Maximum Matching in Koch Snowflake and Sierpinski Triangle

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Abstract The goal of this paper presents the method of finding the maximum matching cardinality in the fractal graph. A fractal is uneven or irregular sets of well-defined structure that has to be broken up into small pieces, the property of having each piece analogous or identical to the overall structure at random ranges. Fractal graph is an excellent construction of well-defined objects. It provides a general frame work-study of regular self-similarity sets and natural phenomena of the fractal graphs. Matching is the collection of non-touching edges in the graph. Maximum matching is the largest possible collection of non-touching edges in the graph. This paper is used to determine the constant ratio of maximum matching cardinality in all the iteration of self-similarity Fractal Graph. This paper is used to find the constant ratio between the number of edges in each iteration of the Fractal Graph and the Maximum Cardinality number of Maximum Matching. This provides general formulae for all the iteration of Koch snowflake and Sierpinski triangle. By using mathematical induction method, it finds the maximum matching constitutive number in all the iterations of them.

Keywords Graph · Matching · Maximum matching · Fractals · Mathematical induction

AMS Classification Key 05C · 05C70 · 28A80 · 51N3

1 Introduction

Graph theory is the study of the graphs which can be made by vertices or dots or points which are joined by edges or sinks or lines or curves [1]. It takes an essential

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part of nature. The graph can be implemented from nature. It implies more basis ways which are used to mathematical structures. It is used to solve many difficult practical problems. It acts as an essential role in many fields like physical, medical, social, chemical and informative system. Fractal is one of the mathematically defined genesis structures or appearances. The special case of the fractal is it has the same characteristics in each iteration. It has a simple and repetitive defined fine structure at approximate scales. Set and functions are not enough to describe sufficiently smooth irregular objects. Mathematician discovered fractal graph which is the smooth self-similarity object. It has a simple and recursive defined fine structure at arbitrary scales. Despite that it has been divided or spitted or enlarged or minimized, it has the following same properties and similar mathematically defined structure. That means, it contains self-similarity [2] in each iteration. Recently, the paper entitled maximum matching in fractal graph [3] obtained the maximum matching constitutive number of Von Koch curve and star fractal graph motivated and connected the above result. This paper is used to obtain the Maximum Matching constitutive number [4] of Koch Snowflake and Sierpenski Triangle [5].

2 Koch Snowflake

Koch snowflake is one of the earliest fractals having been described. It is the next level of the von Koch curve. It has been invented by a Swedish Mathematician Hedge Von Koch [6]. He dissatisfied Karl Weierstrass abstract which is the opponent of similarity mathematical object and gives more geometric definition of self-similarity [7] which is called Koch curve, Koch snowflake and so on. Structure of the Koch snowflake is very interesting. It starts with an equilateral triangle in the initial iteration. In the first iteration, it can be implemented into large. Each side of the triangle split into three segments. The middle segment can be moved into outward and form a triangle. It shows that each side of the triangle spit into four lines. This process is continued in the upcoming iteration. In each iteration, edges of the Koch snowflake increase in the common ratio. In each iteration, Koch snowflake has $3(4^{n})$ number of edges where n is increased by one and starts at zero.

3 Sierpinski Triangle

It is the world-famous fractal graph in classical geometry [8]. It consists of full of triangles inside. It consists of overall shape of an equilateral triangle which has each side of the triangle that has an equal length [9]. It has self-similarity. It is used in the structure of jewels and building construction [10]. Hedge Von Koch is the father of the Sierpinski triangle.

Sierpinski triangle can be started with an equilateral triangle. In each side of the triangle, fix the center point. Now joining the center points of the triangle, it may form an equilateral triangle [11]. It can be covered ¾ of the original triangle. It can be removed from the original triangle. Each side decreased 1/3 of the original length. Repeat the same procedure for every line to form the new structure, and repeating forever, got a new structure in turn. It is shown in Fig. 7. Its diameter is infinite and its area is finite. By using Theorem 2, maximum matching cardinality [12] of the Sierpinski triangle can be calculated.

4 Maximum Matching in Koch Snowflake

Matching [13] is the set of independent non-touching edges that do not share a common vertex. Maximum matching is the matching that has the largest collection of non-touching edges. Maximum matching constitutive number or cardinality is the number of collections of non-touching edges. The number of edges in the graph and number of edges in maximum matching is denoted by *N(G)* and *V(G)*. Here Maximum Matching Cardinality is calculated for all the iterations of Koch Snowflake in Fig. 3 by using the mathematical induction method [14]. It follows by an Algorithm 1.

Algorithm 1

Step 1: Analyze the theorem for initial iteration of the fractal.

Step 2: Assume the theorem for $n = k$.

Step 3: To find the maximum matching cardinality for $n = k + 1$.

