EOQ inventory model for supplier – retailer with price reduction and shortage

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Abstract

This study examines how a supplier's temporary price reduction changes a retailer's restocking strategy when there is a shortage. When selecting whether to implement a regular or special order procedure amid a transitory deals season, retailers construct a decision-making process. The model is distinctive in that it considers shortages, unlike past attempts that did not. The best special arranges amount is chosen by minimizing ad up to fetched of special and ordinary orders. This is demonstrated by a number of numerical examples and sensitivity testing of the best answer.

Keywords: EOQ, Inventory, Special order quantity, Price discount

1. INTRODUCTION

Inventory appears to play a significant role in the academic and industry across several businesses. All business must keep some inventory to ensure the smooth operation of the business. No company asserts that they do not maintain an inventory. In fact, a supplier may provide clients a special price discount to buy a special amount if they encounter severe showcase immersion, an unanticipated the flood of stock or a alter within the product's fabricating run. The offer motivates retailers to make bigger purchases and provide customers with discounts in an effort to increase demand and profitability.

A combined pricing decaying stocking model that accounts for the life cycle of the product and advance payment with a discount offer was created by Abu Hashan Md Mashud et al. in 2021. Chandra and Sujan looked at a modelling strategy using ramp-type request, time-varying associated expenses, and price concessions on backorders (2017). Chen and Teng (2015) employed discounted evaluation of cash flow to emphasise inventories and financing alternatives for time-varying degrading products involving downstream as well as upstream trade credit services. Goyal and Giri (2011) investigated current developments in inventory degradation modelling. Inventory management was made by Hasan, Md. Rakibul et al. (2020), who also offered advance savings and online purchase.

Khouja (2000) investigated the single-period problem's ideal scheduling, discounting, and selling. In 2021, Latha et al. developed the double financial lot-sizing problems using geometric supply plan reorder price drops and an optimal investment to reduce ordering costs. Leopoldo Eduardo Cardenas-Barron et al. (2010) evaluated the perfect order size to keep ahead of just one special offer with approved backorders. An overall quantity of economic order model with inventory level and warehouse limitations was read by Muniappan et al. in 2020. Naik and Patel investigated a model for decaying products using varying rates of degradation for goods of varying quality and shortages.

Rajan and Uthayakumar evaluated the lowest prices and replenish approaches for quickly depreciating goods with backlogs and trade credit when inflation is taken into account. In their 2019 study published Ravithammal et al. constructed an EOQ inventory methodology uses an algebraic technique with a stock levels constraint. A generalised approach for discounting pricing plans was created by Rubin and Benton in 2003 Shah et al. created inventory techniques for cost stock-dependent supply and pricing policies (2018). San-Jose et al. built a periodic supply cycle, time-varying demand, partial backordering, and profitability inventory system in 2021. Teng et al. looked at inventory lot-size rules for goods with timelines and deposits (2016).

2. NOTATIONS AND SUSPICIONS

The model uses the taking after notations and presumptions.

2.1 Notations

d	Demand rate/ units/ time
r_1	Buyer's ordering cost / order /unit
h_1	Buyer's unit carrying cost /unit, $h_1 = Ci$
b	Buyer's Shortage cost / unit
i	Inventory carrying cost rate
C	Product unit cost/unit
u	Percentage of defecting items /unit
v	Percentage of scrap items /unit
d_c	Disposed cost / unit
S_c	Buyer's unit screening cost / unit
Q	Economic Order Quantity
Q_1	Back Order Level
Q'	Special Order Quantity
Z	Purchase additional units over Q to benefit from the discount
α	A predetermined discount offered by the supplier

2.2 Assumptions

- Demand is constant and unsurprising.
- The arranging skyline is one year.
- There is an infinite supply of cash and inventory storage space.
- The unit cost is steady unless the provider offers a lower retail cost.

