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Research paper

Bipolar Fuzzy Soft Hyperideals and Homomorphism of Gamma-Hypersemigroups

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Abstract

In this paper, we introduce the concept of bipolar fuzzy soft gamma hyperideals in gamma hyper semigroups. We define bipolar fuzzy soft hyper ideals, bi-ideals and interior ideals of gamma hyper semigroups and discuss some properties.

Keywords: *Soft set, Γ- hyper semigroups, bipolar valued fuzzy set, hyper ideal, homomorphism.*

1. Introduction

Zadeh [18] introduced the concept of fuzzy sets in 1965. Algebraic hyper structures represent a natural extension of classical algebraic structures, and they were originally proposed in 1934 by Marty [7]. One of the main reasons which attract researchers towards hyperstructures is its unique property that in hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an element. Zhang [19] introduced the notion of bipolar fuzzy sets. Lee [4] used the term bipolar fuzzy sets as applied to algebraic structures. Bipolar fuzzy Γ-hyperideals in Γ-hyper semigroups was studied by Naveed Yaqoob et al [14]. Soft set theory was introduced by Molodtsov [8] in 1999, and its a new mathematical model for dealing with uncertainty from a parameterization point of view. Maji et al [6] studied the some new operations on fuzzy soft sets. Aygunoglu and Aygun [3] introduced the notion of a fuzzy soft group. The concept of bipolar fuzzy soft sets has been introduced by Naz et al [12]. Aslam et al [2] worked on bipolar fuzzy soft sets and their special union and intersection. Bipolar fuzzy soft Γ-semigroups was introduced by Muhammad Akram et al [10]. Γ-semigroups was introduced by Sen and Saha [16]. In this paper, we define a new notion of bipolar fuzzy soft Γ- hyper semigroups and investigate some of its properties with examples.

2. Preliminaries

In this section, we list some basic definitions.

Definition 2.1[16]
Let $S = \{a, b, c, \dots\}$ and $\{\alpha, \beta, \gamma, \dots\}$ be two non-empty sets. Then S is called a Γ-semigroup if it satisfies the conditions
(i) $a\alpha b \in S$,
(ii)$(a\beta b)\gamma c = a\beta(b\gamma c)$ $\forall$ $a, b, c \in S$ and $\alpha, \beta, \gamma \in \Gamma$.

Definition 2.2
A map $\circ : H \times H \to P^*(H)$ is called a hyper operation or join operation on the set H, where H is a non-empty set and $P^*(H) = P(H)\backslash\{\phi\}$ denotes the set of all non-empty subset of H. A hypergroupoid is a set H together with a (binary) hyperoperation.

Definition 2.3
A hypergroupoid $(H,\circ)$, which is associative, that is $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$, is called a hyper semigroup. Let A and B be two non-empty subsets of H. Then we define

$$A \circ B = \begin{cases} \bigcup_{a\in A, b\in B} a \circ b, & a \circ B = \{a\} \circ B \\ & A \circ b = A \circ \{b\} \end{cases}$$

Definition 2.4[1]
Let S and Γ be two non-empty sets. S is called a Γ-hypersemigroup if every $\gamma \in \Gamma$ is a hyperoperation on S that is $x\gamma y \subseteq S$ for every $x, y \in S$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in H$ we have $x\alpha(y\beta z) = (x\alpha y)\beta z$. If every $\gamma \in \Gamma$ is a hyper operation, then S is a Γ-semigroup. If (S, γ) is a hypergroup for every $\gamma \in \Gamma$, then S is called a Γ-hypergroup. Let A and B be two non-empty subsets of S and $\gamma \in \Gamma$. We define $A\gamma B = \cup\{a\gamma b | a \in A, b \in B\}$. Also $A\Gamma B = \cup\{a\gamma b | a \in A, b \in B \text{ and } \gamma\Gamma\} = \bigcup_{A\gamma B}$. Let S be a Γ-hypersemigroup and let $\gamma \in \Gamma$. A non-empty subset A of S is called a Γ-hypersubsemigroup of S if $a_1\gamma a_2 \subseteq A$ for every $a_1, a_2 \in A$. A Γ-semihypergroup S is called commutative if for all $x, y \in S$ and $\gamma \in \Gamma$ we have $x\gamma y = y\gamma x$.

