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Secure - Vertex - Edge Domination of Certain Named Special Graphs

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Abstract:

Applications using domination in graphs can be found across multiple domains.When there is a set number of resources (such as fire departments and healthcare facilities) and the goal is to reduce the distance that someone must travel in order to reach the most nearby facility, domination emerges in facility positioning problems.Domination notions can also be found in land mapping problems (e.g., limiting the quantity of places where an assessor has to visit in order to obtain measurements of elevation for an entire region), tracking telecommunications or electrical infrastructure, and tasks involving spotting squads of senators. A comparable issue arises when efforts are made to minimize the quantity of facilities needed to serve every individual and the ideal distance to service is established. Considering the graph G = [{V}, {E}]. Let the set I V {G} is a secure - vertex - edge dominating set of G, suppose every edge, y E [G], then there exists a vertex V I so that V stands up for y . i.e., The vertex in I defends the edges incident on that vertex and the edges which lie next to the incident edges. A secure - vertex - edge dominating set I of a graph G has the characteristic of being a dominant set where every vertex z V – I either follows a vertex or a vertex adjacent to the incident edges of z, x I such that (I- {x}) {z} is a dominating set. The secure - vertex - edge domination number in G is the least cardinality of secure - vertex - edge domination and is depicted by . We have commenced researching this new parameter and have found the secure - vertex - edge dominance number of several standard graphs and the middle graphs of some standard graphs. In the current analysis, the secure - vertex - edge dominance number of a few designated specific graphs such as Bull Graph, Durer Graph, Heawood Graph, Moser Spindle Graph and etc., was discovered.

Keywords: Dominating set, Secure - vertex - edge dominating set, Domination number, Bull Graph, Butterfly Graph, Durer Graph, Heawood Graph, Moser Spindle Graph.

1. Introduction

Graph theory is one of a great deal of active fields in modern mathematics, having developed during the last trilogy of decades. Numerous domains, including discrete optimization problems, sequential problems, and ancient algebraic challenges are among its applications. [1]. In addition, the fields of languages, biology, physics, and sociological studies all bear the mark of graph theory's imprint. Computational frameworks are in handy whenever an extensive issue arises. One of the easiest and most efficient methods to approach a challenge is to lay out it as a visual representation graph. One of the main focuses of graph theory is exploration the concept of dominance in graphs. The goals of the dominance hypothesis include identifying dominant groups in graphs, investigating their characteristics, and comprehending the practical applications associated with those collections. When De Jaenisch examined the bare minimum of monarchs needed to wrap around or rule an NXN checkers board in 1862, the doctrine of domination originated [2]. During 1892, chess players studied three basic types of problems, according to W. W. Rouse Ball [3]. [4] Berge initially introduced the dominant number of a graph in 1958; It was alternatively known as the "coefficient of external stability." In 1962, Ore used the phrases "dominating set" and "domination number" to convey a similar concept [5]. Nevertheless, dominating sets in graph theory weren't well investigated until about 1960.

Once Cockayne and Hedetniemi presented a comprehensive study of the research on dominant sets in graphs at that period 1977 [6], they significantly advanced the discipline's understanding of dominating sets. They created the graph notation's domination number (G), which was subsequently adopted by many. The pioneering poll results by Hedetniemi and Cockayne caused a spike in the amount of studies being done on graph domination. Over 1200 academic papers on this fascinating intricate issue were produced in the two decades of which followed the research study. Secure domination is a theory that was developed by Cockayne et al. in 2003, [7]. To defend an unprotected vertex u, within a graph G = [{V}, {E}], each vertex in a subset S of V has to have an enforcer assigned to it. This thorough analysis seeks to explore the multifaceted nature of vertex dominations in graph theory, illuminating several topics like dominance insets, different kinds of dominance, typical minimal dominance, theoretical findings, results, and applications of dominance in graphs[8].Several results on vertex-edge and edge-vertex properties in graphs were put forth in 2007 by J. R. Lewis [9] in his dissertation for doctoral study. The cardinal value of v is represented by $deg(v)$, which is the degree of v. An independent vertex or a pendant vertex has a degree of zero or one, correspondingly.