3. MODEL FORMULATION

Two cases are taken into account in this section. Case I illustrates what happens when a buyer decides to keep buying Q units despite not taking benefits of the reduced pricing. A customer places an unusually large order quantity to take advantage of the decreased cost, as in case II.

Case - I: Ordinary Order Amount with no rebate and shortage

The total cost for buyer contains as following cost

TC_b = Ordering cost + Carrying cost + Shortage cost + Screening cost + Disposed cost + Purchasing cost

i.e., $TC_b = \frac{r_1 d}{Q} + \frac{h_1(Q-Q_1)^2}{2Q} + \frac{bQ_1^2}{2Q} + \frac{QS_c}{2} + \frac{Quvd_c}{2} + Cd$

Q

For optimality $\frac{\partial TC_b}{\partial Q_1} = 0$ and $\frac{\partial^2 TC_b}{\partial Q_1^2} > 0$ and $\frac{\partial TC_b}{\partial Q} = 0$ and $\frac{\partial^2 \mathrm{TC_b}}{\partial \mathrm{Q}^2} > 0$ we get,

$$Q_1 = \frac{\square_1 Q}{(\square_1 + b)}$$

$$Q = \sqrt{\frac{2dr_1[h_1 + b]}{b\square_1 + [h_1 + b][S_c + uvd_c]}}$$

Case -II: Special Quantity with a rebate for ordinary and extra units and shortages

The entire cost for a buyer is contained as taking after fetched

TC_{b1} = Ordering cost + Carrying cost + Shortage cost + Screening cost + Disposed cost + Purchasing cost

$$\text{i.e., } TC_{b1} = r_1\left[1 + \frac{d-Q-Z}{Q}\right] + \frac{h_1}{2d}[1-\alpha][Q - Q_1 + Z]^2 + \frac{h_1(Q-Q_1)^2}{2Q}\left[\frac{d-Q-Z}{d}\right] + \frac{bQ_1^2}{2Q} + \frac{bQ_1^2}{2Q}\left[\frac{d-(Q+Z)}{d}\right] + \frac{S_cQ}{2} + \frac{S_cQ}{2}\left[\frac{d-(Q+Z)}{d}\right] + \frac{uvd_cQ}{2} + \frac{uvd_cQ}{2}\left[\frac{d-(Q+Z)}{d}\right] + C\,[1-\alpha][Q+Z] + C\,[d-(Q+Z)]$$

By tolerating the offer, this contrast appears in the investment funds that will be made.

$D(Z) = TC_b - TC_{b1}$

$$= \frac{h_1}{2d}[\alpha - 1]Z^2 + \left[\frac{r_1}{Q} - \frac{h_{1Q}}{2d} - \frac{h_{1\alpha Q_1}}{d} + \frac{h_{1\alpha Q}}{d} + \left[\frac{h_1+b}{2dQ}\right]Q_1^2 + \frac{S_cQ}{2d} + \frac{uvd_cQ}{2d} + C\alpha\right]Z$$

$$- \frac{bQ_1^2}{2Q} - \frac{S_cQ}{2} - \frac{uvd_cQ}{2} + \frac{h_1\alpha Q^2}{2d} + \frac{h_1\alpha Q_1^2}{2d} - \frac{h_1\alpha QQ_1^2}{2d} + \frac{bQ_1^2}{2d} + \frac{S_cQ^2}{2d} + \frac{uvd_cQ^2}{2d} + C\alpha Q$$

$$D'(Z) = \frac{h_1}{d}[\alpha - 1]Z + \left[\frac{r_1}{Q} - \frac{h_{1Q}}{2d} - \frac{h_{1\alpha}Q_1}{d} + \frac{h_{1\alpha Q}}{d} + \left[\frac{h_1 + b}{2dQ}\right]Q_1^2 + \frac{S_cQ}{2d} + \frac{uvd_cQ}{2d} + C\alpha\right]$$

$$D''(Z) = \frac{h_1}{d}[\alpha - 1]$$

For Optimality $D'(Z) = 0$ and $D''(Z) > 0$, we get additional units

$$Z = \left[\frac{\alpha}{1-\alpha}\right]\left[Q + \frac{Cd}{h_1} - \frac{h_1Q}{[h_1 + b]}\right] + \frac{1}{[1-\alpha]}\left[\frac{r_1d}{h_1Q} + \frac{S_cQ}{2h_1} + \frac{uvd_cQ}{2h_1} + \frac{h_1Q}{2[h_1 + b]} - \frac{Q}{2}\right]$$