Definition 2.5 [8] Let U be an universel set and E be the set of parameters. P(U) denote the power set of U. Let A be a non empty subset of E then the pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \to P(U)$.

Definition 2.6
[18] Let X be a non-empty set. A fuzzy subset μ of X is a function from X into the closed unit interval [0,1]. The set of all fuzzy subset of X is called the fuzzy power set of X and is denoted by FP(X).

Definition 2.7[4]
A bipolar fuzzy set A in a universe U is an object having the form $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$, where $\mu_A^+ : X \to [0,1]$ and $\mu_A^- : X \to [-1,0]$. Here $\mu_A^+(x)$represents the degree of satisfaction of an element x to the property and $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$ and $\mu_A^-(x)$ represents the degree of satisfaction of x to some implict counter property of A. For the simplicity the symbol $\langle \mu_A^+, \mu_A^- \rangle$ is used for the bipolar fuzzy set $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

Definition 2.8 [2]
Let U be the universe set and E be the set of parameter. Let $A \subseteq E$ and BF^U denotes the set of all bipolar fuzzy subsets of U. Then a pair (F, A) is called a bipolar fuzzy soft sets over U, where F is a mapping given by $F: A \to BF^U$.
It is defined as $(F, A) = \{\langle x, \mu_a^+(x), \mu_a^-(x)\rangle : x \in U \text{and } a \in A\}$ For any

$$a \in A, F(a) = \{\langle x, \mu_{F(a)}^+(x), \mu_{F(a)}^-(x)\rangle : x \in U\} = \langle \mu_{F(a)}^+(x), \mu_{F(a)}^-(x)\rangle.$$

Definition 2.9 [2]
Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U, then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ is defined as $(F, A) \wedge (G, B) = (H, C)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in A \times B$.

Definition 2.10 1[2]
Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U, then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$ is defined as $(F, A) \vee (G, B) = (H, C)$ where $C = A \times B$ and $H(a, b) = F(a) \cup G(b)$, for all $(a, b) \in A \times B$.

Definition 2.11 [2]
Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U then their extended union is a bipolar fuzzy soft set over U denoted by $(F, A) \cup_\varepsilon (G, B)$ and is defined by $(F, A) \cup_\varepsilon (G, B) = (H, C)$ where $C = A \cup B$, $H: C \to BF^U$ and

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cup G(c) & \text{if } c \in A \cap B \end{cases}$$

Definition 2.12 [2]
Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U then their extended intersection is a bipolar fuzzy soft set over U denoted by $(F, A) \cap_\varepsilon (G, B)$ and is defined by $(F, A) \cup_\varepsilon (G, B) = (H, C)$ where $C = A \cup B$, $H: C \to BF^U$ and

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cap G(c) & \text{if } c \in A \cap B \end{cases}$$

Definition 2.13[13]
Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$. The restricted union of (F, A) and (G, B) is defined to be a bipolar fuzzy soft set (H, C) over U where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$, for all $c \in C$. This is denoted by $(H, C) = (F, A) \cup_R (G, B)$.

Definition 2.14 [11]
Let (F, A) and (G, B) be two bipolar fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$. The restricted intersection of (F, A) and (G, B) is defined to be a bipolar fuzzy soft set (H, C) over U where $C = A \cap B$ and $H(c) = F(c) \cap G(c)$, for all $c \in C$. This is denoted by $(H, C) = (F, A) \cap_R (G, B)$.

Definition 2.15 [9]
Let (F, A) be a bipolar fuzzy soft set over U for each $t \in [0,1]$ and $s \in [-1,0]$ the set $(F, A)^{(t,s)} = (F^{(t,s)}, A)$ where $(F, A)_a^{(t,s)} = \{x \in U | \mu_{F(a)}^P(x) \geq t, \mu_{F(a)}^N(x) \leq s\}$ for all $a \in A$.

Definition 2.16[17]
Let $\phi: H_1 \to H_2$ and $h: E_1 \to E_2$ be two maps, $A \subseteq E_1$ and $B \subseteq E_2$, where E_1 and E_2 are sets of parameters viewed on H_1 and H_2, respectively. The pair (ϕ, h) is called a fuzzy soft map from H_1 to H_2. If ϕ is a hypergroup homomorphism, then (ϕ, h) is called a fuzzy soft homomorphism from H_1 to H_2.

