MULTI-OBJECTIVE OPTIMIZATION OF GEAR DRIVE USING EVOLUTIONARY ALGORITHM

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Abstract:

Multi-objective optimal design for an engineering optimization problem involves various decision variables and constraints. In engineering applications, objectives under consideration conflict with each other, and optimizing a meticulous solution with respect to a single objective can result in objectionable results with respect to the other objectives. A practical solution to a multi-objective problem is to examine a set of solutions, each of which satisfies the objectives at a satisfactory level without being conquered by any other solution. In this paper, a new population based evolutionary algorithm used to optimize the gear drive with combined objective function which maximises the power, efficiency and minimises the overall weight, centre distance has been considered. The performance of the proposed algorithms is validated with three gear materials and results are compared.

Keywords: Design Optimization, Evolutionary Algorithm, Gear Drive and Multi-objective Optimization

Introduction:

Mechanical engineering design can be defined as the preference of materials and geometry, which satisfies, specified functional requirements of that design. A good design has to reduce the most significant difficult result and to exploit the main significant desirable result. Optimization algorithms are more flexible and ever-increasing in field engineering design problems, rightfully because of the availability and affordability of today's technical world. A population based heuristic algorithm offers well-organized ways of creating and comparing a novel design solution in order to complete an optimal design. Gears, as a class of mechanical mechanism, are used to transmit relative motion between two shafts. The design of gears is critical for smooth functioning of any mechanism, automobile and machinery. Gear drive design starts with the need of optimizing the gear width, module and number of teeth etc., it creates huge challenges to designer.

Hai Huang *et al.* [1] described a goal programming optimization mathematical model in order to improve efficiency of designing point-line meshing gears and also for the bending strength composite stiffness at the point-line meshing gear. Quancai Li *et al.* [2] developed a multi-objective for gear transmission with design variables and choice restrictive constraints. Majid and Esmaile [3] evaluated an engineering optimization problem with continuous design variables with two New Harmony search heuristic algorithms. Vipin and Chauhan [4] discussed, minimized the surface fatigue life factor and volume of gear box with classical SQP algorithm and other non-traditional NSGA-II with other geometric conditions. Kwon Soon *et al.* [5] developed a genetic algorithm for a gear pump for minimizing the wear rate proportional factor.

Ya Feng Li *et al.* [6], a design of three-stage wind turbine gear box was optimized with genetic algorithm with more optimized result. Savsani *et al.* [7] developed the particle swarm optimization and simulated annealing heuristic algorithms to minimize the weight of a spur gear train. Chong and Lee [8] showed a two-stage gear train and the simple planetary gear train for a minimization volume with genetic algorithm and discussed that genetic algorithm is better than the conventional algorithms. Padmanabhan *et al.* [9] used a Real Coded Genetic Algorithm to optimize the design of helical gear pair with combined objective function to maximize the Power, Efficiency and minimize Weight, Centre distance and proposed algorithm results was compared with LINGO Software. Bozca and Fietkau [10] proposed an empirical model based optimization for an automotive gearbox to reduce rattle noise with geometric design parameters. Parks and Miller [11] described selective breeding strategies in a multiobjective Genetic Algorithm with parents being selected from the current population. Sriramya *et al.* [12] proposed a new population based evolutionary algorithm as selective breeding algorithm for the Bin Packing application and the optimal results were compared with other packing algorithms. In this work, a new population based evolutionary algorithm, Selective Breeding Algorithm (SBA) was used to optimize the spur gear drive with combined objective function which maximises the power, efficiency and minimises the overall weight, centre distance has been considered.

Gear Drive Design Optimization

The Spur gear drive design consists of determining the design variable such as module, gear thickness and number of teeth in order to optimize the design. A number of objective functions by which optimality of gear drive design are include:

(1) Maximization of Power transmitted (f_1)

(2) Minimization of Weight (f_2)

(3) Maximization of Efficiency (f_3) and

(4) Minimization of Centre distance (f_4).

Several design constraints should be considered in the design of gear drive like bending stress, compressive stress, module and centre distance etc.

1. Objective functions for Spur Gear Drive

The equations (1), (2), (3) and (7) represent the maximization of Power, minimization of Weight, maximization of Efficiency and minimization of Centre distance. The efficiency equation (3) has been adopted from Dudley [13].

$$\text{Maximize, } f_1 = P \tag{1}$$

$$\text{Minimise, } f_2 = [[\frac{\pi}{4} \times d_1^2 \times b \times \rho] + [\frac{\pi}{4} \times d_2^2 \times b \times \rho]] \tag{2}$$

$$\text{Maximize, } f_3 = 100 - P_L = \eta \tag{3}$$

$$P_L = \frac{50f}{\text{Cos}\Phi} \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \tag{4}$$

$$H_t = \frac{(i+1)}{i} \times \sqrt{\left(\left[\frac{r_0}{r}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \tag{5}$$

$$H_s = (i + 1) \times \sqrt{\left(\left[\frac{R_0}{R}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \tag{6}$$

$$\text{Minimise, } f_4 = \frac{(d_1 + d_2)}{2} = \frac{m}{2}\ [Z_1 + Z_2] \tag{7}$$