The lowest degree, $\delta\ (G)$, and highest degree, $\Delta\ (G)$, are depicted correspondingly. A regular graph is one where $\Delta(G) = \delta(G)$ were discussed in [10]. The secure dominant set of graph G is defined as a collection of vertices in V-S, where each vertex is secured by a vertex in S. This is a secure dominant set's minimal cardinality. Arumugam et al. (2014) provide some conclusions regarding co-secure and secure dominations in graphs [11]. Additionally, this new guard arrangement forges a strong group. Motivated by all these results, we have consequently developed a new attribute called secure - vertex - edge domination of graphs [12]. [13] – [15] used some special graphs in their research. We kick off the current study by investigating the novel parameter and ascertaining the secure - vertex - edge domination number of some named special graphs such as Bull graph, Butterfly graph, Diamond graph, House graph, Wagner Graph, Golomb Graph and etc.,

2. Preludes

Graph: A graph G = ({V},{E}) is composed of a collection of vertices V(G) and a set of edges E(G). Let represent the order of the group G. The degree of a vertex in a graph is determined by the number of edges that are connected to the vertex, taking into account loops as two separate edges. The degree of a vertex is represented by the notation deg(v). The maximum and least degree of a graph are indicated by $\Delta(G)\ and\ \ \delta(G)$ respectively.

Neighborhood: The open neighborhood of a vertex w $\in V$ is denoted by $N(\mathrm{w}) = \{u \in V : uw \in E\}$ and its closed neighborhood is $N[\mathrm{w}] = N(\mathrm{w}) \bigcup \{\mathrm{w}\}$.

Floor Function: The biggest integer less than or equal to *l* is the floor function of a real number *l*, and it can be expressed as $\lfloor l \rfloor$. Suppose that $m \le l < m+1$, where m is an integer, then $\lfloor l \rfloor = m$.

Ceiling Function: The smallest integer larger than or equal to a real number l is its ceiling function, and it is symbolized by $\lceil l \rceil$. Suppose that $m-1 < l \le m$, where n is an integer, then $\lceil l \rceil = m$.

Domination: A dominating group in which, each vertex of G is either in D or has at least one neighbor who is also in D. D is a set of vertices. The dominance number of G, $\gamma(G)$ is the minimal cardinality of such a set.

Secure Domination: Assume that G = [{V}, { E}] is an elementary graph of order n. If every vertex in V- S is next to a vertex in S, then set S $\subseteq$ V is a dominant set of set G. A secure dominating set S inside a graph G is characterized by its ability to include all vertices u $\in$ V - S close to a vertex v$\in$ S, so that [(S - {u})$\cup$ {v}] is moreover a dominating set. The secure domination number, represented by $\gamma_s(G)$, is the minimal cardinality of a secure dominating set of G.

Secure - vertex - edge Domination: If for every edge, y $\in$ E (G), there is a vertex V $\in$ *I* such that V defends y, then the collection *I* $\subseteq$V (G) is a safe vertex edge dominating set of G. In this regard, an edge that is incident on a vertex in *I* defends that vertex's surrounding edges as well as the incident edges themselves. The dominant set *I* of a graph G is defined as a collection of vertices such that every vertex z in V - *I* is either adjacent to a vertex in *I* or has a vertex in *I* adjacent to its incident edges, such that removing any vertex x from *I* and adding z to the resulting set, [(*I* - {x})$\cup$z] forms another dominating set. The secure - vertex - edge domination number, indicated as $\gamma_{sve}(G)$, refers to the lowest cardinality of secure - vertex - edge domination in graph G.

Bull Graph: The Bull graph is a planar undirected network consisting of five vertices and five edges. Its form resembles a triangle with two additional edges attached to other vertices.

Butterfly Graph: The butterfly graph is a planar, undirected graph having six edges and five vertices. The graph is created by linking two duplicates of the cycle graph C_3 at a shared vertex and is identical to the friendship graph F_2.

Diamond Graph: A diamond graph refers to a planar, undirected network that consists of four vertices and five edges. It is comprised of a complete K_4 graph with one edge eradicated.

Durer Graph: With twelve vertices and eighteen edges, the Durer graph is a free hexagonal graph. Six vertices create an exterior hexagon in the Durer graph, despite the other six vertices which compose an inverted 2 triangle region. Only one vertex on the hexagon is next to any of the six vertices of the interior triangles.