Definition 2.17 [3]
Let (f, A) and (g, B) be two fuzzy soft sets over H_1 and H_2, respectively, and (ϕ, h) be a fuzzy soft function from H_1 to H_2
(i) The image of (f, A) under the soft function (ϕ, h) denoted by $(\phi, h)(f, A)$, is a fuzzy soft set over H_2 defined by $(\phi, h)(f, A) = (\phi(f), h(A))$, where for all $b \in h(A)$ and for all $y \in H_2$, then

$$\Phi(f)_b(y) = \begin{cases} \bigvee_{\phi(x)=y} \bigvee_{h(a)=b} f_a(x), & \text{if} x \in \phi^{-1}(y) \\ 0 & \text{otherwise} \end{cases}$$

(ii) The inverse image of (g, B) under the fuzzy soft function (ϕ, h) denoted by $(\phi, h)^{-1}(g, B)$, is a fuzzy soft set over B defined by $(\phi, h)^{-1}(g, B) = (\phi^{-1}(g), h^{-1}(A))$, where for all $a \in h^{-1}(A)$ and for all $x \in H_1$, $\phi^{-1}(g)_a(x) = g_{h(a)}(\phi(x))$. If ϕ and h is injective(surjective), then (ϕ, h) is said to be injective (surjective).

Definition 2.18 [15]
Let (ϕ, ψ) be a fuzzy soft Γ-function from X to Y. If ϕ is a homomorphism function from X to Y, then (ϕ, ψ) is said to be fuzzy soft Γ-homomorphism. If ϕ is isomorphism function from X to Y and ψ is one to one mapping from N to M, then (ϕ, ψ) is said to be fuzzy soft Γ-isomorphism.

3.Bipolar Fuzzy Soft Γ- Hyper Ideals

In this section, we introduce the notion of bipolar fuzzy soft gamma hyperideals in gamma semigroups and discuss some of its properties S denotes the Γ- hyper semigroup.

Definition 3.1
A bipolar fuzzy soft set (F, A) over a Γ- hypersemigroups S is called a bipolar fuzzy soft Γ-subhypersemigroup over S if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \min\{\mu_{F(a)}^+(y), \mu_{F(a)}^+(z)\}$
(ii) $\sup_{x \in y\gamma z} \mu_{F(a)}^-(x) \leq \max\{\mu_{F(a)}^-(y), \mu_{F(a)}^-(z)\}$ for all $x, y, z \in S$, $\gamma \in \Gamma$ and $a \in A$.

Definition 3.2
A bipolar fuzzy soft set (F, A) over a Γ- hypersemigroups S is called a bipolar fuzzy soft left Γ-hyperideal over S if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \mu_{F(a)}^+(z)$
(ii) $\sup_{x \in y\gamma z} \mu_{F(a)}^-(x) \leq \mu_{F(a)}^-(z)$ for all $x, y, z \in S$, $\gamma \in \Gamma$and $a \in A$.

Definition 3.3A bipolar fuzzy soft set (F, A) over a Γ-hypersemigroups S is called a bipolar fuzzy soft right Γ-hyperideal over S if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \mu_{F(a)}^+(y)$
(ii) $\sup_{x \in y\gamma z} \mu_{F(a)}^-(x) \leq \mu_{F(a)}^-(y)$ for all $x, y, z \in S$, $\gamma \in \Gamma$ and $a \in A$.

Definition 3.4
A bipolar fuzzy soft set (F, A) over a Γ- hypersemigroups S is called a bipolar fuzzy soft Γ-hyperideal of S if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \max\{\mu_{F(a)}^+(y), \mu_{F(a)}^+(z)\}$

(ii) $\sup_{x \in y\gamma z} \mu^{-}_{F(a)}(x) \leq \min\{\mu^{-}_{F(a)}(y), \mu^{-}_{F(a)}(z)\}$ for all $x, y, z \in S$, $\gamma \in \Gamma$ and $a \in A$.

Definition 3.5

A bipolar fuzzy soft set (F, A) over a Γ- hypersemigroups S is called a bipolar fuzzy soft Γ- hyperbi-ideal over S if

(i) $\inf_{p \in x\alpha y\beta z} \mu^{+}_{F(a)}(p) \geq \min\{\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(z)\}$

(ii) $\sup_{p \in x\alpha y\beta z} \mu^{-}_{F(a)}(p) \leq \max\{\mu^{-}_{F(a)}(x), \mu^{-}_{F(a)}(z)\}$ for all $x, y, z \in S$, $\alpha, \beta \in \Gamma$ and $a \in A$.