- Here, the image is divided into regular meshes.
- To calculate the matching cardinality for each image.
- Applying the concept of permutation, merge the divided images [15].
- To get the maximum matching cardinality for $n = k + 1$.

Step 4: The theorem is proved for all *n*.

Theorem 1 *Koch snowflake consists of* $t(4^n)$ *number of edges in each iteration. Then, it has* $(2t)4^{n-1}$ *number of edges in the maximum matching in the corresponding iteration where t is the number of edges in the equilateral triangle in the initial iteration.*

Proof This theorem can be proved by an induction method.

Step 1:

In the initial iteration, it consists of an equilateral triangle. It has only 3 edges.

It is shown in Fig. 1. Obviously, it is an equilateral triangle; any single edge is the maximum matching cardinality.

Number of edges in Koch snowflake $N(G_0) = 3(4^0) = 3$.

Maximum matching number $V(G_0) = (2 * 3)4^{0-1} = 1$ (consider integer only).

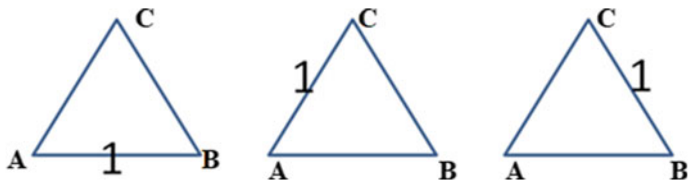
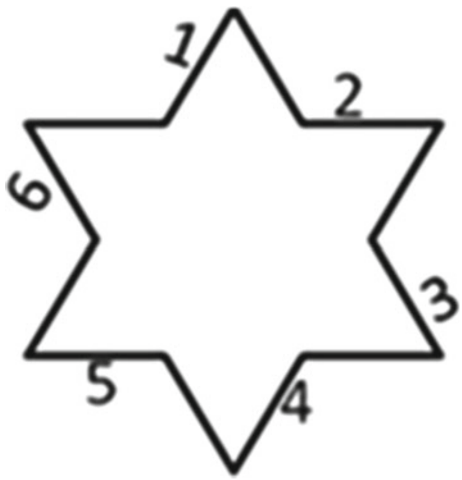


Fig. 1 Initial iteration of Koch snowflake G_0

Fig. 2 First iteration of Koch snowflake G_1



Step 2:

In the first iteration of Koch snowflake, it consists of 12 edges.

It is shown in Fig. 2.

Here $t = 3$ and $n = 1$. Number of edges in the graph $N(G_1) = t\ 4^{\boldsymbol{n}} = 3 * 4 = 12$.

Total number of edges in maximum matching in the first iteration $V(G_1) = (2t)\ 4^{n-1} = (2*3) * 4^{1-1} = 6$.

Step 3:

Let us assume that this theorem is true for all $n = k$.

For Koch snowflake consisting of $t(4^k)$, edges have $(2t)4^{k-1}$ number of edges in maximum matching.

Step 4:

To prove that the theorem is true for all $n = k + 1$.

In Koch snowflake in $(k + 1)$, iteration has a number of edges t 4^{k+1}.

It can be divided into two components $t\ 4^{\,k}$ and 4.

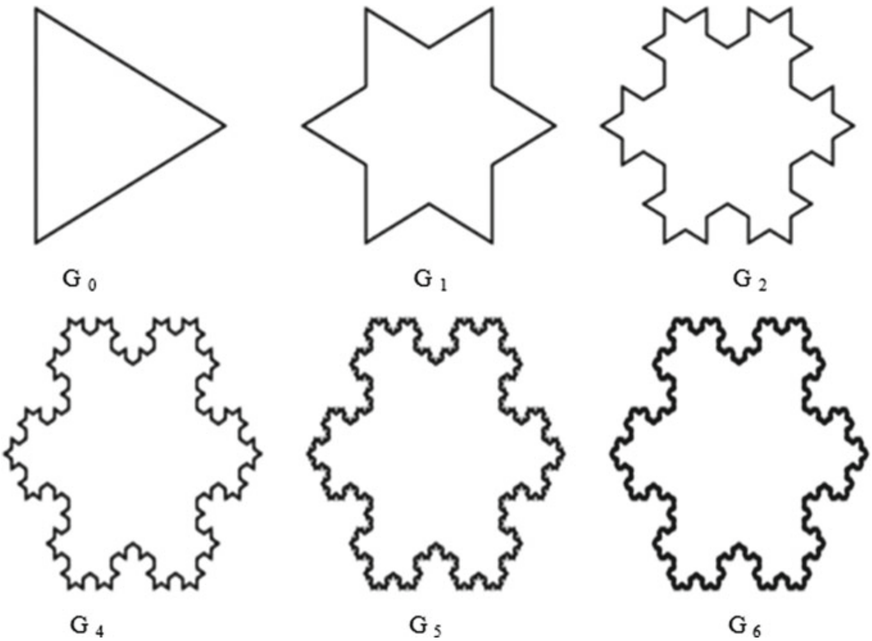


Fig. 3 Different iterations of Koch snowflake

In the first component t 4^k edges of Koch Snowflake has $(2t)\ 4^{k-1}$ edges of Maximum Matching and 4 edges from the second component connected with the first component by permutation method.

