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Generation of prime graphs using someunary graph products on star graphs

R. Vijayan[1] and K. Rajendran[21]
Research Scholar Department of Mathematics,
Vels Institute of Science Technology and Advanced StudiesChennai, India
plrvjn@gmail.com
[2]Department of Mathematics
Vels Institute of Science Technology and Advanced StudiesChennai, India
gkrajendra59@gmail.com

Abstract

A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots, |V|$ such that for each edge xy the labels assigned to x and y are relatively prime. In this paper, we prove that sub division and shadow of star graphs preserve prime labeling.Mathematics Subject Classification: 05C78;05C05

Keywords: prime labeling; prime graphs; unary product of graphs;

1 Introduction

Graphs we considered here are finite, simple and undirected graphs. For the graph theoretic terminologies, we refer the book [7]. Rosa [5] introduced various graph labeling and after that many researchers introduced various graph labeling for to decompose graphs. One such graph labeling is prime labeling introduced by Entringer. A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots, |V|$ such that for each edge xy the labels assigned to x and y are relatively prime. In the year 1980, Entringer conjectured that all trees have a prime labeling. Seoud et.al., [6] proved the necessary and sufficient conditions for a graph to be prime. They also gave a procedure to determine whether or not a graph is prime. Deretsky et al., [1] proved that cycles and disjoint union of cycles are prime. Lee et. al. [4] proved that complete graph does not have a prime labeling for $n \geq 4$ and wheel graphs W_n are prime if and only if n is even. For an exhaustive survey on Graceful Tree Conjecture, refer the excellent survey by Gallian [3]. In this paper, we prove that graphs $C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even.

2 Prime labeling of sub-division of star graphs

From the literature of prime labelings, we know that star graphs $K_{1,n}$ are prime graphs. In this section, we prove that sub division and shadow of star graphs allow prime labeling.

Theorem 1. *Edge sub-division of star graphs admit prime labeling.*

Proof. Consider a star graph $K_{1,n}$ with u as a central vertex and the pendant vertices as $v_1, v_2, \ldots, v_n$ along with its prime labeling given by the function $f(v_i) = i + 1$ for any $1 \leq i \leq n$ and $f(u) = 1$. It is clear that, the function f gives the prime labeling for the star graph $K_{1,n}$. To obtain the sub-division of star graphs, let us sub-divide every edge uv_i by introducing a new vertex w_i so that uw_i and w_iv_i are the edges of the sub-division graph of the star graph $K_{1,n}$. Let us define the prime labeling for the sub-division graph of the star graph as follows:

$$g(u) = 1$$

$$g(w_i) = 2i, \text{ for } 1 \leq i \leq n \; g(v_i) = 2i + 1, \text{ for } 1 \leq i \leq n$$

For the definition of the function g, the vertex labels of two end vertices of every edge of sub-division of star graphs are relatively prime. Thus, the sub-division of star graphs admit prime labeling. □

Theorem 2. *Shadow of star graphs admit prime labeling.*

Proof. Consider a star graph $K_{1,n}$ with u as a central vertex and the pendant vertices as $v_1, v_2, \ldots, v_n$ along with its prime labeling given by the function $f(v_i) = i + 1$ for any $1 \leq i \leq n$ and $f(u) = 1$. It is clear that, the function f gives the prime labeling for the star graph $K_{1,n}$. To obtain the shadow of star graphs, let us consider the copies of the vertices v_i as w_i correspondingly and the copy of the central vertex u be w. Because of the shadow of the star graph, along with the original edges of the star graph, let us introduce the new edges as follows: add edges between w and v_i for every i and add edges between u and w_i for every i, $1 \leq i \leq n$. Let us define the prime labeling for the shadow graph of the star graph as follows:

$$g(u) = 1$$

$$g(w) = 2$$

$$g(v_i) = 2i + 1, \text{ for } 1 \leq i \leq n g(w_i) = 2i + 2, \text{ for } 1 \leq i \leq n$$

For the definition of the function g, the vertex labels of two end vertices of every edge of shadow of star graphs are relatively prime. Thus, the shadow of star graphs admit prime labeling. □

3 Conclusion

In this paper, we prove that unary operations like sub-division and shadow of star graphs admit prime labeling. In this direction, we raise a question of what are the other unary products on star graphs that preserve the prime labeling.

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