Definition 3.6

A bipolar fuzzy soft set (F, A) over a Γ- hypersemigroups S is called a bipolar fuzzy soft Γ-hyperinterior ideal over S if

(i) $\inf_{p \in x\alpha y\beta z} \mu^{+}_{F(a)}(p) \geq \mu^{+}_{F(a)}(y)$

(ii) $\sup_{p \in x\alpha y\beta z} \mu^{-}_{F(a)}(p) \leq \mu^{-}_{F(a)}(y)$ for all $x, y, z \in S$, $\alpha, \beta \in \Gamma$ and $a \in A$.

Theorem 3.7

Let (F, A) and (G, B) be two bipolar fuzzy soft Γ-hypersubsemigroups over S, then $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ are bipolar fuzzy soft Γ- hypersubsemigroup of S.

Proof. Let (F, A) and (G, B) be two bipolar fuzzy soft Γ-hypersubsemigroups over S defined as $(F, A) \wedge (G, B)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in C = A \times B$, $x, y, z \in S\gamma \in \Gamma$

$$\inf_{z \in x\gamma y}\{\mu^{+}_{H(a,b)}(z)\} = \inf_{z \in x\gamma y}\{\min\{\mu^{+}_{F(a)}(z), \mu^{+}_{G(b)}(z)\}\}$$
$$= \min\{\inf_{z \in x\gamma y} \mu^{+}_{F(a)}(z), \inf_{z \in x\gamma y} \mu^{+}_{G(b)}(z)\}$$
$$\geq \min\{\min\{\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(y)\}, \min\{\mu^{+}_{G(b)}(x), \mu^{+}_{G(b)}(y)\}\}$$
$$= \min\{\{\min\{\mu^{+}_{F(a)}(x), \mu^{+}_{G(b)}(x)\}, \min\{\mu^{+}_{F(a)}(q), \mu^{+}_{G(b)}(q)\}\}$$
$$= \min\{(\mu^{+}_{F(a)} \cap \mu^{+}_{G(b)})(x), (\mu^{+}_{F(a)} \cap \mu^{+}_{G(b)})(y)\}$$
$$= \min\{\mu^{+}_{H(a,b)}(x), \mu^{+}_{H(a,b)}(y)\}.$$

and

$$\sup_{z \in x\gamma y}\{\mu^{-}_{H(a,b)}(z)\} = \sup_{z \in x\gamma y}\{\max\{\mu^{-}_{F(a)}(z), \mu^{-}_{G(b)}(z)\}\}$$
$$= \max\{\sup_{z \in x\gamma y} \mu^{+}_{F(a)}(z), \inf_{z \in x\gamma y} \mu^{+}_{G(b)}(z)\}$$
$$\geq \max\{\max\{\mu^{-}_{F(a)}(x), \mu^{-}_{F(a)}(y)\}, \min\{\mu^{-}_{G(b)}(x), \mu^{-}_{G(b)}(y)\}\}$$
$$= \max\{\{\max\{\mu^{+}_{F(a)}(x), \mu^{+}_{G(b)}(x)\}, \max\{\mu^{-}_{F(a)}(y), \mu^{-}_{G(b)}(y)\}\}$$
$$= \max\{(\mu^{-}_{F(a)} \cup \mu^{-}_{G(b)})(x), (\mu^{-}_{F(a)} \cup \mu^{-}_{G(b)})(y)\}$$
$$= \max\{\mu^{-}_{H(a,b)}(x), \mu^{-}_{H(a,b)}(y)\}.$$

Hence $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft Γ-hypersubsemigroup over S.Similarly it can be shown that $(F, A) \vee (G, B)$ are bipolar fuzzy soft Γ-hypersub semigroup over S.

Theorem 3.8

Let (F, A) and (G, B) be two bipolar fuzzy soft Γ-hyperleft (resp.right) ideals over S, then $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ are bipolar fuzzy soft Γ-hyperleft (resp.right) ideals of S.