Therefore, The special order quantity $Q' = Q + Z$

4. NUMERICAL EXAMPLE

Example 1:

Let $d = 60000$, $r_1 = 2000$, $i = 0.2$, $C = 5$, $S_c = 0.3$, $\alpha = 0.1$, $u = 0.1$, $v = 0.2$, $d_c = 3$, $b = 0.3$.

The Optimal Solution is

$Q = 20156$, $TC_b = 311910$, $Z = 33850$, $Q' = 54006$, $TC_{b1} = 292750$.

Example 2:

Let $d = 5000$, $r_1 = 100$, $i = 0.2$, $C = 3$, $S_c = 0.3$, $\alpha = 0.2$, $u = 0.2$, $v = 0.4$, $d_c = 5$, $b = 0.2$.

The Optimal Solution is

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$Q = 1084, TC_b = 15922, Z = 6317, Q' = 7402, TC_{b1} = 12740.$

Example 3:

Let $d = 25000$, $r_1 = 500$, $i = 0.1$, $C = 4$, $S_c = 0.2$, $\alpha = 0.3$, $u = 0.2$, $v = 0.3$, $d_c = 5$, $b = 0.3$.

The Optimal Solution is

$Q = 5516, TC_b = 104530, Z = 63091, Q' = 68608, TC_{b1} = 67193.$

Sensitive Analysis

The sensitivity investigation regularly takes each limitation independently while holding the extra limit steady. The comes about are shown in Table 1.

