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Even Vertex Odd Edge Root Square Mean Labelling of Path Related Graphs

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Abstract:

Consider G be a graph with p vertices and q edges. A graph G is said to be even vertex odd edge root square mean labelling if there exist an injective map $f : V(G) \to \{0,1,2,3, \dots, 2q\}$ such that the induced edge labels are odd and distinct which can be obtained by $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$. Any graph which admits even vertex odd edge root square mean labelling then it is called as even vertex odd edge root square mean labelling graph.

Keywords: EVOE – RSML, Graph, Ladder, Corona graph

1.Introduction:

Throughout the work, we consider a simple graph. The terminology and the definitions of graph were followed in [1]. For a complete analysis of labeling studied by Gallian [1]. In 1960's graph labeling was introduced by Rosa. Odd vertex even edge root square mean labeling discussed in [5] which motivated to construct the above said labeling pattern. We study a few paths related graphs which satisfies the above pattern. Mean labeling, root square mean and super root square mean labeling were discussed in [4,6,7,8].

2 Preliminary:

Definition 2.1 [5]

A graph G is said to be odd vertex even edge root square mean labelling if there exist an injective map $f : V(G) \to \{1,2,3, \dots, 2q + 1\}$ such that the induced edge labels are odd and distinct which can be obtained by $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$. Any graph which admits even vertex odd edge root square mean labelling then it is called as odd vertex even edge root square mean labelling graph.

Definition 2.2

A graph G is said to be even vertex odd edge root square mean labelling if there exist an injective map $f : V(G) \to \{0,1,2,3, \dots, 2q\}$ such that the induced edge labels are odd and distinct which can be obtained by $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$. Any graph which admits even vertex odd edge root square mean labelling then it is called as even vertex odd edge root square mean labelling graph.

Definition 2.3 [6]:

A graph G with p vertices and q edges, is a mean graph if there is an injective function f from the vertex set to { 1, 2, 3…q} when each edge uv is labelled with $\frac{f(u)+f(v)}{2}$ if f(u)+f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u)+f(v)+1 is odd then the labelling edges are distinct.

Definition 2.4 [4]:

A graph G with p vertices and q edges is called a Root Square Mean graph if is possible to label the vertices v with distinct labels f(x) from 1, 2, 3,… q+1 in such a way that when each edge e = uv is labelled with f(e) =$\lfloor\sqrt{(\frac{f(u)^2+f(v)^2}{2})}\rfloor$ or $\lceil\sqrt{(\frac{f(u)^2+f(v)^2}{2})}\rceil$ then each labels are distinct.

Definition 2.5 [4]:

The corona graph is obtained by taking one copy of path Pn and n copies of K1 then joining the ith vertex of Pn with an edge to every vertex in the ith copy of K1. It is denoted by Pn K1.

Definition 2.6 [7]:

TW(Pn) is a graph which is obtained from a path by identifying k1,2 to all the vertices of the path except one pendent vertex (Twing graph) A path with a least vertex is connected and has two terminal vertices while all other vertices have degree 2.

Theorem 3.1: A path P_n is an even vertex odd edge root square mean labelling graph.

Proof:

Let G be a Path P_n, where V(G)= $\{u_i : 1 \le i \le n\}$ and E(G) = $\{u_i, u_{i+1} : 1 \le i \le n-1\}$ It is evident that G allows an even-vertex odd-edge root square mean labeling, as labels can be assigned to its vertices and edges in a sequential manner. Hence, G is classified as an even-vertex odd-edge root square mean graph.

Illustration: Figure 1 shows G = P_7 is Even vertex odd edge root square mean labelling.

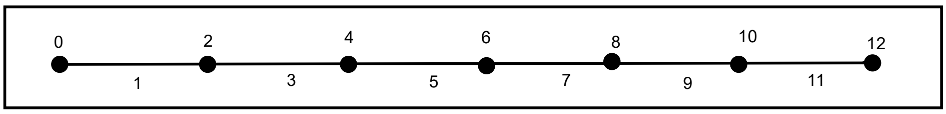

Figure 1 Even vertex odd edge root square mean labelling of P_7

Theorem 3.2: $P_n \odot K_1$ is an even vertex odd edge root square mean labelling graph.

Proof: Let G = $P_n \odot K_1$

The vertex set and edge set of G is defined as follows;

V(G) = $\{u_i , u'_i ; 1 \le i \le n\}$ and

E(G) = $\{u_i u'_i ; 1 \le i \le n$ and $u_i u'_{i+1} ; 1 \le i \le n-1 \}$

We define a map f: V(G) → {0,1, 2, 3…….2q}.