2. Design Constraints

The above gear drive objectives should satisfies with the design constraints of allowable bending stress, allowable compressive stress, minimum module and minimum centre distance etc. the below equations (8), (9), (10) and (11) have been adopted from [14].

$$\text{Bending stress: } \sigma_b = \frac{(i+1)}{(a\ m\ b\ y)} \times [M_t] \quad \le [\sigma_b] \tag{8}$$

Compressive stress: $\sigma_c = 0.74\left(\frac{i+1}{a}\right) \times \sqrt{\left[\left(\frac{i+1}{ib}\right)\times E\times[M_t]\right]} \leq [\sigma_c]$ (9)

Centre distance: $a_{min} = (i+1)\sqrt[3]{\left[\left(\frac{0.74}{[\sigma_c]}\right)^2 \times\left(\frac{E[M_t]}{i\Psi}\right)\right]} \leq a$ (10)

Module: $m_{min} = 1.26 \times \sqrt[3]{\frac{[M_t]}{(y\,\sigma_b\Psi_m Z_1)}} \leq m$ (11)

The gear drive design problem has four different parameters in the objectives considered in this work. i.e., power, weight, efficiency and centre distance. Since all these objectives are on different scales, these factors are to be normalized to the same scale. The normalized objective function is obtained as follows:

$$COF = \left[\left(\frac{power}{\max.power}xNW_1\right)+\left(\frac{\min.weight}{weight}xNW_2\right)+\left(\frac{efficiency}{\max.efficiency}xNW_3\right)+\left(\frac{\min.cent.dist}{cent.dist}xNW_4\right)\right]$$

(12)

Where NW_1, NW_2, NW_3 and $NW_4 = 0.25$.

3. A Spur Gear Reducer

In this paper, a spur gear reducer problem was adopted form V. B. Bhandari [15]. Design a spur gear reducer for a compressor running at 250 rpm drive by a 7.5 kW, 1000rpm electric motor. The gears are made of carbon steel C45. This problem is to be solved with C45, Cast Iron (Grade 25) and Alloy Steel (40Ni 2Cr1Mo28) gear materials. The material properties of gear drive are tabulated [14] in Table 1.

Table 1: Material Properties of Gear drive

Material	Density (ρ) kg/mm^3	Bending Stress (σ_b) kg/mm^2	Compressive Stress (σ_c) kg/mm^2	Young's Modulus (E) N/mm^2
C-45	7.85×10^{-6}	140	500	2.1×10^5
CI G25	7.4×10^{-6}	60	600	1.7×10^5
Alloy Steel	8.839×10^{-6}	400	1100	2.15×10^5

The complete optimized problem of spur gear drive in terms of design variables P, m, b and Z_1 for the above problem with C45 material, after simplification is,

$$\text{Maximize} \quad f_1 = P \quad \text{where, } P^{(L)} \leq P \leq P^{(U)} \tag{13}$$

$$\text{Minimize} \quad f_2 = 104.81 \times 10^{-6} \times b \times (mZ_1)^2 \tag{14}$$

$$\text{Maximize} \quad f_3 = 100 - P_L \tag{15}$$

$$\text{Where, } P_L = 4.257 \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \tag{16}$$

$$H_s = 5 \times \left\{ \left[\left(1 + \frac{0.5}{Z_1}\right)^2 - 0.883 \right]^{0.5} - 0.342 \right\} \tag{17}$$

$$H_t = 1.25 \times \left\{ \left[\left(1 + \frac{2}{Z_1}\right)^2 - 0.883 \right]^{0.5} - 0.342 \right\} \tag{18}$$

$$\text{Minimize } f_4 = 2.5\, mZ_1 \tag{19}$$

Subject to,

$$m Z_1\, b^{0.5}\, P^{-0.5} \geq 337.94 \tag{20}$$

$$m^2 (Z_1 + 5)\, b\, P^{-1} \geq 725.87 \tag{21}$$

$$m\, Z_1\, P^{-0.333} \geq 53.4 \tag{22}$$

$$m^3 (Z_1+5)^{0.333}\, P^{-0.333} \geq 4.17 \tag{23}$$

Selective Breeding Algorithm

Selective breeding is the course of action of breeding plants and animals for testing character. Usually, plants strains are selectively bred are cultivated, and the breeding is occasionally through by an professional breeder. Bred animals are known as breeds, whereas bred plants are known as varieties, cultigens. The cross bred of animals result is called a crossbreed, and plants are called hybrids. The term selective breeding is tantamount with artificial selection [16]. The new selective breeding algorithm performs step by step flow code is show in figure 1. [12].

