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On δgα Closure and δgα Interior S In TSS

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Abstract:
The purpose of this research article is to explain the new notions of δgα - derived, δgα-closure, δgα-interior, δgα-nbd., and moreover, the connections among them are identified.

Keywords: δgα-O, δgα-closure, δgα-interior, δgα-nbd., δgα-Derived, δgα-border, δgα-frontier, δgα-exterior and δgα-saturated.

1. Introduction

In many application domains, like data mining, the significance of general TSs is growing quickly [13]. Mathematizing both quantitative and qualitative data is possible with topological structures on the data collection serving as appropriate mathematical models. Nowadays, a large number of topologists worldwide are studying generalized Os because they are crucial to general topology.

A widely recognized concept that serves as a source of inspiration is the idea of αδ-O [12], which was first presented by R. Devi et al., We shall carry out the analysis of related functions with αδ-O and αδ-C s in this research. We present and define the terms "αδ-D," "αδ-exterior," as well as deduce their relationship. Furthermore, we present a brand-new function known as αδ-Totally-Continuous Functions. Additionally, as delineated and examined in these works by D. Sivaraj et al., [1-4], A Study on Beta Generalized C s in TS, Soft α–O s, [19–25] On soft regular star generalized star C s in soft TSs and [5-10] semi-closure, a note on soft g-C s Hildebrand S. K. et al., Regarding very αδ super irresolute functions in TSs, Benchalli S et al., On RW-C s in TSs, [11–12] V. Kokilavani et al., the αδ-kernal and αδ-closure via αδ-O s in TSs, D–αδ -s and related separation axioms in TSs, [15–18] Davis A. S., In addition, the fundamental characteristics of these functions as well as TS preservation theorems are presented and examined.

2. Preliminaries

Let X be a TS and A be X's subset. A's interior and closure are represented, respectively, by the symbols cl(A) and int(A).

Definition 2.1: A sub A of a space (X, τ) is called

(1) Regular-O [15] if A = int(cl(A)).

(2) semi-O [15] if A ⊆ cl(int(A)).

(3) α-O [2] if A ⊆ int(cl(int(A))).

(4) δ-semi-O [12] A ⊆ cl(Int $_{\delta}$ (A)).

Levine's g-Cs have been compared to other generalized closure operators or classes of generalized-O s to generate a variety of ideas. A useful tool for characterizing TSs is the generalize-C. The union of all regular Os of X contained in A is the δ-interior [10] of a sub A of X, and it is represented by Intδ(A). If A = Intδ(A), then the sub A is referred to as δ-O [10]. That is, if an is the union of regular-O s, then it is δ-O. A δ-C is the complement of a δ-O.

Detailed study in this regard by many investigators has enriched the field of generalized C s to a considerable extent. Nbd. is one of the core concepts of topology. Nbd. in topology have significant role in the applications of mathematics.

3. δgα –Closure (δgα –cl) and δgα –Interior (δgα –int) In TSS

In this paper we establish the notion of δgα – closure, δgα – interior in the TSs.

Definition 3.1. A subset M of a TS X is called a δgα -O (briefly, δgα-O) if M^c is δgα -C. The family of all δgα - O s in a TS X is represented by δgα-O(X).

Example 3.2. Let X= {*e*, *f*, *g*, *h* }, τ = {X, φ, {*e*}, {*f*}, {*e*, *f*}, {*e*, *g*}, {*e*, *h*}, {*e*, *f*, *g*}, {*e*, *f*, *h*}, {*e*, *g*, *h*}} then the δgα C s are {X, φ, {*e*}, {*f*}, {*g*}, {*e*, *f*, *g*}} and δgα -O s are { X, *φ*, {*f*, *g*, *h*}, {*e*, *g*, *h*},{*e*, *f*, *h*} {*h*}}.

Definition 3.3. The δgα – cl of a subset *A* of (*X*, τ) is denoted by δgα– *cl*(*A*) and is defined as the intersection of all δgα – C s containing *A* and is denoted by δgα-*cl*(*A*). δgα-*cl*(*A*) is the smallest δgα-C containing *A*. Therefore**,** δgα – *cl*(*A*)=∩ {*M* ⊆ *X*: *A*⊆ *M* and *M* is δgα – C}.

Definition 3.4. The δgα – int of subset *A* of (*X*, τ) is denoted by δgα –int(*A*) and is defined as the union of all δgα – O contained in *A* and is denoted by δgα-int(*A*). δgα-int(*A*) is the largest δgα O sub of *A*. Therefore, δgα-int(*A*) =∪ {*N* ⊆ *X*: *N* ⊆ *A* and *N* is δgα – O}.