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The labelling pattern of vertices can be defined as follows;

$f(u_{2i-1}) = 8i - 6$ and $f(u_{2i}) = 8i - 4;\ 1 \le i \le n$

$f(u'_{2i-1}) = 8(i-1)$ and $f(u'_{2i}) = 8i-2;\ 1 \le i \le n$

Then the induced edge labels are

$f'(u_i u'_i) = 4i - 3;\ 1 \le i \le n$

$f'(u_i u'_{i+1}) = 4i - 1;\ 1 \le i \le n-1$

Thus $G = P_n \odot K_1$ admit even vertex odd edge root square mean labelling graph.

Illustration: Figure 2 illustrate the $G = P_4 \odot K_1$ is even vertex odd edge root square mean labeling

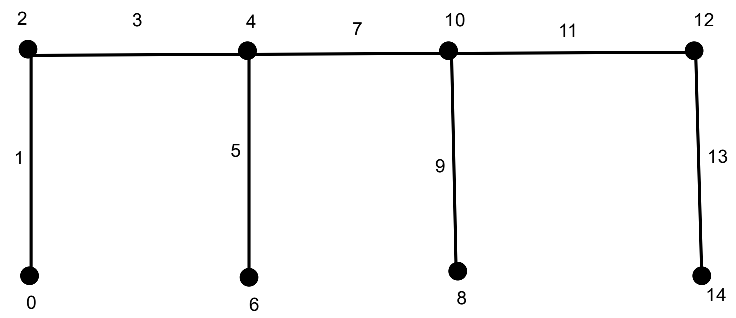


Figure 2 $G = P_4 \odot K_1$

Theorem 3.3: $P_n \odot K_{1,2}$ is an even vertex odd edge root square mean labelling graph.

Proof: Let $G = P_n \odot K_{1,2}$ is a graph formed by attaching a $K_{1,2}$ (a star with one central vertex and two leaves) to each vertex of the path P_n.

Let $V(G) = \{u_i, u'_i, u''_i\ ;\ 1 \le i \le n\}$ and

$E(G) = \{u_i u'_i, u_i u''_i : 1 \le i \le n$ and $u_i u'_{i+1} : 1 \le i \le n\text{-}1\}$

This describes the standard labeling of the vertices and edges of G.

We define a map $f: V(G) \to \{0, 1, 2, \ldots\ldots 2q\}$ by

$f(u_i) = 6i - 4 : i \le i \le n$

$f(u'_i) = 6(i-1)$ and $f(u''_i) = 6i - 2: 1 \le i \le n$

It is evident that the included edge labels are both odd and distinct.

Hence $G = P_n \odot K_{1,2}$ is an even vertex odd edge root square mean labelling.

Illustration: Figure 3 illustrate the $G = P_4 \odot K_{1,2}$ is even vertex odd edge root square mean labeling

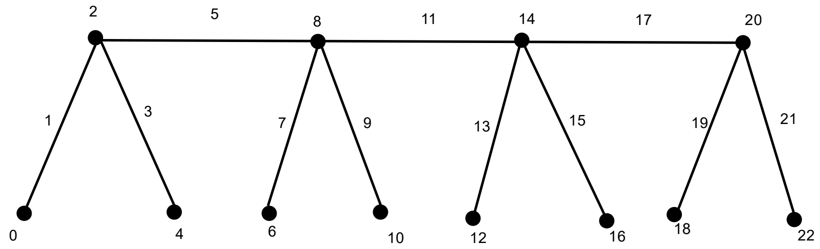


Figure 3 G = $P_4 \odot K_{1,2}$

Theorem 3.4: The graph SL_n is an even vertex odd edge mean square labelling graph.

Proof: Let G = SLn

Let the vertex set V(G) = $\{u_i, v_i : 1 \leq i \leq n\}$ and edge set

$$E(G) = \begin{cases} u_i\ u_{i+1} : 1 \leq i \leq n-1 \\ v_i\ v_{i+1} : 1 \leq i \leq n-1 \\ v_{i+2}\ v_i : 2 \leq i \leq n-1 \end{cases}$$

Define a map f: V(G) → {0,1, 2, ……2q} by f (u_1) = 0, f(v_n) = $2q$, further

$$f(u_i) = 6i - 10 : 2 \leq i \leq n,\ f(v_i) = 6i - 2 : 1 \leq i \leq n-1$$

Then the induced edge labels are defined as follows;

$f'(v_i v_{i+1}) = 6i + 1 : 1 \leq i \leq n-2,$

$f'(v_{n-1} v_n) = 2q - 1 : 1 \leq i \leq n-2$ and

$f'(u_{i+1} v_i) = 6i - 3 : 1 \leq i \leq n-1.$

Hence, SL_n admits even vertex odd edge mean square labelling graph.

