,d(k)} + \alpha\,(\mathrm{F}Q_c - \mathrm{X}) + \beta\,(\mathrm{p}Q_c - \mathrm{W}) \quad (12)$$

Equation (12) can be composed as

$$\mathrm{TC_s} = Q_c\left[\frac{\mathrm{H_b+s_c+H_v km}+2\alpha\mathrm{F}+2\beta \mathrm{p}}{2}\right] + \frac{1}{Q_c}\left[\mathrm{D}\left(\mathrm{R_1} + \frac{\mathrm{R_2}}{\mathrm{km}}\right)\right] - \alpha\mathrm{X} - \beta W + \mathrm{pD\,d(k)} \quad (13)$$

Equation (13) is of the structure $a_1 Q_c + \frac{a_2}{Q_c} + a_3$.

$Q_c$will be taken as, $Q_c = \sqrt{\frac{a_2}{a_1}}$

$$Q_c = \sqrt{\frac{2\mathrm{D}\left(\mathrm{R_1}+\frac{\mathrm{R_2}}{\mathrm{km}}\right)}{(\mathrm{H_b+s_c+H_v km}+2\alpha\mathrm{F}+2\beta \mathrm{p})}} \quad (14)$$

Where, $\alpha = \frac{2\mathrm{DF^2}\left(\mathrm{R_1}+\frac{\mathrm{R_2}}{\mathrm{km}}\right) - \mathrm{X^2[H_b+s_c+H_v km}+2\beta\mathrm{p}]}{2\mathrm{FX^2}}$; $\beta = \frac{2\mathrm{Dp^2}\left(\mathrm{R_1}+\frac{\mathrm{R_2}}{\mathrm{km}}\right) - \mathrm{W^2[H_b+s_c+H_v km}+2\alpha\mathrm{F}]}{2\mathrm{pW^2}}$

3. 1 Solution Procedure for integrated system cost without coordination

Step 1. Ignoring the two constraints and deciding order quantity Qby utilizing Eqn. (5). If Qsatisfies both constraints, then Q is the optimal value to minimize the integrated expected total cost and go to step 5.

Step 2. Else determining order quantity Qby using Eqn. (8). If Qsatisfies inventory level constraint, then Q is the optimal value to minimize the integrated expected total cost and go to step 5.

Step 3. Else deciding the order quantity Qby using Eqn. (11). If Qsatisfies warehouse capacity constraint, then Q is the optimal value to minimize the integrated expected total cost and go to step5.

Step 4. If none of the above steps are satisfied then both constraints are active. Now, determining order quantity Qby using Eqn. (14). Then Q is the optimal value to minimize the integrated expected total cost and go to step 5.

Step 5. Stop.

4. NUMERICAL EXAMPLE

Example 1:Let $D = 3000$, $R_1 = 100$, $R_2 = 300$, $H_b = 0.2$, $H_v = 0.4$, $s_c = 0.03$, $p = 0.3$, $m =$

$3, n = 2, F = 4, X = 2000, W = 150, k = 4, d(k) = 20\%$.

The Optimal Solution is,
$Q_c = 376, TC_s = 2.0537\ X\ 10^3$ satisfies the constraints $FQ \leq 2000$ and $pQ \leq 150$.

Example 2: Let $D = 5000$, $R_1 = 150$, $R_2 = 200$, $H_b = 2$, $H_v = 4$, $s_c = 0.1$, $p = 0.2$, $m = 3$, $n = 2$, $F = 3$, $X = 1000$, $W = 300$, $k = 4$, $d(k) = 20\%$.

The Optimal Solution is,
$Q_c = 182$, $TC_s = 9.3378\ X\ 10^3$ satisfies the constraints $FQ \leq 1000$ and $pQ \leq 300$.

Sensitivity Analysis

The sensitivity analysis is performed by taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in following table.