Proof. Let (F, A) and (G, B) be two bipolar fuzzy soft Γ -hyperleftideals over S defined as $(F, A) \wedge (G, B)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in C = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\inf_{z \in x\gamma y}\{\mu^{+}_{H(a,b)}(z)\} = \inf_{z \in x\gamma y}\{\min\{\mu^{+}_{F(a)}(z), \mu^{+}_{G(b)}(z)\}\}$$
$$= \min\{\inf_{z \in x\gamma y} \mu^{+}_{F(a)}(z), \inf_{z \in x\gamma y} \mu^{+}_{G(b)}(z)\}$$
$$= \min\{\mu^{+}_{F(a)}(y), \mu^{+}_{G(b)}(y)\}$$
$$= \mu^{+}_{H(a,b)}(y)\}.$$

and

$$\sup_{z \in x\gamma y}\{\mu^{-}_{H(a,b)}(z)\} = \sup_{z \in x\gamma y}\{\max\{\mu^{-}_{F(a)}(z), \mu^{-}_{G(b)}(z)\}\}$$
$$= \max\{\sup_{z \in x\gamma y} \mu^{-}_{F(a)}(z), \inf_{z \in x\gamma y} \mu^{-}_{G(b)}(z)\}$$
$$\leq \max\{\mu^{-}_{F(a)}(y), \mu^{-}_{G(b)}(y)\}$$
$$= \mu^{-}_{H(a,b)}(y)$$

Hence $(F, A) \wedge (G, B)$ are bipolar fuzzy soft Γ-left (resp.right) hyperideals over S.

Similar proof shows that $(F, A) \vee (G, B)$ is a bipolar fuzzy soft Γ-left (resp.right) hyperideals over S.

Theorem 3.9

Let (F, A) and (G, B) be two bipolar fuzzy soft Γ-hyperbi-ideals over S, then $(F, A) \wedge (G, B)$and $(F, A) \vee (G, B)$ are bipolar fuzzy soft Γ-hyperbi-ideals of S.

Proof. Let (F, A) and (G, B) be two bipolar fuzzy soft Γ -hypersemigroups over S defined as $(F, A) \wedge (G, B)$ where $C = A \times B$, $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in C = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\inf_{z \in x\alpha y\beta z}\{\mu^{+}_{H(a,b)}(z)\} = \inf_{z \in x\gamma y}\{\min\{\mu^{+}_{F(a)}(z), \mu^{+}_{G(b)}(z)\}\}$$
$$= \min\{\inf_{z \in x\gamma y} \mu^{+}_{F(a)}(z), \inf_{z \in x\gamma y} \mu^{+}_{G(b)}(z)\}$$
$$\geq \min\{\min\{\mu^{+}_{F(a)}(x), \mu^{+}_{F(a)}(z)\}, \min\{\mu^{+}_{G(b)}(x), \mu^{+}_{G(b)}(z)\}\}$$
$$= \min\{\{\min\{\mu^{+}_{F(a)}(x), \mu^{+}_{G(b)}(x)\}, \min\{\mu^{+}_{F(a)}(z), \mu^{+}_{G(b)}(z)\}\}$$
$$= \min\{(\mu^{+}_{F(a)} \cap \mu^{+}_{G(b)})(x), (\mu^{+}_{F(a)} \cap \mu^{+}_{G(b)})(z)\}$$
$$= \min\{\mu^{+}_{H(a,b)}(x), \mu^{+}_{H(a,b)}(z)\}.$$

and

$$\sup_{z \in x\alpha y\beta z}\{\mu^{-}_{H(a,b)}(z)\} = \sup_{z \in x\gamma y}\{\max\{\mu^{-}_{F(a)}(z), \mu^{-}_{G(b)}(z)\}\}$$
$$= \max\{\sup_{z \in x\gamma y} \mu^{+}_{F(a)}(z), \inf_{z \in x\gamma y} \mu^{+}_{G(b)}(z)\}$$
$$\leq \max\{\max\{\mu^{-}_{F(a)}(x), \mu^{-}_{F(a)}(z)\}, \min\{\mu^{-}_{G(b)}(x), \mu^{-}_{G(b)}(z)\}\}$$
$$= \max\{\{\max\{\mu^{-}_{F(a)}(x), \mu^{-}_{G(b)}(x)\}, \max\{\mu^{-}_{F(a)}(z), \mu^{-}_{G(b)}(z)\}\}$$
$$= \max\{(\mu^{-}_{F(a)} \cup \mu^{-}_{G(b)})(x), (\mu^{-}_{F(a)} \cup \mu^{-}_{G(b)})(z)\}$$
$$= \max\{\mu^{-}_{H(a,b)}(x), \mu^{-}_{H(a,b)}(z)\}.$$

Hence $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft Γ-hyperbi-ideal over S.