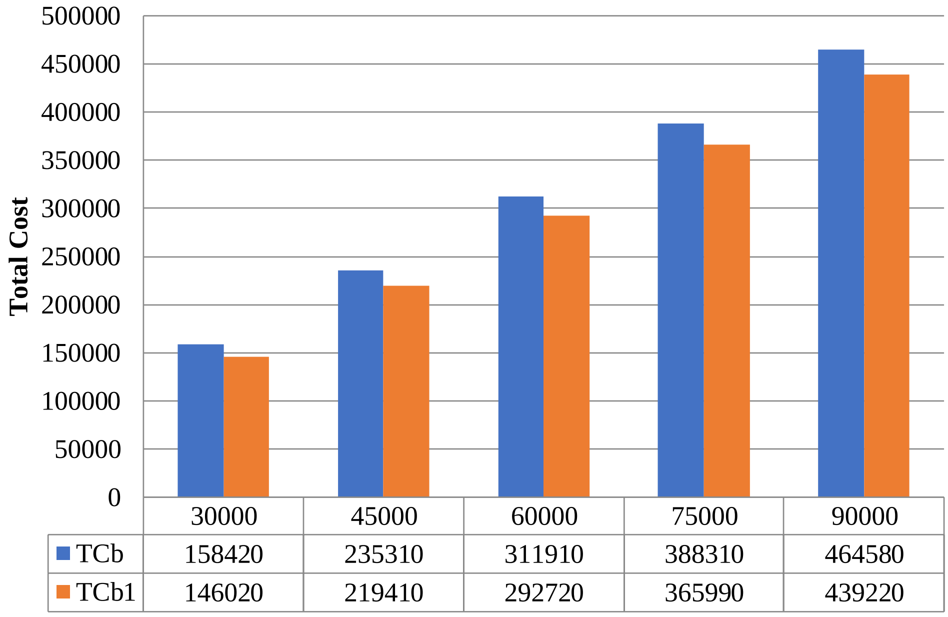
Table 1: Effects of Changes

Decision Variables	**Cost / Unit**	Q	Q_1	TC_b	Z	Q'	TC_{b1}
	30000	14252	10963	158420	17032	31284	146020
	45000	17455	13427	235310	25448	42903	219410
d	60000	20156	15504	311910	33850	54006	292720
d	75000	22535	17334	388310	42244	64779	365990
d	90000	24686	18989	464580	50633	75318	439220
d	1000	14252	10963	308420	33699	47951	292810
d	1500	17455	13427	310310	33781	51236	292800
r_1	2000	20156	15504	311910	33850	54006	292720
r_1	2500	22535	17334	313310	33911	56446	292590
r_1	3000	24686	18989	314580	33966	58652	292430
r_1	0.1	20937	13086	311460	67539	88476	278630
r_1	0.15	20443	14602	311740	45093	65536	287990
i	0.2	20156	15504	311910	33850	54006	292720
i	0.25	19968	16103	312020	27096	47064	295590
i	0.3	19835	16529	312100	22590	42425	297510
i	2.5	20937	13086	161460	34206	55143	150770
i	3.75	20443	14602	236740	33982	54425	221850
C	5	20156	15504	311910	33850	54006	292720
C	6.25	19968	16103	387020	33763	53731	363490
C	7.5	19835	16529	462100	33701	53536	434190
S_c	0.15	23335	17950	310290	33932	57266	289950
S_c	0.225	21571	16593	311130	33886	55458	291430

	0.3	20156	15504	311910	33850	54006	292720
	0.375	18986	14605	312640	33820	52807	293870
	0.45	18000	13846	313330	33795	51795	294910
	0.05	20156	15504	311910	16034	36190	306480
α	0.075	20156	15504	311910	24701	44857	300240
α	0.1	20156	15504	311910	33850	54006	292720
α	0.125	20156	15504	311910	43522	63677	283810
α	0.15	20156	15504	311910	53762	73918	273390
α	0.05	20688	15914	311600	33864	54552	292220
u	0.075	20417	15705	311760	33857	54273	292480
u	0.1	20156	15504	311910	33850	54006	292720
u	0.125	19905	15311	312060	33844	53748	292960
u	0.15	19663	15125	312210	33838	53500	293200
u	0.1	20688	15914	311600	33864	54552	292220
v	0.15	20417	15705	311760	33857	54273	292480
v	0.2	20156	15504	311910	33850	54006	292720
v	0.25	19905	15311	312060	33844	53748	292960
v	0.3	19663	15125	312210	33838	53500	293200
v	1.5	20688	15914	311600	33864	54552	292220
d_c	2.25	20417	15705	311760	33857	54273	292480
d_c	3	20156	15504	311910	33850	54006	292720
d_c	3.75	19905	15311	312060	33844	53748	292960
d_c	4.5	19663	15125	312210	33838	53500	293200
d_c	0.15	22122	19236	310850	33654	55775	290130
b	0.225	21011	17151	311420	33762	54773	291580
b	0.3	20156	15504	311910	33850	54006	292720
b	0.375	19476	14164	312320	33924	53399	293640
b	0.45	18922	13049	312680	33986	52907	294390