Franklin Graph: Eighteen edges and twelve vertices make up the three-regular Franklin graph. In addition, the graph has three edges and three vertices that are connected.

Golomb Graph: The Golomb Graph is a non-planar polyhedral shape graph that may be represented as a unit distance graph. It is composed of 10 vertices and 18 edges.

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Moser Spindle Graph: The Moser spindle is a seven-vertex, eleven-edge undirected graph. The shape of it resembles a triangle that is enclosed in a hexagon. The Hajos graph is another name for the Moser spindle.

Herschel Graph: The bipartite, undirected Herschel graph has eighteen edges and eleven vertices. The smallest polyhedral graph without a Hamiltonian cycle is this one, which is also polyhedral.

Petersen Graph: The Petersen Graph is an untamed graph that holds ten vertices and fifteen edges. It is the complement of the K5 line graph.

Fish Graph: A unique type of graph that is comprised of six vertices and seven edges could be referred to as a fish graph.

Wagner Graph: There are just eight vertices and twelve edges in the three-regular Wagner graph. This is an octahedral Mobius ladder graph. The Wagner graph, a particular kind of circulant graph in which the vertices can be organized in a cycle, is a cubic Hamiltonian graph.

Heawood Graph: Having 21 edges and 14 vertices, the Heawood Graph is an undirected graph. There are six edges or more on each cycle in the cubic graph. This graph is 6-cage because the cycles of any smaller cubic graph are shorter.

Pappus Graph: Pappus Graph is a bipartite 3-regular graph featuring 18 vertices and 27 edges.

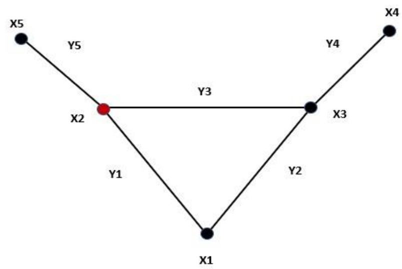
House Graph: The graph consists of five nodes and six edges. The term "house graph" is derived from its resemblance to a schematic illustration of a home with a roof.

Cricket Graph: The Cricket graph is a triangular-shaped planar free graph. It has five vertices and five edges. These edges are linked to a unique node by two different pendant vertices.

3. Main Results

Theorem 1: The secure - vertex - edge domination of a Bull graph is 1.

Proof:

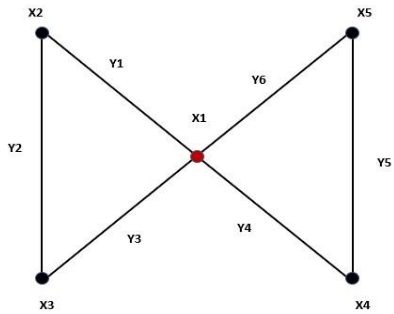


Bull Graph (BG)

The Bull graph is a graph comprised of 5 vertices and 5 edges. It is shaped like a triangle with two separate pendant edges. Two vertices possess a degree of 3, while one vertex possesses a degree of 2. The remaining vertices possess a degree of 1. The secure - vertex - edge dominating set *I*= {x2} or{x3}. The vertex in the set *I* have degree three, which defends the edges that are connected to this particular apex and the edges that's are adjacent to that incident edges. So, all the edges of the Bull graph will be defended by the set *I*. Hence the secure - vertex - edge domination number of the Bull graph is one.

Theorem 2: Let ByG be the Butterfly graph, the secure - vertex - edge domination number of the graph ByG is one.

Proof:

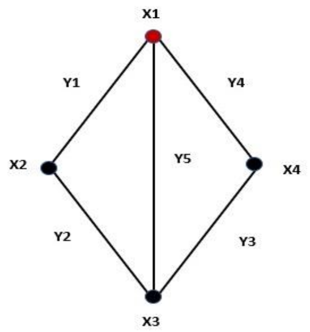


Butterfly Graph (ByG)

The Butterfly Graph is a planar graph made up of five vertices and six edges. The structure is formed by combining two instances of the cycle graph C3, with a shared vertex. The common vertex has degree four and all the remaining vertices have degree two. The secure - vertex - edge dominating set *I* = {x1}, which is the common vertex with highest degree. This common vertex protects all the incident edges and the edges adjacent to that incident edges, so that all the edges of the graph are protected by the vertex x1. Hence the secure - vertex - edge domination number of the Butterfly Graph is one.