Remark 3.5. (i). Every O is δgα -O.

(ii). Finite intersection of δgα-O s need not be δgα-O.

(iii). Finite union of δgα-O s need not be δgα-O.

Theorem 3.6. A subset M of a space Z is δgα -O ⟺ F⊆ αint(M) whenever F⊆M where F is δ-C.

Proof: Let M be a δgα -O subset of X and suppose F⊆ M where F is δ-C. Then Z-M is δgα-C and Z-M ⊆ Z-F where Z-F is δ-O in Z. By Definition of δgα -C, αcl(Z-M)⊆ Z-F. Since αcl (Z - M) = Z - αint(M), then Z - αint(M) ⊆ Z - F. Therefore F ⊆ α int(M).

Conversely, let F ⊆ α int(M) be true whenever F ⊆ M and F is δ-C in Z, then Z - α int(M) ⊆Z - F. That is, αcl(Z - M) ⊆ Z - F. Thus Z - M is δgα – C and M is δgα -O.

Theorem 3.7. If F is δgα-O sub of a space Z whereas αint(F) ⊆G⊆F, then V is δgα -O.

Proof: From the Definition 3.1 and δgα – C.

Theorem 3.8. If S is any δgα -O sub of a space X whereas αint(S) ⊆N, then S⋂N is δgα -O.

Proof: Let S be any δgα-O sub of X and αint(S) ⊆ N, then S ⋂ αint(S) ⊆ S ⋂ N ⊆ S. Since αint(S) ⊆ S, then αint(S) ⊆ S ⋂ N ⊆ S and from Theorem 3.5, S ⋂ N is δgα-O in X.

Theorem 3.9. Let M be any δgα-C subset. Then αcl(M)-M is δgα -O.

Proof: Let M be a δgα-C and F be a δ-C in X whereas F ⊆ αcl(M) – M, Then by Theorem M be a δgα -C sub of a space X, then αcl(M)-M contains no non empty δ-C ., F = ∅ and hence F ⊆αint(αcl(M) - M). Therefore, by Theorem 3.4, αcl(M) - M is δgα -O in X.

Lemma 3.10. Let Y be a δgα-subspace of X. If U is δgα O in Y and Y is δgα O, then U is δgα-O.

Proof: Given U is δgα O in Y, U = Y ⋂ G for some G δgα O in X. But Y and G are both δgα O in X so Y ⋂G is also δgα O in X.

Theorem 3.11. Assume that *M* and *N* be any two subs of a TS. Then

the succeeding properties hold.

1. E is δgα – C iff δgα – $cl(E) = E$.

2. δgα – $cl(E)$ is the smallest δgα – C sub of X containing E.

3. δgα – cl (ϕ) is empty, δgα – cl (X)= X.

4. δgα – $cl(E)$ is a δgα – C in (X, τ).

5. If $E \subseteq F$, then δgα – $cl(E) \subseteq$ δgα – $cl(F)$

6. δgα – cl ($E \cup F$) = δgα – $cl(E) \cup$ δgα – $cl(F)$.

7. δgα – cl ($E \cap F$) = δgα – $cl(E) \cap$ δgα – $cl(F)$.

8. δgα – cl (δgα – $cl(E)$) = δgα – $cl(F)$.

Proof: 1. For any sub E of X we have $E \subseteq$ δgα – $cl(E)$. Assume that E is a δgα – C in (X, τ). But $E \subseteq E$. Also $E \in \{H \subseteq X\colon\ E \subseteq H$ and E is δgα – C}, it gives $E = \cap$ {$H \subseteq X : E \subseteq H$ and H is δgα –C}$\subseteq E$. Then δgα – $cl(E) \subseteq E$. So E = δgα –$cl(E)$.

2. Beginning the definition of δgα – cl, δgα – $cl(E)$ is C. Suppose if F is any δgα – C then δgα – $cl(E) \subseteq F$. Hence δgα – $cl(E)$ is the smallest δgα – C in (X, τ) containing E.

3. Proof is obvious from the definition.

4. Proof is apparent from the definition.

5. If $E \subseteq F$ then $E \subseteq$ δgα – $cl(F)$ because $F \subseteq$ δgα – $cl(F)$ for all F. Hence δgα – $cl(F)$ is the δgα – C containing E. But δgα –$cl(E)$ is smallest δgα – C containing E. So δgα – $cl(E) \subseteq$ δgα – $cl(F)$.