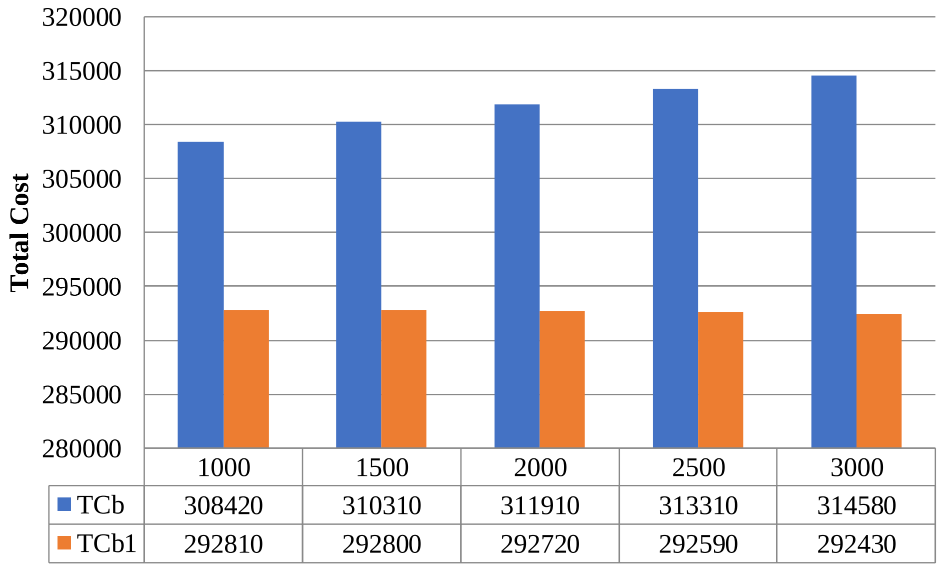
b

Fig 1: Effect of changes when *d* increases



	30000	45000	60000	75000	90000
TCb	158420	235310	311910	388310	464580
TCb1	146020	219410	292720	365990	439220

Fig 2: Effect of changes when r_1 increases



	1000	1500	2000	2500	3000
TCb	308420	310310	311910	313310	314580
TCb1	292810	292800	292720	292590	292430