Number of edges in maximum matching $= (2t)\ 4^{k-1} * 4 = (2t)4^k$.

The theorem is proved for all n.

Induction method is proved.

A various iteration of Koch snowflake is shown in Fig. 3.

Theorem 2 *Sierpinski triangle has 3^n a number of edges in each iteration. It has the common ratio between the number of edges and the cardinality number of maximum matching in each iteration. Then in the first two initial iterations, the Sierpinski triangle has 3^{n-1} edges of maximum matching constitutive number ($1 \leq n \leq 2$) and the next level of iterations, and it has 7*3^{n-3} edges of maximum matching constitutive number where $n \geq 3$.*

Proof This theorem can be proved in two steps. In the first step, analyze about initial iteration where ($1 \leq \boldsymbol{n} \leq 2$).

Step 1:

When $n = 1$, first iteration of the Sierpinski triangle is shown in Fig. 4.

It consists of $3^n = 3$ edges and $3^{n-1} = 3^0 = 1$ edges of maximum matching cardinality.

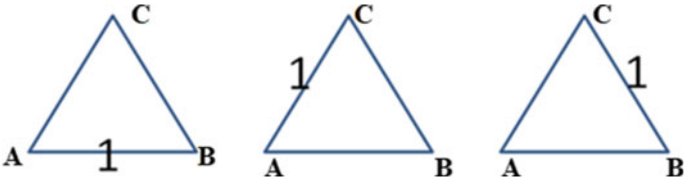
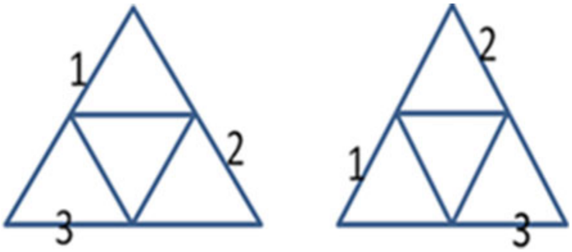


Fig. 4 Maximum matching in first iteration of Sierpinski triangle I_1

Fig. 5 Maximum matching in second iteration of Sierpinski triangle I_2



Here in the initial iteration, start with an equilateral triangle. It has only three edges.

It is a closed-loop structure [12]. It implies any one of the edges which is considered as maximum matching cardinality.

Here, maximum matching cardinality $V(G) = 1$.

Step 2:

When $n = 2$, in the second iteration Sierpinski triangle has $3^n = 3^2$ edges and $3^{n-1} = 3^1$ maximum matching cardinality. Here, only three edges are possible to maximum matching set. It is shown in Fig. 5.

Step 3:

After two iterations, the common ratio between the number of edges and maximum matching cardinality has differed.

Hereafter, Sierpinski triangle implies 3^n edges of the fractal graph have $7{*}3^{n-3}$ maximum matching cardinality.

It can be proved by using mathematical induction method.

Case (i):

In the third iteration, Sierpinski triangle is very interesting. Let $n = 3$.

This Sierpinski triangle has $3^n = 3^3 = 27$ edges.

It has $7{*}3^{n-3} = 7{*}3^{3-3} = 7$ number of edges in maximum matching.

It is shown in Fig. 6.