Illustration: Figure 4 illustrate the G = SL_6 is even vertex odd edge root square mean labeling

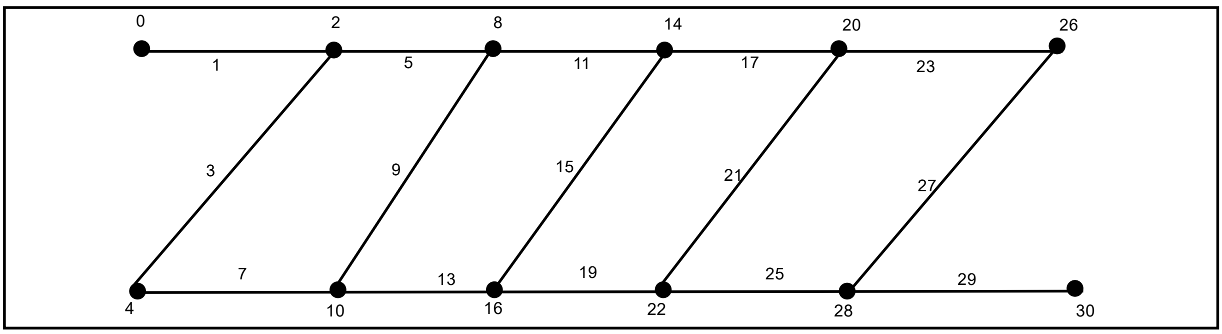


Figure 4 G = SL_6

Theorem 3.5:[5] Let G_p be a graph formed by attaching a pendant edge to each internal vertex of a path. This graph G_p is an even vertex odd edge root mean square labelling graph.

Proof:

Let V(G) = { $u_i : 1 \leq i \leq n$ and $u'_i : 2 \leq i \leq n-1$ and

$$E(G) = \begin{cases} u_i\, u_{i+1} : 1 \leq i \leq n-2 \\ u_i\, u'_i : i = 2,3,4,5 \ldots . n-2 \end{cases}$$

be the vertex set and edge set of G respectively.

Define a map f: V(G) → $\{0,2,4,\ldots 2q\}$ by f(u) where f(u) is determined based on the following two cases.

Case 1: For n is odd

$f(u_i) = 4(i-1)$ and $f(u'_i) = 4i-6$: i = 3, 5, …

$f(u_i) = 4i-6$ and $f(u'_i) = 4(i-1)$: i = 2, 4, …

The labels of the edges are then defined as follows

$f'(u_i\, u'_i) = 4i - 5$, i = 2, 4,…. n-1 and $f'(u_i\, u_{i+1}) = 4i - 3$, i = 1, 2 ,….n-1

Case 2:

For n is even

$f(u_i) = 4i-5$ and

$f(u'_i) = 4i-6$: i = 3, 5, …

$f(u_i) = 4i-6$ and $f(u'_i) = 4(i-1)$: i = 2, 4, …

Then the induced edge labels are defined as follows:

$f'(\mathbf{u_i\, u'_i}) = 4i - 5$, i = 2, 4,….n-1 and $f'(\mathbf{u_i\, u_{i+1}}) = 4i - 3$, i = 1, 2 ,….n-1

Hence f is an even vertex odd edge root square mean labelling of graph G. Therefore, G is classified as an even vertex odd edge root square mean labelling of graph with respect to f.

Theorem 3.6: TW(Pn) is an even vertex odd edge root square mean labelling of graph.

Proof: Let G = TW(Pn)

$$V(G) = \begin{cases} u_i : 1 \leq i \leq n \quad \text{and} \\ u'_i \text{ and } u''_i : 2 \leq i \leq n \end{cases}$$

$$E(G) = \begin{cases} u_i u_{i+1} : 1 \leq i \leq n-1 \quad \text{and} \\ u_i u'_i \text{ and } u_i u''_i : 2 \leq i \leq n \end{cases}$$

be the vertex set and edge set of G respectively. We define a map f: V(G) → $0,2,4,\ldots 2q$ by f(u) = 0;

$f(u'_i) = 2(i + j - 1)$ i = 2 and j = 1, 2; $f(u'_i) = 6(i-1)$, i = 3,4, …n; $f(u''_i) = 6i-10$,

i = 3, 4,…n

Then the induced edge labels are defined as follows;

$f'(u_i\, u'_i) = 6i - 9$, $f'(u_i\, u''_i) = 6i - 7$, i = 2,4, …, n-1 and $f'(u_i\, u_{i+1}) = 6i-5$; i =1, 2,…n-1

Hence G is an even vertex odd edge root square mean labelling of graph since f is an even vertex odd edge root square mean labelling of graph G.