ISBN: 978-93-5777-720-9

Fig 3: Effect of changes when *i* increases

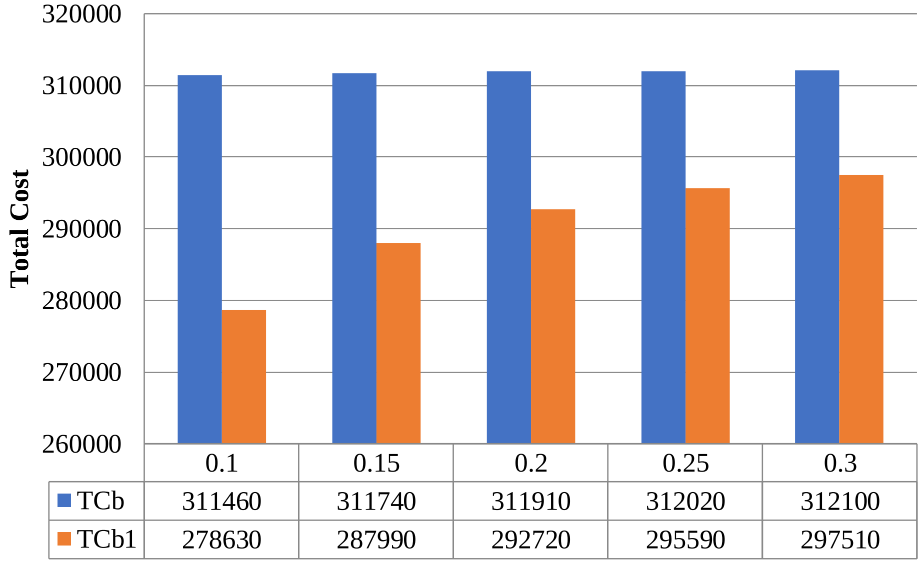


Fig 4: Effect of changes when *C* increases

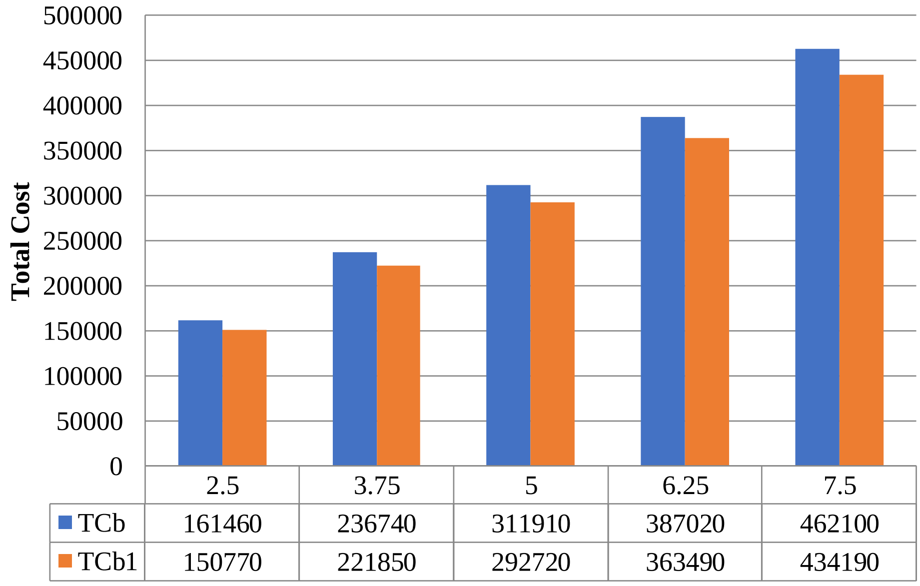


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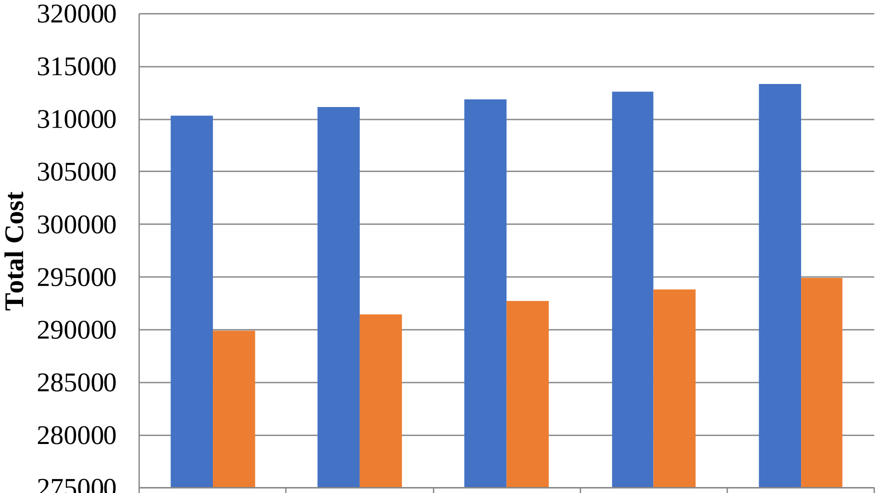
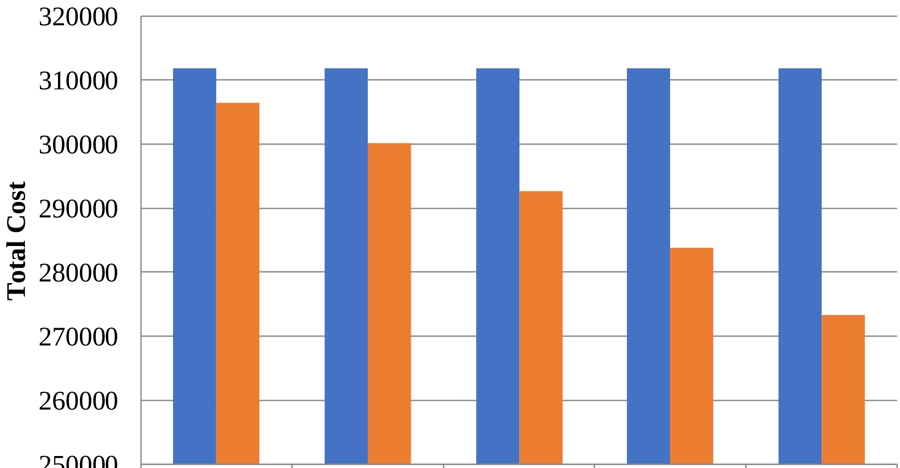
Fig 5: Effect of changes when S_c increases