Theorem 3: Let DG be the Diamond graph whose secure - vertex - edge domination number is one.

Proof:



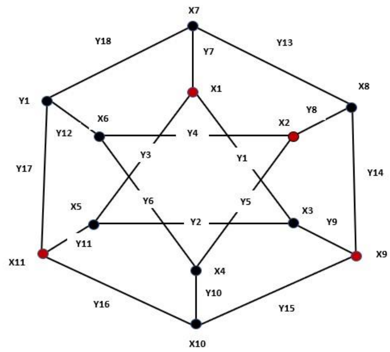
Diamond Graph (DG)

A graph with four vertices and five edges constitute a Diamond graph. In that four vertices, two vertices have their degree three and the remaining two vertices of degree two. Secure - vertex - edge dominating set *I* = {x1} or {x3}. The vertices x1 and x3 are having the highest degree, one of these two vertices can secure all the incident edges and the adjacent edges of that two vertices. So that, all the edges of the graph are secured. Thus, the secure - vertex - edge domination number of the Diamond graph is one.

Theorem 4: The secure - vertex - edge domination number of the Durer graph is calculated through

$\gamma_{sve}(DrG) = \frac{n}{3}$.

Proof:



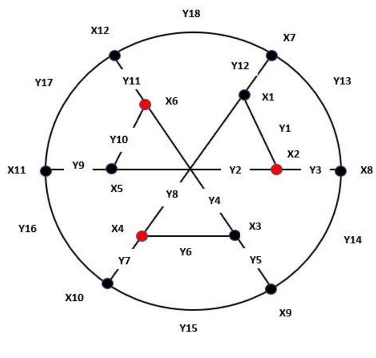
Durer Graph (DrG)

The Durer graph is a graph that is not directed and has a total of 12 vertices and 18 edges. Every single vertex in the graph has a degree of 3. It is a well covered 3- connected cubic graphs. The secure - vertex - edge domination number of the graph that comes from

$\frac{n}{3}$, i.e. $\frac{12}{3} = 4$. The set *I* = {x1, x2, x9, x11} is the secure - vertex - edge dominating set of this Durer graph which protects all the edges which are incident on the vertices of the set *I* and the edges which are adjacent to that incident edges of that vertices. We can select any four vertices of the graph to the set *I* so that it will conserve every edge of the graph. Hence $\gamma_{sve}(DrG) = \frac{n}{3}$.

Theorem 5: If FkG is the Franklin graph then the secure - vertex - edge domination number of the graph is 3.

Proof:



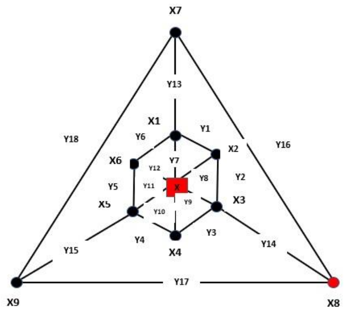
Franklin Graph (FkG)

The Franklin graph is a graph that consists of twelve nodes and 18 lines. The graph is trivalent. The set *I* = {x2, x4, x6} contains three vertices, which will protect the edges which are incident on the vertices and the edges which are adjacent to that incident edges. The vertex x2 protects the edges y1, y2, y3 and the adjacent edges y8, y13, y9, y10, y14, y12. The vertex x4 defend the following edges y6, y7, y8 which are incident and the adjacent edges y1, y4, y5, y12, y15, y16 and the vertex x6 safeguards the edges y4, y11, y10, adjacent edges y5, y6, y17, y18, y9, y2. Hence the set *I* will defend all the edges of the graph. Thus, the secure - vertex - edge domination of the Franklin graph $\gamma_{sve}(FkG) = 3$.

Hence the proof.

Theorem 6: The secure - vertex - edge domination number of the Golomb Graph is two.