Table 1

Decision Variable	Cost/Unit/Dollar	Q	TC_s
D	1000	217	1.1711 X10^3
	2000	307	1.6679 X10^3
	3000	376	2.0537 X10^3
	4000	434	2.3822 X10^3
	5000	485	2.6739 X10^3
R_1	80	344	1.8837 X10^3
	90	360	1.9723 X10^3
	100	376	2.0537 X10^3
	110	390	2.1320 X10^3
	120	405	2.2073 X10^3
R_2	240	368	2.0135 X10^3
	270	372	2.0337 X10^3
	300	376	2.0537 X10^3
	330	379	2.0736 X10^3
	360	383	2.0932 X10^3
H_b	0.10	379	2.0348 X10^3
	0.15	377	2.0443 X10^3
	0.20	376	2.0537 X10^3
	0.25	374	2.0631 X10^3
	0.30	372	2.0725 X10^3
H_v	0.2	500	1535
	0.3	427	2.8136 X10^3
	0.4	376	2.0537X10^3
	0.5	339	2.2679 X10^3
	0.6	312	2.4631 X10^3

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s_c	0.1	383	2.0158 X10^3
	0.2	379	2.0348 X10^3
	0.3	376	2.0537 X10^3
	0.4	372	2.0725 X10^3
	0.5	369	2.0910 X10^3
p	0.10	376	2.0237 X10^3
	0.15	376	2.0387 X10^3
	0.20	376	2.0537 X10^3
	0.25	376	2.0687 X10^3
	0.30	376	2.0837 X10^3
m	1	500	1635
	2	472	1.8071 X10^3
	3	376	2.0537 X10^3
	4	321	2.2773 X10^3
	5	284	2.4818 X10^3
n	1	376	2.0537 X10^3
	1.5	376	2.0537 X10^3
	2	376	2.0537 X10^3
	2.5	376	2.0537 X10^3
	3	376	2.0537 X10^3
F	2	376	2.0537 X10^3
	3	376	2.0537 X10^3
	4	376	2.0537 X10^3
	5	376	2.0537 X10^3
	5.5	333	2.0683 X10^3
X	1000	250	2.2225 X10^3
	1500	375	2.0538 X10^3
	2000	376	2.0537 X10^3
	2500	376	2.0537 X10^3
	3000	376	2.0537 X10^3
W	120	376	2.0537 X10^3
	135	376	2.0537 X10^3
	150	376	2.0537 X10^3
	165	376	2.0539 X 10^3
	180	376	2.0537 X 10^3
K	2	500	1685
	3	441	1.8711 X 10^3
	4	376	2.0537 X 10^3
	5	332	2.2233 X 10^3
	6	301	2.3816 X 10^3
$d(k)$	0.06	376	2.0297 X 10^3

	0.08	376	$2.0417 \text{ X } 10^3$
	0.10	376	$2.0573 \text{ X } 10^3$
	0.12	376	$2.065 \text{ X } 10^3$
	0.14	376	$2.0777 \text{ X } 10^3$