	0.15	0.225	0.3	0.375	0.45
TCb	310290	311130	311910	312640	313330
TCb1	289950	291430	292720	293870	294910

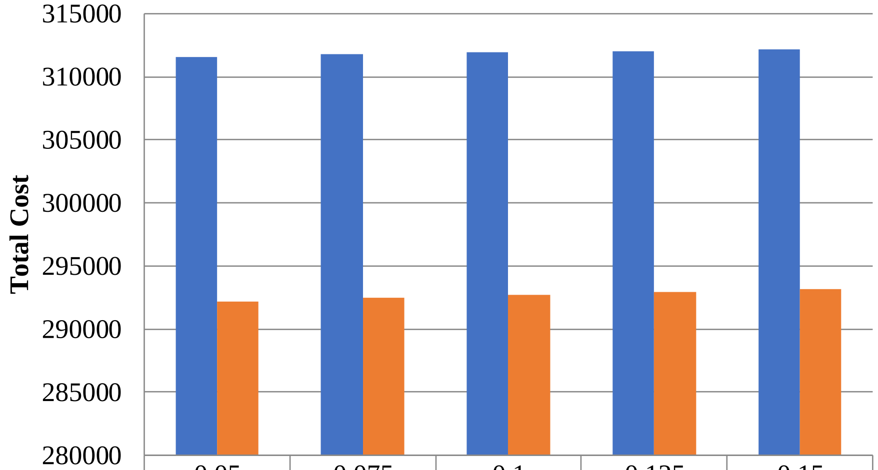
Fig 6: Effect of changes when α decreases



	0.05	0.075	0.1	0.125	0.15
TCb	311910	311910	311910	311910	311910
TCb1	306480	300240	292720	283810	273390

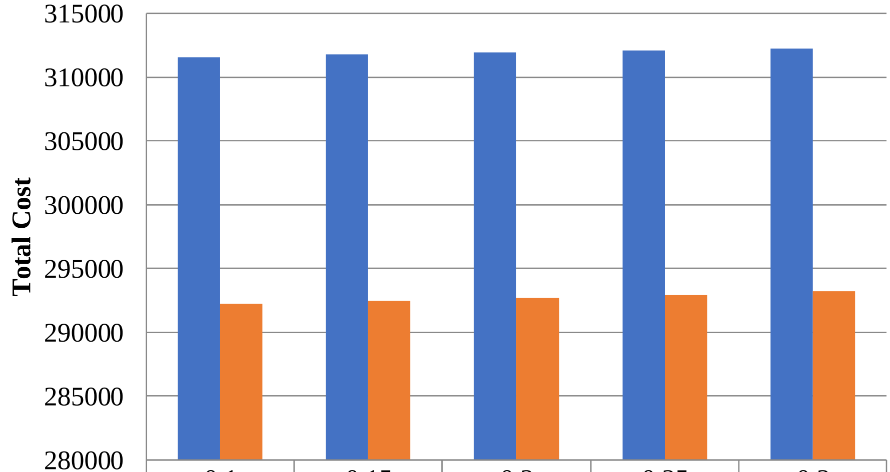
ISBN: 978-93-5777-720-9

Fig 7: Effect of changes when *u* increases



	0.05	0.075	0.1	0.125	0.15
TCb	311600	311760	311910	312060	312210
TCb1	292220	292480	292720	292960	293200