Proof:



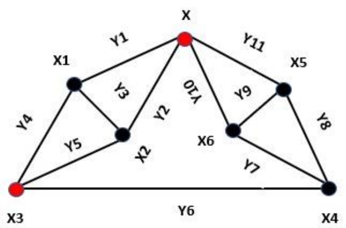
Golomb Graph (GoG)

A graph that involves 10 vertices and eighteen edges has been referred to as a Golomb graph. The middle vertex of the graph has a degree of six. The outer vertices have a degree of four, whereas the inner three vertices have a degree of three. The remaining three vertices also have a degree of four. A set *I* = {x, x8} is a secure - vertex - edge dominating set which contains two vertices; these two vertices will defend all the edges of the Golomb graph. The set *I* should contain the centre vertex and any of the outer vertex, then only the vertices in *I* can protect all the edges of the graph. Thus, the secure - vertex - edge domination number of the Golomb graph is two.

Hence the proof.

Theorem 7: Let MSG be the Moser Spindle Graph with seven vertices, the secure - vertex - edge domination number of the graph $\gamma_{sve}(MSG) = 2$.

Proof:

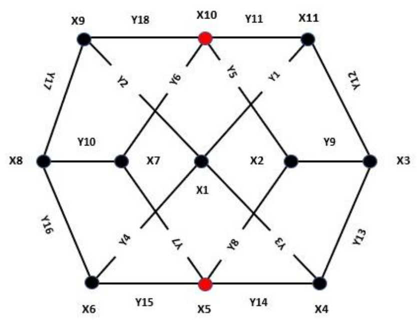


Moser Spindle Graph (MSG)

A Moser Spindle Graph may be constructed by using a graph that has no direction comprising Seven vertices and 11 edges. MSG is of the form of a triangle which is inscribed within a hexagon. The centre vertex of the graph having the degree 4 and all the other vertices having degree 3. The set *I* = {x, x3} be the secure - vertex - edge dominating set which will defend all the edges which are incident and the edges which are adjacent to that incident edges. In the set *I* there should be a centre vertex which has the highest degree and the other vertex can be any one from the remaining vertices. Therefore, these two vertices guarantee the safety of every edge in the graph. Thus, the secure - vertex - edge domination number of the MSG is 2.

Theorem 8: The secure - vertex - edge domination number of the Herschel graph is two.

Proof:

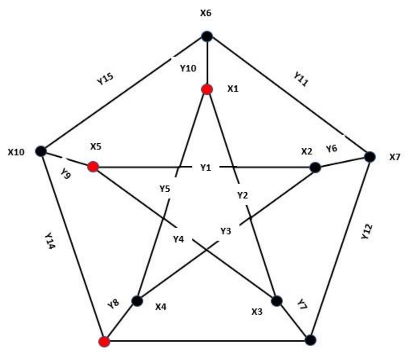


Herschel Graph (*HlG*)

The Herschel graph is a bipartite graph with eleven vertices and eighteen edges, and it is free of direction. The network consists of three vertices with a degree of 4, while the remaining eight vertices have a degree of 3. The secure - vertex - edge dominating set *I* = {x5, x10}. The vertex x5 will safeguard the following edges y7, y8, y14, y15, y3, y13, y9, y5, y10, y6, y4, y16 and the vertex x10 will be a defender to the following edges y18, y11, y5, y6, y2, y1, y17, y12, y9, y8, y10, y7. Hence these two vertices in set *I* defend all the edges which are incident and the edges which are adjacent to that incident edges. Thus, all the edges of the HlG are protected. So, the secure - vertex - edge domination number of the Herschel graph $\gamma_{sve}(HlG) = 2$.

Theorem 9: Let P*n*G be the Petersen Graph with n =10 vertices for which the secure - vertex - edge domination number is given by $\left\lceil \frac{n}{3} \right\rceil$.

Proof:

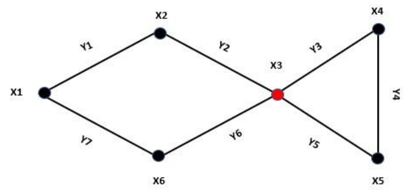


Petersen Graph (*PnG*)

A graph formed by 10 vertices and fifteen edges is referred to as a Petersen graph. The Petersen graph is of the form of a star inscribed in a Pentagon. The degree of each vertex in the Petersen graph is three. The secure - vertex - edge domination number of the P*n*G is given by $\left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{10}{3} \right\rceil = 3$. Let *I* = { x1 , x5, x9 } be the secure - vertex - edge dominating set, in which x1 vertex protects the edges y2, y5, y3, y8, y4, y11, y7, y10, y15 the vertex x5 defends the edges y1, y4,y2, y9,y3, y6, y14, y7,y15 and the vertex x9 protects the edges y5, y12,y7, y14,y8, y2,y9, y13, y15. Thus, all the edges of the graph are protected. Hence the secure - vertex - edge domination number of the Petersen graph is $\gamma_{sve}(PnG) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 10: The Fish graph possesses a secure - vertex-edge dominance number as one.