6. We know the result $E \subseteq (E \cup F)$ and $N \subseteq E \cup F$, from the above result, δgα – $cl(E) \subseteq$ δgα – cl ($E \cup F$) also δgα – $cl(F) \subseteq$ δgα – cl ($E \cup F$) and δgα – $cl(F) \subseteq$ δgα – cl (E $\cup F$). So δgα – $cl(E) \cup$ δgα – $cl(F) \subseteq$ δgα – cl ($E \cup F$). But δgα – $cl(E)$ is δgα – C containing E and δgα – $cl(F)$ is δgα – C containing F. Hence δgα – $cl(E) \cup$ δgα – cl(E) is δgα – C containing $E \cup F$. Here δgα – cl ($E \cup F$) is

δgα − C containing ($E \cup F$). Therefore δgα − *cl* (E) ∪ δgα − *cl*(F) ⊇ δgα − *cl*(E ∪ F). Therefore we get δgα − *cl*($E \cup F$) = δgα − *cl*(E) ∪ δgα − *cl*(F).

7. We know that ($E \cap F$) ⊆ E and ($E \cap F$) ⊆ F. By (v) δgα − *cl*($E \cap F$) ⊆ δgα − *cl*(E) and δgα − *cl*($E \cap F$) ⊆ δgα − *cl*(E) ∩ δgα − *cl*(F).

8. δgα − *cl*(E) is a δgα - C in (X, τ). Let then K is δgα -C δgα − *cl*(E) = K, in (X, τ). Using (i) δgα − *cl*(K) = K, which gives δgα − *cl* (δgα − *cl*(E)) = δgα − *cl*(E).

Remark 3.12. For any sub A ⊆ X,

1. δgα −int(E) is the largest δgα − O ⊆ E.

2. A is δgα − O, iff δgα −int(A) = A.

3. δgα − int(X) = X.

4. δgα − int(ϕ) = ϕ.

4. δgα - NBD In TSS:

In this paper we establish the notion of δgα − nbd. in the TSs.

Definition 4.1. Let N be a sub of TS (X, τ), then N is said to be δgα − nbd. of point $x \in X$ if there exist a δgα − O (G) where as $x \in G \subseteq N$. The group of all δgα − nbd. of an element $x \in X$ called δgα − nbd. of x and is signified by δgα −N(x).

Example 4.2. Let X= {*e*, *f*, *g*, *h* }, τ = {X, φ, {*e*}, {*f*}, {*e*, *f*}, {*e*, *g*}, {*e*, *h*}, {*e*, *f*, *g*}, {*e*, *f*, *d*}, {*e*, *g*, *h*}} then the δgα C s are {X, φ, {*e*}, {*f*}, {*g*}, {*e*, *f*, *g*}} and δgα - O s are { X, *φ*, {*f*, *g*, *h*}, {*e*, *g*, *h*},{*e*, *f*, *h*} {*h*}}. Let b∈ X, if there exist a δgα − O G whereas $f \in G \subseteq N$, then δgα-nbd. of an element b∈ X, That is δgα −N(f) ={X, *φ*, {*f*, *g*, *h*}, {*e*, *f*, *h*}}.

Theorem 4.3. A sub P of (X, τ) is δgα − C and p∈ δgα − *cl*(P) iff $Y \cap P$ is not empty for any δgα − nbd. Y of p in (X, τ).

Proof: Assume p is not an element of δgα − *cl*(P). Then there exits δgα −C E of X whereas $P \subseteq E$ and p is not an element of E. Hence p∈($X \setminus E$) is δgα − O in X. But $P \cap (X \setminus E)$ is empty. This is a contradiction. Thus p∈ δgα − *cl*(P).

Conversely assume that there is a δgα − nbd. Y of a pt. p∈ X where as $Y \cap P$ is empty. Then there is a δgα − O E of X whereas p∈ $E \subseteq Y$. Hence $E \cap P$ is empty, p∈(X / E). So δgα − ∈($X \setminus E$) and p is not an element of δgα − *cl*(P). This is a contradiction to p∈ δgα − *cl*(P). Thus, the intersection of Y and P is not empty.

Theorem 4.4. If B is δgα − O then it is δgα − nbd. of each of its pts.

Proof: Consider a δgα − O of (X,τ). Then by definition for all $b \in B$, $b \in$B ⊆ M. So M is δgα − nbd. of each of its pts.

Theorem 4.5. If $B \subseteq X$ is a δgα − C, b∈B^c, then there is a δgα − nbd. M of b whereas M ∩B =ϕ .