Create an Initial population of 'N' haploids (N is the size of population);
C= 0; (C is the number of cycles);
For each generation do;
C= C+1;
Find the Objective function for each haploid;
Sort the generation in M (separate storage);
do the above process up to C iterations;

Check the design constraints for each haploid in M;
Verified haploid with best object function is an output;
haploids based on objective function;
Divide the haploids into two equal sets based on sorting;
(first set named Dominant set and remaining set named Recessive set)
Form the diploid set which contains one dominant and one recessive in order (H1h1,H2h2...etc);
Do breeding process for all the combination diploid sets;
Perform fusion process for breed diploid set by interchange genes between each diploid set;
Separate diploid set into haploids;
Do "In Breeding Depression" into haploids (add "B" percentage of new haploids);
Sort the haploids based on objective function and take 'N' number of best haploids for the next generation;
Place best one haploid from each generation in M (separate storage);
do the above process up to C iterations;
Check the design constraints for each haploid in M;
Verified haploid with best object function is an output;

Figure 1: Flow of selective breeding algorithm

Result and Discussion

The above algorithm was developed using Microsoft Visual C#, with design parameters (P, m, b and Z_1) boundary values as inputs. This program was designed to solve any gear ratio, driver speed, pressure angle and gear material properties. The window developed by visual C# for gear drive with selective breeding algorithm is shown in figure.2.

After iteration of SBA for the three different gear materials for the specified spur gear reducer, the optimized results were tabulated in table 2 in compared with existing design [15]. The SBA result shows, significantly enhanced values with respect to power and weight reduction for the spur gear drive.

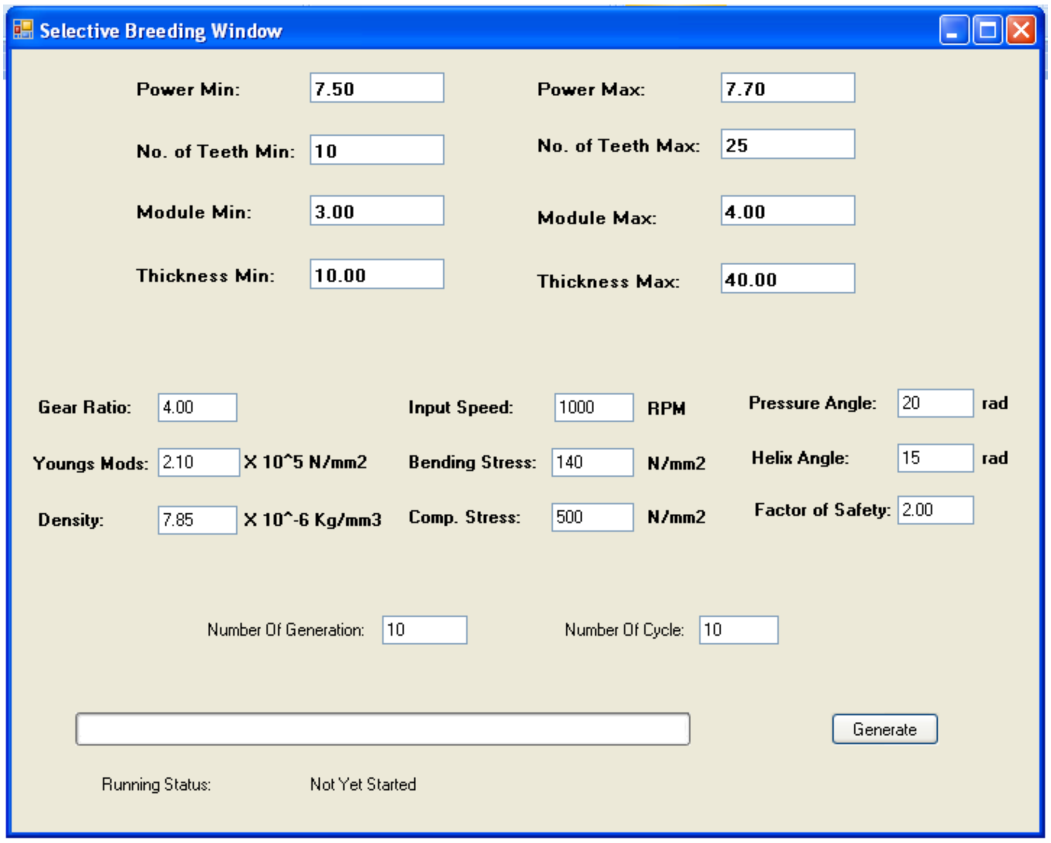

Figure 2: Window of selective breeding algorithm for Spur gear drive

Table 2: Comparasion of gear drive opimized results by SBA

Parameters / Material	Existing System	Selective Breeding Algorithm		
	C-45	C-45	Cast Iron Grade 25	Alloy Steel 40Ni 2Cr1Mo28
Power (P) kW	7.5	7.62	7.58	7.62
Module (m) mm	4.00	3.69	3.82	3.75
Gear Thickness (b) mm	40.00	34.36	36.64	34.98
No. of teeth on pinion (Z_1)	25	25	25	25
Centre Distance (a) mm	250.00	230.625	238.75	234.75
Gear Weight (kg)	41.92	30.65	33.02	36.28
Efficiency (%)	98.91	98.91	98.91	98.91