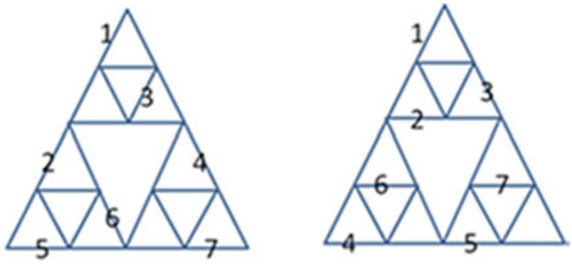


Fig. 6 Maximum matching in third iteration of Sierpinski triangle I_3

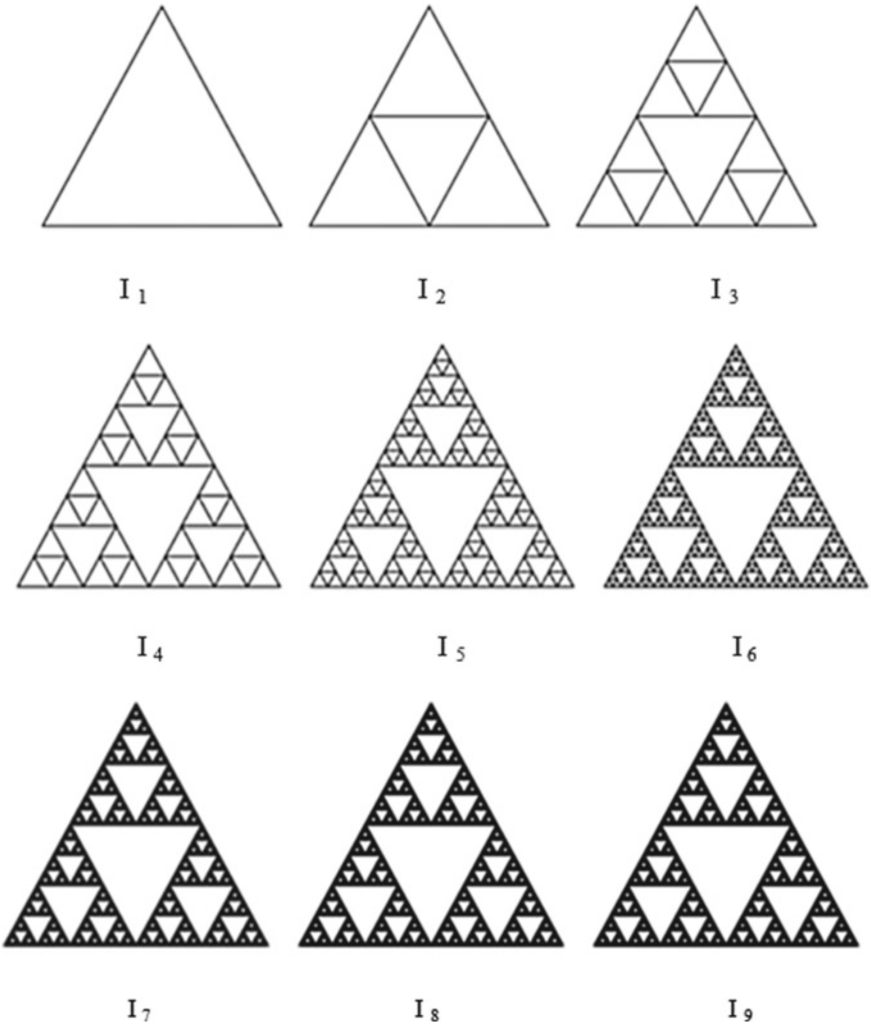


Fig. 7 Different iteration of Sierpinski triangle

Case (ii):

Let us assume that the theorem is true for all $n = k$.

Assume that in kth iteration Sierpinski triangle has 3^k edges and it has $7{*}3^{k-3}$ number edges in maximum matching.

Case (iii):

To prove this theorem is true for $n = k + 1$, i.e., to prove that in this Sierpinski triangle has 3^{k+1} edges and $7{*}3^{k-2}$ edges of a maximum matching set.

In this, Sierpinski triangle has 3^{k+1} edges. It can be divided into two components where one has 3^k edges and other has 3 edges. Assumption of the case (ii) implies the first component of 3^k has $7{*}3^{k-3}$ edges of maximum matching and by the way of permutation method, the second component of three edges can be added to the first component. In this way, permutation implies $7{*}3^{k-2}$ edges of maximum matching cardinality. This theorem is proved for all n.

Various iterations of the Sierpinski triangle are shown in Fig. 7.

Note:

1. In each iteration of Sierpinski triangle, number of edges as well as a number of vertices can be increased.
2. Each iteration number of triangles has to be increased.
3. Way of permutation method is applied for the selection of edges in maximum matching.
4. The number of edges of the Sierpinski triangle consists of only power of 3 values.
5. The diameter of the Sierpinski triangle is increased in the upcoming iteration.
6. The structure of the Sierpinski triangle plays an essential role in designing the new structure of jewels.

5 Conclusion and Future Work

In this paper, maximum matching is applied in self-similarity fractal graph. It is deeply noted it implies a constant ratio between the maximum matching cardinality and the number of edges of each iteration of the fractal graph. The relation and constant ratio between maximum matching cardinality and the edges of the self-similarity fractal graph in the Koch snowflake and Sierpinski triangle are implemented by Theorems 1 and 2. This paper used the concept of coloring of graphs, domination of graphs, creation of new fair ornament, finding injured cells for injecting drugs in medical science, placing stones in jewels, plumping work, designing fractal antennas and so on. This concept is very useful for the architecture field, structures of the building, networking, finding influential people in a social network such as media, Facebook, Instagram, WhatsApp, Google Map and so on.

In the upcoming paper, maximal matching will be analyzed in various types of the graph as well as fractal graph. It will show the role of 'cardinality of maximal matching' in day-to-day real-life problems. The importance of matching in fractals can be explained in various engineering fields like computer science, electrical and biomedical. In the future work, the application of matching in fractals will be explained in medical filed, computer designing, coloring and domination.

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