Illustration: Figure 5 illustrate the Gp on 6 vertices is even vertex odd edge root square mean labeling

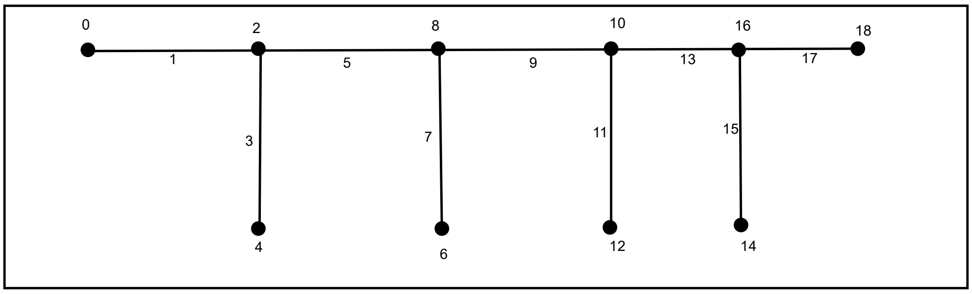
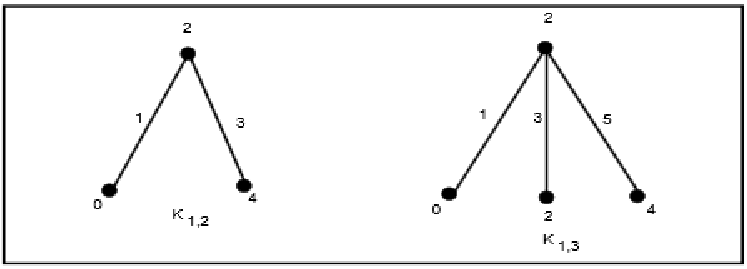


Figure 5 Gp on 6 vertices

Theorem 3.5: $K_{1,n}$ an even vertex odd edge mean square labelling graph. if $n \leq 3$.

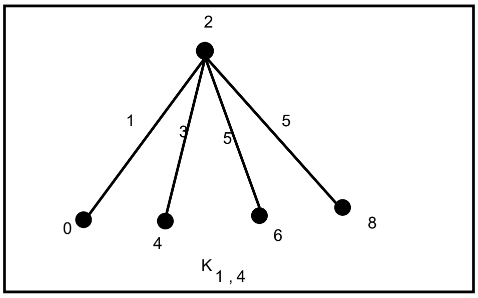
Proof:

Let the vertex and edge sets of $K_{1,n}$ be defined as $\{u, v_1, v_2, \dots, v_n\}$and $\{e_1, e_2, \dots, e_n\}$ respectively. For $n \leq 3$, it exhibits an odd vertex even edge mean square labelling graph. This demonstrates the figures that are admitted as an odd vertex even edge mean square labelling graph.



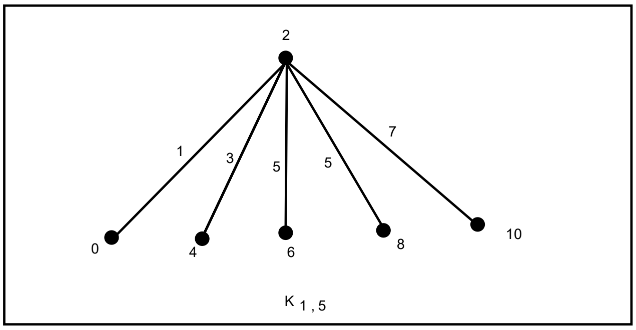
If n = 4

$K_{1,4}$ is not even vertex and odd edge labelling graph



Since the graph depicts that the edge label 5 repeated two times

If n = 5



Since the graph depicts that the edge label 5 repeated two times

In general, n > 3

Case (i)

Let f(u) = 0, then f (u_i) = {2, 4, 6, …, 2q}

In this case, (0,6), (0, 8), will get same labels, which is contradiction to the definition.