Proof:



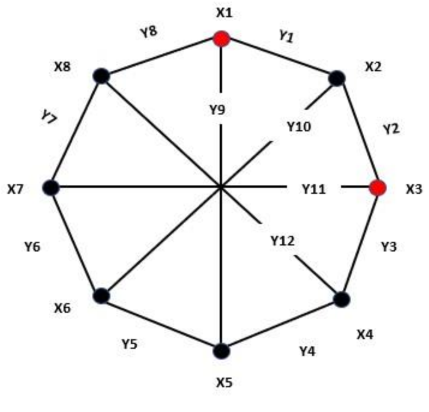
Fish Graph (*FiG*)

Fish graph comprises six vertices and 7 edges in which one vertex has its degree 4 and the other vertices has degree 2. The secure vertex-edge dominant set I = {x3}, where x3 is the vertex with the greatest degree, will safeguard the edges that are connected to x3 as well as the edges next to those connected to x3. So that, every edge of the fish graph is safe. Thus, the secure - vertex - edge domination of the fish graph is given by $\gamma_{sve}(FiG) = 1$.

Hence the proof.

Theorem 11: The Wagner Graph, designated as WrG, has a secure - vertex-edge dominance number as 2.

Proof:



Wagner Graph (Wr*G*)

Wagner Graph is a graph which consists of eight vertices and 12 edges. The graph is a regular graph with a degree of 3. Since it is symmetric, one vertex from each side will safeguard each of the edges of the graph. Two vertices in *I* should not be in the same straight line. The secure - vertex - edge dominating set *I* = {x1, x3}, which will protect all the 12 edges of the graph. Thus, the secure - vertex - edge domination of Wagner graph is $\gamma_{sve}(WrG) = 2$.

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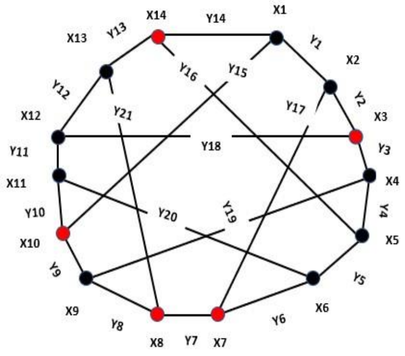
Hence the proof.

Theorem 12: The Heawood graph's secure-vertex-edge dominance number is identified by $\gamma_{sve}(HdG) = \left\lceil \frac{n}{3} \right\rceil$.

Proof:

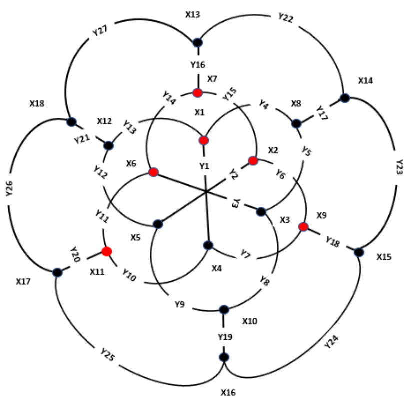


Heawood Graph (H*dG*)

The Heawood graph (HdG) is a graph featuring Fourteen nodes and twenty-one edges, where all vertices are linked. It is a 3- regular graph. The secure - vertex - edge domination number of this Heawood graph is determined through $\left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{14}{3} \right\rceil = \lceil 4.6 \rceil = 5$. The set I contains five vertices namely x3, x7, x8, x10, x14. These five vertices secure all the twenty-one edges of the graph. The condition to select these vertices is that the vertices in the set I should not be in the same line. i.e., In a straight line there will be two vertices at each end of the line, no two vertices of this straight line should be in the set I. Therefore, all the edges that are connected to the vertices and the edges that are connected to the connected edges will be safeguarded. Hence the secure - vertex - edge domination number of the Heawood graph is $\gamma_{sve}(HdG) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 13: Consider a network with n = 18 vertices that is a Pappus Graph. The secure-vertex-edge dominance number of this graph is given by $\left\lceil \frac{n}{3} \right\rceil$.