Proof: Assume that B is a δgα – C, then B^c is δgα – O. By definition B^c is δgα – nbd. of each of its points. Let us assume that $b \in B^c$ then there is a δgα – O M whereas $b \in M \subseteq B^c$. So $M \cap B = \phi$.

Theorem 4.6. If x is an element in the TS (X, τ) then

1. δgα – $N(x)$ is non empty.

2. If a sub B∈ δgα – $N(x)$ then $x \in B$.

Proof: (1) Since $X \in$ δgα – $N(x)$ and δgα-$N(x)$ is not empty.

(2) Assume that B∈ δgα – $N(x)$, then there is a δgα – O M whereas $x \in M \subseteq$ B. Hence $x \in$B.

Theorem 4.7. If a sub $B \in$ δgα – $N(x)$ and $B \subseteq$ A, then A∈ δgα – $N(x)$.

Proof: Assume that $B \in$ δgα – $N(x)$, then there is a δgα – O U whereas $x \in U \subseteq B$. Given B ⊆ A, then $x \in U \subseteq$ A. Hence A∈ δgα – $N(x)$.

Theorem 4.8. Let (X, τ) be a TS. If N is a nbd. of $t \in X$, then N is

a δgα – nbd. of X.

Proof: Assume that N is a nbd. of $t \in X$. By definition there exist an O H whereas $t \in F \subseteq N$. But we know that all O s are δgα – O whereas $t \in F \subseteq N$. Thus, N is δgα – nbd. of X.

5. δgα-Derived

In this paper we establish the notion of δgα – derived in TSs.

Definition 5.1: If M is a sub of a TS (X, τ), then a pt. p∈ X is called an δgα – limit point of a M⊆ X if every δgα-O S⊆ X containing p, contains a pt. of M other than p. The set of all δgα – limit pt. of M is called an δgα-derived set of M and is signified by δgα-D(M).

Theorem 5.2: The following five results are true. If M and S are two subs of a TS (X, τ).

(i). If M ⊆ S, then δgα-d(M) ⊆ δgα-d(S).

(ii) M is an δgα-C if and only if it contains each of its δgα-limit point.

(iii). δgα-cl(M) = M ∪ δgα-d(M).

(iv). δgα-d(M∪S) ⊇ δgα-d(M) ∪ δgα-d(S).

(v). δgα-d(M∩S) ⊆ δgα-d(M) ∩ δgα-d(S).

Proof: (i) By definition 5.1, we have p∈ δgα-d(M) if and only if E ∩ (M-{p})≠ φ, for every δgα-O E containing p. But, M⊆S, then E ∩ (S-{p})≠ φ, for every δgα-O E containing p. Hence p∈δgα-d(S). Therefore, δgα-d(M) ⊆ δgα-d(S).

(ii). Let M be an δgα-C and p∉M then p ∈ (X-M) which is an δgα-O , hence there exist an δgα-O (X-M) whereas (X-M) ∩ M = φ. So p ∉ δgα-d(M), therefore, δgα-d(M) ⊆ M.

Conversely, suppose that δgα-d(M) ⊆ M and p∉M. Then p∉δgα-d(M), hence there exist an δgα-O E containing p whereas E ⋂ M = φ and hence X-M = $U_{p \in M}$ {E, E is δgα-O}. Therefore, M is δgα- C.

(iii). Since δgα-d(M) ⊆ δgα-cl(M) and M ⊆ δgα-cl(M). δgα-d(M) ∪ M ⊆ δgα-cl(M).

Conversely, suppose that p∉ δgα-d(M) ∪ M. Then p∉ δgα-d(M), p∉ M and hence there

exist an δgα-O E containing p whereas E⋂M = φ. Thus p ∉ δgα-cl(M)). δgα-cl(M) ⊆ δgα-d(M) ∪ M, therefore, δgα-cl(M) = δgα-d(M) ∪M.

(iv). Since M ⊆ M ∪ S and S ⊆ M ∪ S. We have, δgα-d(M) ⊆ δgα-d(M∪S) and δgα-d(S) ⊆ δgα-d(M∪S). Therefore, δgα-d(M) ∪ δgα-d(S) ⊆ δgα-d(M∪S).

(v). Since M ⊇ M ⋂ S and S ⊇ M⋂S. We have, δgα-d(M) ⊇ δgα-d(M⋂S) and δgα-d(S) ⊇ δgα-d(M⋂S). Therefore, δgα-d(M) ⋂ δgα-d(S) ⊇ δgα-d(M⋂S).

6. Conclusion

In this study, different idea of closure and interior sets namely, δgα-closure, δgα-interior was established and also discussed about δgα-nbd, δgα-derived sets and also about their properties in topological spaces.

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