Diagrams

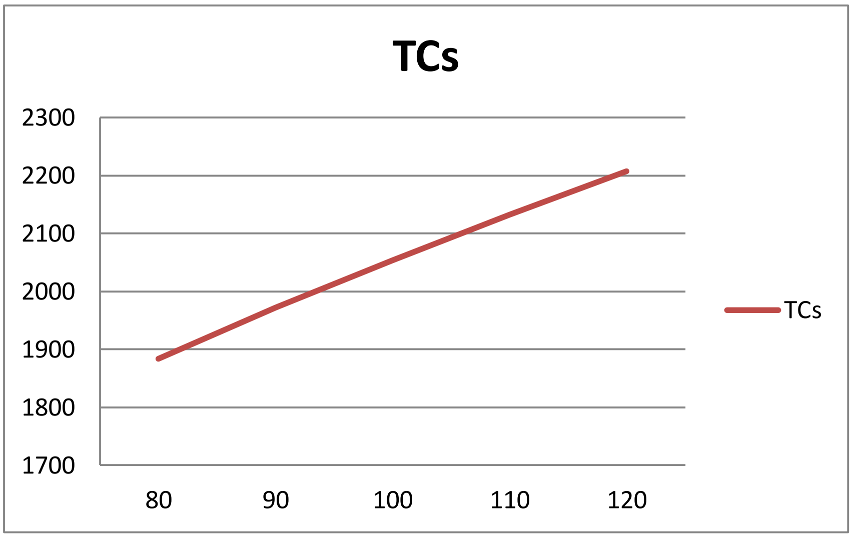

Figure 1 - Effect of changes when R_1 Increases

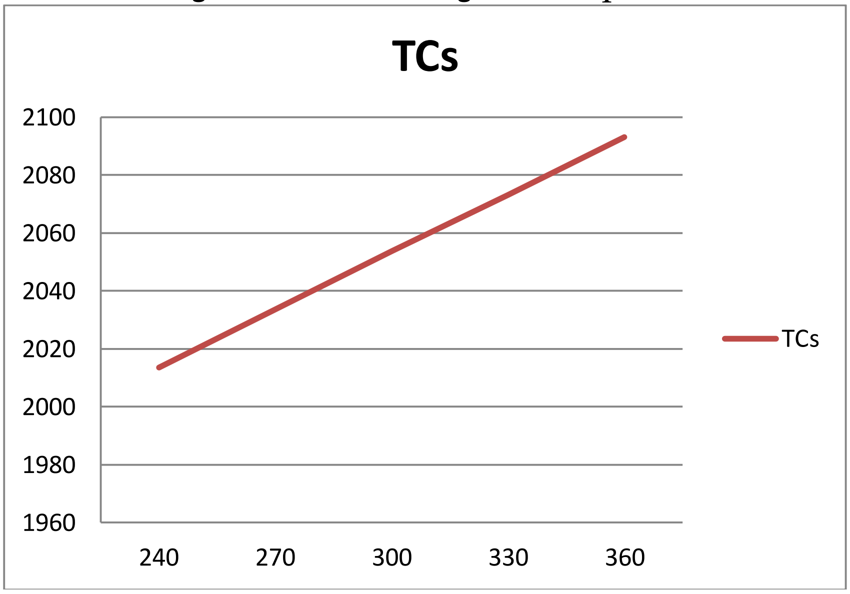

Figure 2 - Effect of changes when R_2 Increases

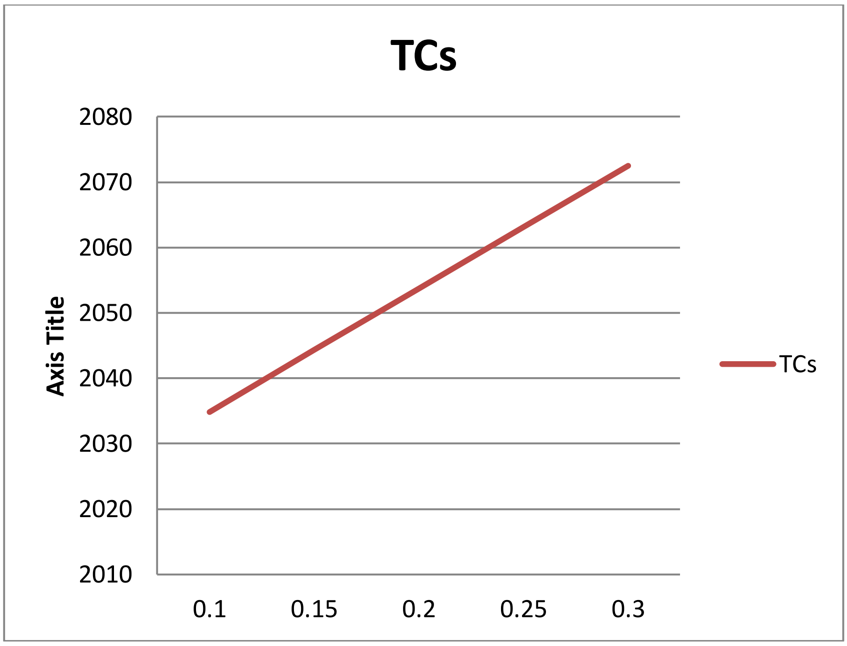

Figure 3 - Effect of changes when H_b Increases

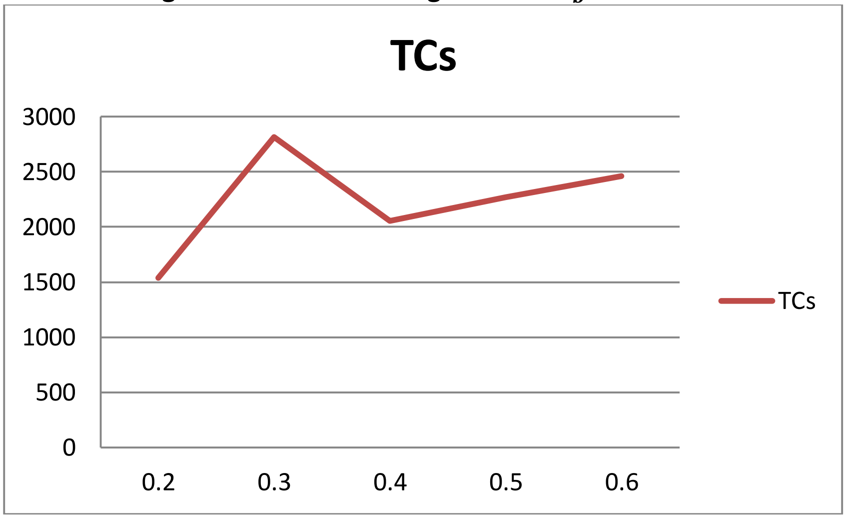

Figure 4 - Effect of changes when H_v Increases

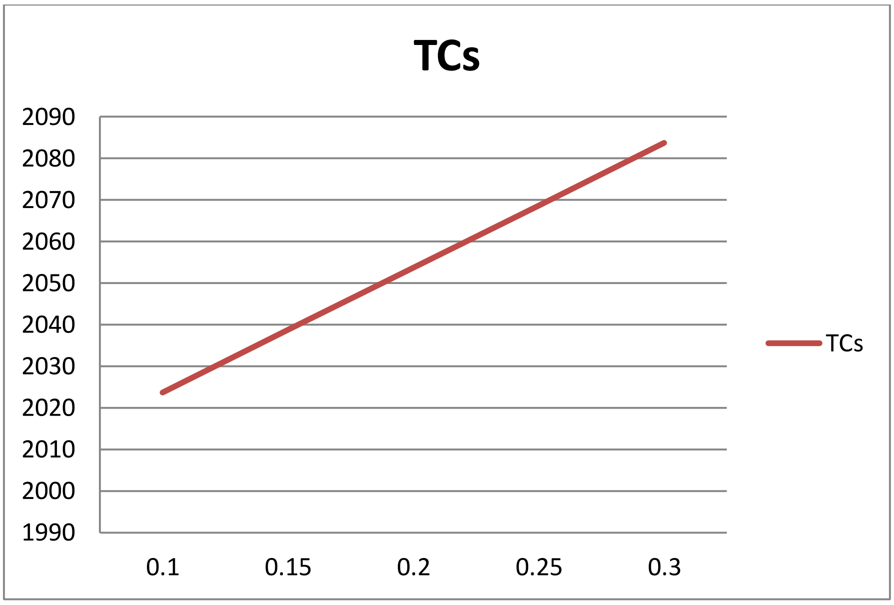

Figure 5 - Effect of changes when p Increases

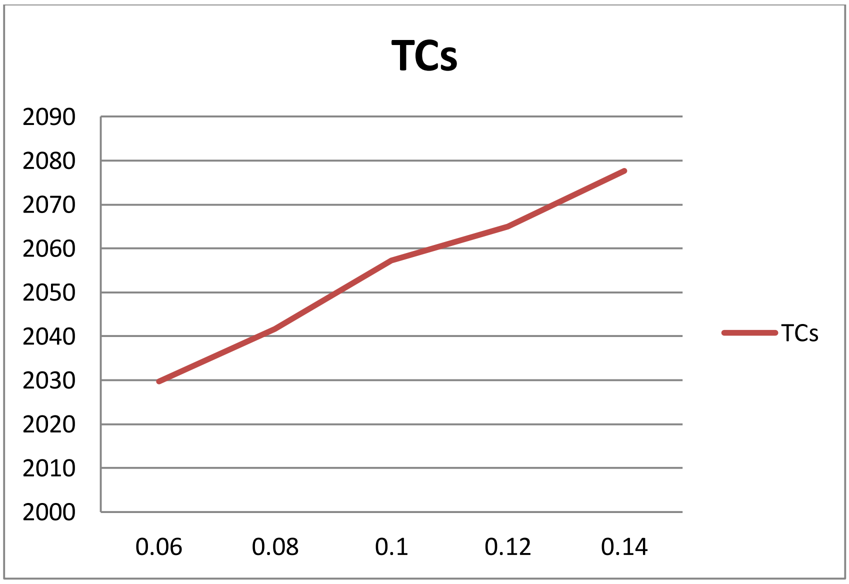

Figure 6 - Effect of changes when $d(k)$ Increases

5. CONCLUSION

In this paper, we dissected the vendor-buyer inventory model under the quantity discounting technique. In particular, the cost of an integrated system is calculated using an algebraic approach and basic analytical geometry. Additionally, it complies with the restrictions on budget level and floor space. To address this problem, Lagrange's multiplier method is used. Finally, a numerical model is provided to support the theory. The model can be coupled to multiple things, various demand, shortages with partial backlogs, random planning horizons, etc., for additional investigations.

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