Proof:

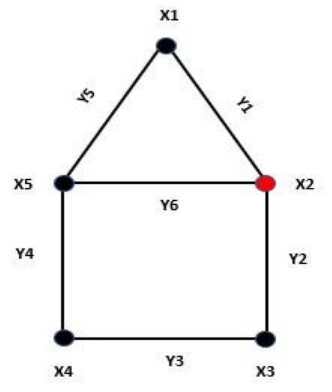


Pappus Graph (Pps*G*)

The Pappus Graph is a graph that has a degree of 3, meaning that each vertex is connected to exactly 3 other vertices. It consists of 18 vertices and 27 edges. The Pappus graph resembles like a flower with 3 layer and 6 petals. The secure - vertex - edge domination number of the Pappus graph provided by $\left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{18}{3} \right\rceil = 6.$ The set I contains 6 vertices in which, any two vertices of each layer can be in the set I or 3 vertices of the inner two layers. These 6 vertices will defend all the 27 edges of the graph. Let I = {x1, x2, x6, x8, x10, x12} be the secure - vertex - edge dominating set of the Pappus graph which secures all the incident edges of that vertices and the edges which are adjacent to that incident edges. Therefore, every edge in the graph is protected. Therefore, the Pappus graph has a secure-vertex-edge dominance number is $\gamma_{sve}(PpsG) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 14: The secure - vertex - edge domination number of the House Graph is one.

Proof:

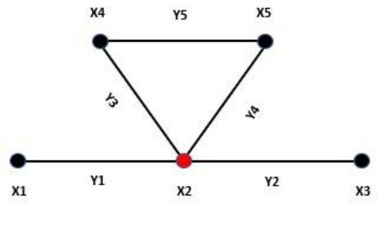


House Graph (H*G*)

The House graph is composed of five vertices labeled x1, x2, x3, x4, and x5, and six edges labeled y1, y2, y3, y4, y5, and y6. Two vertices, x2 and x5, have a degree of three, whereas all of the rest have a degree of two. The secure - vertex - edge dominating set I ={x2} or{x5}. Since x2 and x5 have the maximum degree, these vertices may effectively secure all the incident edges and the neighboring edges connected to those incident edges in the graph. Therefore, all the edges in the graph are safeguarded. Hence the secure - vertex - edge domination number of the house graph is one.

Theorem 15: The cricket graph, denoted as CG, has a secure-vertex-edge dominance number as one.

Proof:



Cricket Graph (C*G*)

The cricket graph consists of five vertices labeled as x1 to x5, and five edges labeled as y1 to y5, in which one vertex x2 has the highest degree four, two vertices x4, x5 with degree two and the remaining vertices x1 and x3 having their degree one. The secure - vertex - edge dominating set I ={x2}. Since x2 has the highest degree, it can safeguard all the incident edges y1, y2, y3, y4 of x2 and the edges which are adjacent to that incident edges y5. So that, all the edges of the graph are protected. Hence the secure - vertex - edge domination number of the cricket graph is one, i.e., $\gamma_{sve}(CG) = 1$.

4. Conclusion

In graph theory, the idea of dominance states that a group of vertices S dominates a graph G if every vertex in the graph is either a member of S or adjacent to a vertex in S. The dominance number of G is determined by the size of the smallest dominating set. This led to the introduction of a new dominance parameter "secure - vertex - edge domination" before. These pervasive concepts may be used to numerous industries, such as radio programming, computer communication networks, school bus routing, social networks, and interconnection systems. This paper's significant contribution is its investigation of the secure - vertex - edge dominance number of a few designated specific graphs, including Bull Graph, Butterfly Graph, diamond Graph, Durer Graph, Franklin Graph, Golomb Graph, Moser Spindle Graph, Herschel Graph, Petersen Graph, Fish Graph, Wagner Graph, Heawood Graph, Pappus Graph, House Graph and Cricket Graph.

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