It can be similarly proved that $(F, A) \vee (G, B)$ is a bipolar fuzzy soft Γ-hyperbi-ideal over S.

Theorem 3.10

Let (F, A) and (G, B) be two bipolar fuzzy soft Γ-hypersubsemigroups over S, then $(F, A) \cap_{\varepsilon} (G, B)$ is a bipolar fuzzy soft Γ-hypersubsemigroups of S.

Proof. Let (F, A) and (G, B) be two bipolar fuzzy soft Γ-hypersubsemigroups over S as defined

$(F, A) \cap_{\varepsilon} (G, B) = (H, C)$ where $C = A \cup B$

$H(c) = (F(c)$

$$H(c) = \begin{cases} F(c) & if \quad c \in A \setminus B \\ G(c) & if \quad c \in B \setminus A \\ F(c) \cap G(c) & if \quad c \in A \cap B \end{cases}$$

Case(i) $c \in A \backslash B$ and $\gamma \in \Gamma$

$$\inf_{z \in x\gamma y}\{\mu^{+}_{H(c)}(z)\} = \inf_{z \in x\gamma y} \mu^{+}_{F(c)}(z)$$
$$\geq \min\{\mu^{+}_{F(c)}(x), \mu^{+}_{F(c)}(y)\}$$
$$= \min\{\mu^{+}_{H(c)}(x), \mu^{+}_{H(c)}(y)\}$$

and

$$\sup_{z \in x\gamma y}\{\mu^{-}_{H(c)}(z)\} = \sup_{z \in x\gamma y} \mu^{-}_{F(c)}(z)$$
$$\leq \max\{\mu^{-}_{F(c)}(x), \mu^{-}_{F(c)}(y)\}$$
$$= \max\{\mu^{-}_{H(c)}(x), \mu^{-}_{H(c)}(y)\}$$

Case(ii) $c \in B \backslash A$ and $\gamma \in \Gamma$.

$$\inf_{z\in x\gamma y}\{\mu^{+}_{H(c)}(z)\} = \inf_{z\in x\gamma y}\mu^{+}_{G(c)}(z)$$
$$\geq \min\{\mu^{+}_{G(c)}(x),\mu^{+}_{G(c)}(y)\}$$
$$= \min\{\mu^{+}_{H(c)}(x),\mu^{+}_{H(c)}(y)\}$$
and
$$\sup_{z\in x\gamma y}\{\mu^{-}_{H(c)}(z)\} = \sup_{z\in x\gamma y}\mu^{-}_{G(c)}(z)$$
$$\leq \max\{\mu^{-}_{G(c)}(x),\mu^{-}_{G(c)}(y)\}$$
$$= \max\{\mu^{-}_{H(c)}(x),\mu^{-}_{H(c)}(y)\}$$
Case (iii) $C\in A\cap B$ and $\gamma\in\Gamma$ then $H(c)=F(c)\cap G(c)$ then by theorem 3.7,
$$\inf_{z\in x\gamma y}\{\mu^{+}_{H(c)}(z)\}\geq \inf_{z\in x\gamma y}\{\mu^{+}_{H(c)}(x),\mu^{+}_{H(c)}(y)\}$$
$$=\min\{\mu^{+}_{H(c)}(x),\mu^{+}_{H(c)}(y)\},$$
and
$$\sup_{z\in x\gamma y}\{\mu^{-}_{H(c)}(z)\}\leq \sup_{z\in x\gamma y}\{\mu^{-}_{H(c)}(x),\mu^{-}_{H(c)}(y)\}$$
$$=\max\{\mu^{-}_{H(c)}(x),\mu^{-}_{H(c)}(y)\}.$$
Hence $(F,A)\cap_{\varepsilon}(G,B)$ is a bipolar fuzzy soft Γ-hypersubsemigroup over S.