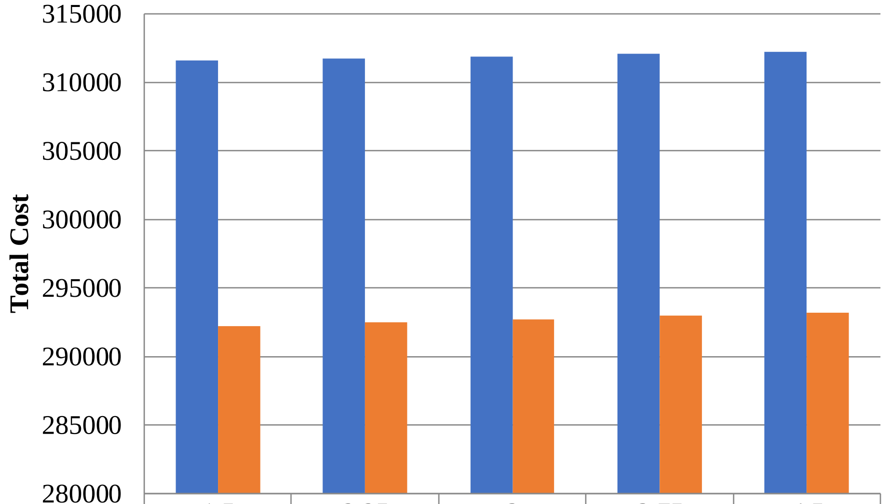
Fig 8: Effect of changes when *v* increases



	0.1	0.15	0.2	0.25	0.3
TCb	311600	311760	311910	312060	312210
TCb1	292220	292480	292720	292960	293200

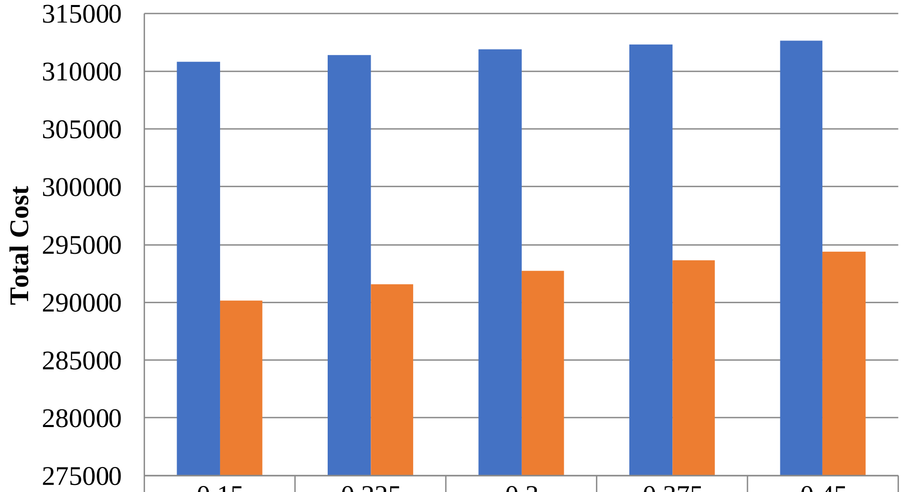
ISBN: 978-93-5777-720-9

Fig 9: Effect of changes when d_c increases



	1.5	2.25	3	3.75	4.5
TCb	311600	311760	311910	312060	312210
TCb1	292220	292480	292720	292960	293200

Fig 10: Effect of changes when b increases



	0.15	0.225	0.3	0.375	0.45
TCb	310850	311420	311910	312320	312680
TCb1	290130	291580	292720	293640	294390

CONCLUSION

The issue of a supplier's temporary price decrease affecting a retailer's restocking strategy when there is a shortage is examined in this article. The goal of this study is to create a choice method to assist merchants decide between a regular order policy and an special order policy. Two cases are taken into consideration in this segment. Case I outlines what happens when a buyer chooses to keep buying Q units despite not taking advantage of the sale pricing. A situation where a customer places an extraordinary order quantity to advantage from the lower unit fetched is illustrated by Case II. The ideal answer is eventually put through a sensitivity analysis with regard to the crucial factors when the study's perfect solution is discovered, the hypothetical results are shown with a variety of numerical situations, and so on.

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