Case (ii)

Let f(u) = 2q, then no edge can get the label 1.

Since (0, 2q) = $\sqrt{2(q)^2} = \sqrt{2}$ q, which is not possible.

Case (iii)

If f(u) = {2 or 4 or 6 or …. 2(q-1)}, then we get the repetition of odd edge labelling.

So, clearly one can check labelling of a graph.

Theorem 3.6 A subdivision of $P_n \odot K_1$ is an even vertex odd edge root square mean labelling graph.

Proof:

The vertex set and edge set of,

G, is defined as follows;

V(G) = $\{u_i \; ; 1 \leq i \leq n\} \cup \{v_i \; 1 \leq i \leq n-1\} \cup \{u_i, u''_i \; 1 \leq i \leq n\}$ and

E(G) = $\{ \{u_i u'_i\} \cup \{u_i, u''_i\} \cup \{u_i v_i\} \cup \{u_i v_{i+1}\} : 1 \leq i \leq n\text{-}1 \}$

We define a map f: V(G) → {0,1, 2, 3,…2q}.

The labelling pattern of vertices can be defined as follows;

$f(u_i) = 4i$; i = 1,2 $\quad$ $f(u_i) = 8i - 6$; i = 3,4,5…, n-1

$f(v_i) = 6i$; i = 1,2 $\quad$ $f(v_i) = 8i - 2$; i = 3,4,5…, n

$f(u'_i) = 8i - 6$; i = 1,2 $\quad$ $f(u'_i) = 8(i - 8)$; i = 3,4,5…, n-1

$f(u''_i) = 14(i-1)$; i = 1,2 $\quad$ $f(u''_i) = 8i - 4$; i = 3,4,5…, n-1 and $f(u_n) = 2(q-1)$

Then the induced edge labels are

$f'(u_iv_i) = 6i - 1; i = 1,2$ $\quad$ $f'(u_iv_i) = 8i - 3; i = 3,4,5\ldots, n-1$

$f'(v_iu_{i+1}) = 8i - 1; 1 \leq i \leq n-1$

$f'(u_iu'_i) = 6i - 3; i = 1,2$ $\quad$ $f'(u_iu'_i) = 8i - 7; i = 3,4,5\ldots, n-1$

$f'(u'_iu''_i) = 12i - 11; i = 1,2$ $\quad$ $f'(u'_iu''_i) = 8i - 5; i = 3,4,5\ldots, n-1$

Thus, subdivision of $G = P_n \odot K_1$ admit even vertex odd edge root square mean labelling graph.

Illustration: Figure 6 illustrate a subdivision of $P_n \odot K_1$ on 8 vertices is an even vertex odd edge root square mean labeling

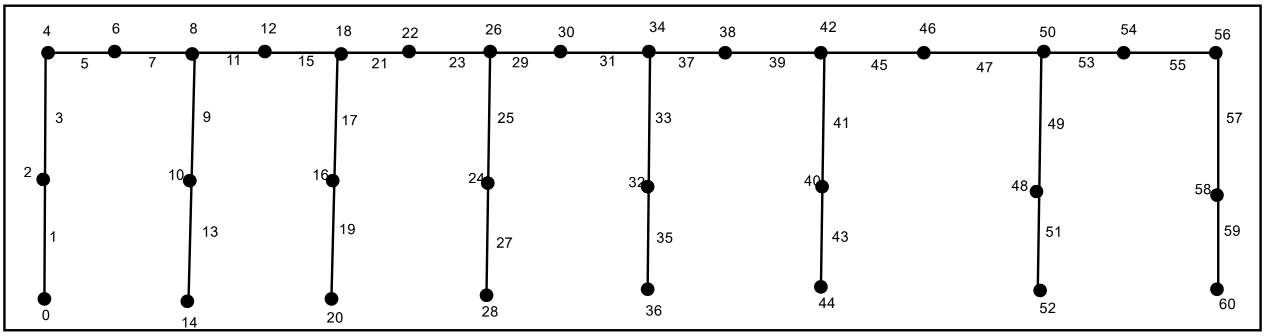


Figure 6 $P_n \odot K_1$ on 8 vertices

Conclusion :

In conclusion, this paper presents a new labeling pattern based on the root square mean, which has been successfully applied to path-related graphs, the subdivision of path corona graphs, and star graphs. The results demonstrate the effectiveness of this approach in enhancing graph labeling techniques and offer potential for further research in related areas.

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