Theorem3.11
Let (F,A) and (G,B) be two bipolar fuzzy soft Γ-hypersubsemigroup over S, then $(F,A)\cup_{\varepsilon}(G,B)$ is a bipolar fuzzy soft Γ-hypersubsemigroup of S.
Proof. Straight forward.

Theorem 3.12
Let (F,A) and (G,B) be two bipolar fuzzy soft Γ-hyperbi(interior) ideal over S, then $(F,A)\cap_{\varepsilon}(G,B)$ is a bipolar fuzzy soft Γ-hyperbi(interior) ideal of S.
Proof. Straight forward.

Theorem 3.13
Let (F,A) and (G,B) be two bipolar fuzzy soft-Γ-hyper bi(interior) ideal over S, then $(F,A)\cup_{\varepsilon}(G,B)$ is a bipolar fuzzy soft Γ- hyperbi(interior) ideal of S.
Proof. Straight forward.

Theorem 3.14
Let (F,A) and (G,B) be two bipolar fuzzy soft Γ-hypersubsemigroup over S, then $(F,A)\cap_{R}(G,B)$ is a bipolar fuzzy soft Γ- hypersubsemigroup of S.
Proof. Let (F,A) and (G,B) be two bipolar fuzzy soft Γ-hypersubsemigroup over S, then $(F,A)\cap_{R}(G,B)=(H,C)$ where $C=A\cap B$ and $H(c)=F(c)\cap G(c)$ for all $c\in C$.
$$\inf_{z\in x\gamma y}\mu^{+}_{H(c)}(z) = \inf_{z\in x\gamma y}\{\min\{\mu^{+}_{F(c)}(z),\mu^{+}_{G(c)}(z)\}\}$$
$$= \min\{\inf_{z\in x\gamma y}\mu^{+}_{F(c)}(z),\inf_{z\in x\gamma y}\mu^{+}_{G(c)}(z)\}$$
$$\geq \min\{\min\{\mu^{+}_{F(c)}(x),\mu^{+}_{F(c)}(y)\},\min\{\mu^{+}_{G(c)}(x),\mu^{+}_{G(c)}(y)\}\}$$
$$= \min\{\min\{\mu^{+}_{F(c)}(x),\mu^{+}_{G(c)}(x)\},\min\{\mu^{+}_{F(c)}(x),\mu^{+}_{G(c)}(x)\}\}$$
$$= \min\{(\mu^{+}_{F(c)}\cap\mu^{+}_{G(c)})(x),(\mu^{+}_{F(c)}\cap\mu^{+}_{G(c)})(y)\}$$
$$= \min\{\mu^{+}_{H(c)}(x),\mu^{+}_{H(c)}(y)\}.$$
and
$$\sup_{z\in x\gamma y}\mu^{-}_{H(c)}(z) = \sup_{z\in x\gamma y}\{\max\{\mu^{-}_{F(c)}(z),\mu^{-}_{G(c)}(z)\}\}$$
$$= \max\{\sup_{z\in x\gamma y}\mu^{-}_{F(c)}(z),\sup_{z\in x\gamma y}\mu^{-}_{G(c)}(z)\}$$
$$\leq \max\{\max\{\mu^{-}_{F(c)}(x),\mu^{-}_{F(c)}(y)\},\max\{\mu^{-}_{G(c)}(x),\mu^{-}_{G(c)}(y)\}\}$$
$$= \max\{\max\{\mu^{-}_{F(c)}(x),\mu^{-}_{G(c)}(x)\},\max\{\mu^{-}_{F(c)}(y),\mu^{-}_{G(c)}(y)\}\}$$
$$= \max\{(\mu^{-}_{F(c)}\cap\mu^{-}_{G(c)})(x),(\mu^{-}_{F(c)}\cap\mu^{-}_{G(c)})(y)\}$$
$$= \max\{\mu^{-}_{H(c)}(x),\mu^{-}_{H(c)}(y)\}.$$
Hence $(F,A)\cap_{R}(G,B)$ is a bipolar fuzzy soft Γ-hypersubsemigroup of S.

Theorem 3.15
Let (F,A) and (G,B) be two bipolar fuzzy soft Γ-hypersubsemigroup over S, then $(F,A)\cup_{R}(G,B)$ is a bipolar fuzzy soft –hypersub semigroup of S.
Proof. Straight forward.