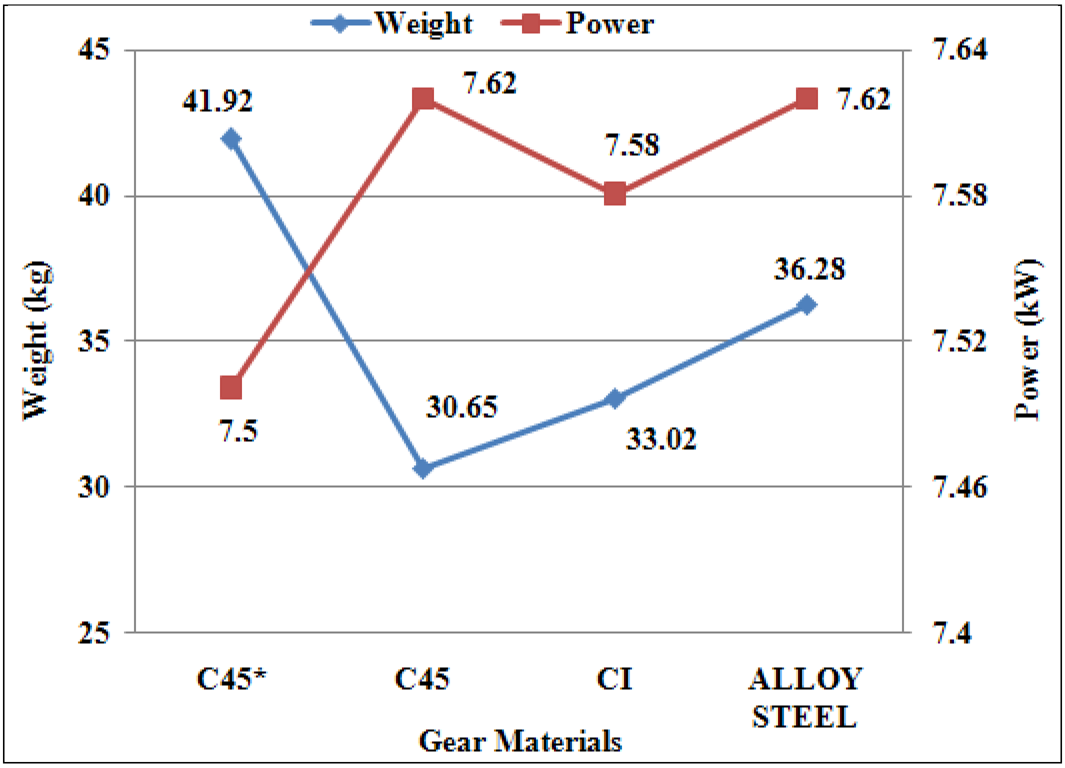

Figure 3: Results of weight and power by SBA for different Gear materials

The above figure 3. shows, 1.6% increase in power and 26.88% of weight reduction for the existing C45 material with respect to SBA results. A slight increment of power and 21.23% of weight reduction for the cast iron grade 25 material and also about 13.5% of weight reduction for alloy steel in compared with existing C45 material by conventional design. From the results, SBA shows a significant improvement in its optimal design values to its objectives.

Conclusion:

The majority of the mechanical engineering design involves extensive calculations and a number of non linear, non-differentiable and multi variables objective functions. It is extremely impracticable to solve by mechanical optimal techniques. Evolutionary algorithm like SBA can be powerfully applied for best solutions in an engineering design. In this work, a spur gear reducer was taken with three different gear materials and optimized results were obtained by selective breeding algorithm. This new evolutionary algorithm shows the substantial decrease in weight of gear drive with all three different gear materials, which is directly proportional to cost of the gear drive. The amount of material consumed is reduced due to reduction in weight of the gear drive and also its shows considerable increase in power with respect to all materials in compare with conventional design. In the designing of machinery gear drive and automobiles takes the advantage of evolutionary, the gear manufacturers can strongly face the sufficient atmosphere of producing at sensible cost. As a future work, the selective breeding algorithm can be used evaluated the range of engineering design application like multispeed gear box, epicyclic gear train, springs and bearings etc.

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