Theorem 3.16
Let (F,A) and (G,B) be two bipolar fuzzy soft Γ-hyperbi(interior)ideal over S, then $(F,A)\cap_{R}(G,B)$ is a bipolar fuzzy soft Γ-hyper bi(interior)ideal of S.
Proof. Straight forward.

Theorem 3.17
Let (F,A) and (G,B) be two bipolar fuzzy soft Γ-hyperbi(interior)ideal over S, then $(F,A)\cup_{R}(G,B)$ is a bipolar fuzzy soft Γ- hyperbi(interior)ideal of S.
Proof. Straight forward.

Example 3.18
Every bipolar fuzzy soft Γ-hyper ideal is bipolar valued fuzzy soft Γ −hypersubsemigroups but converse is not true.
Let $S=\{a,b,c,d,e\}$ and $\Gamma=\{\gamma\}$ then S is Γ-semihypergroup

γ	a	b	c	d	e
a	{a, b}	{b, e}	c	{c, d }	e
b	{b, e }	e	c	{c, d }	e
c	c	c	c	c	c
d	{c, d }	{c,d }	c	d	{c, d}
e	e	e	c	{c, d }	e

Let $E=\{u,v,w,x,y\}$ and $A=\{u,v,y\}$. Define the bipolar fuzzy soft set (F,A) as
$(F,A)=\{F(u),F(v),F(y)\}$, where
$F(u)=\{(a,0.6,-0.5),(b,0.7,-0.6),(c,0.4,-0.2),(d,0.3,-0.1),(e,0.9,-0.8)\}$
$F(v)=\{(a,0.8,-0.4),(b,0.9,-0.7),(c,0.6,-0.3),(d,0.2,-0.1),(e,1,-0.9)\}$
$F(y)=\{(a,0.7,-0.8),(b,0.8,-0.9),(c,0.5,-0.4),(d,0.2,-0.3),(e,1,-0.9)\}$
Hence (F,A) is a bipolar fuzzy soft sub Γ- hypersemigroups but not bipolar valued fuzzy hyperideal. Sinc $\inf_{a\in a\gamma c}\mu^{+}_{F(a)}(a)\geq \max\{\mu^{+}_{F(a)}(a),\mu^{+}_{F(a)}(c)\}$
$$=0.4\ngeq 0.6$$

Example 3.19
Every bipolar fuzzy soft Γ-hyperideal is bipolar valued fuzzy soft Γ hyper bi-ideals but converse is not true.
Let $S=\{a,b,c,d,e\}$ and $\Gamma=\{\alpha,\beta\}$ then S is Γ-hypersemigroup

α	a	b	c	d	e
a	{a, b}	{b, e}	c	{c, d }	e
b	{b, e}	e	c	{c, d }	e
c	c	c	c	c	c
d	{c, d}	{c,d}	c	d	{c, d}
e	e	e	c	{c, d }	e

β	a	b	c	d	e
a	{b, e}	e	c	{c, d}	e
b	e	e	c	{c, d}	e
c	c	c	c	c	c
d	{c, d}	{c,d}	c	d	{c, d}
e	e	e	c	{c, d}	e

Let $E=\{u,v,w,x,y\}$ and $A=\{w,x,y\}$. Define the bipolar fuzzy soft set (F,A) as
$(F,A)=\{F(w),F(x),F(y)\}$, where
$F(w)=\{(a,0.2,-0.1),(b,0.4,-0.3),(c,1,-0.9),(d,0.6,-0.7),(e,0.7,-0.8)\}$
$F(x)=\{(a,0.1,-0.2),(b,0.2,-0.3),(c,0.7,-0.8),(d,0.4,-0.5),(e,0.5,-0.6)\}$

$F(y) = \{(a, 0.3, -0.1), (b, 0.4, -0.2), (c, 0.9, -0.7), (d, 0.6, -0.3), (e, 0.8, -0.5)\}$

Hence (F, A) is a bipolar fuzzy soft Γ-hyperbi-ideal but not bipolar valued fuzzy hyper ideal,

Since $\inf_{a \in d\alpha e} \mu^{+}_{F(a)}(a) \geq \max\{\mu^{+}_{F(a)}(d), \mu^{+}_{F(a)}(e)\}$
$= 0.6 \ngeq 0.7$.

Example 3.20

Every bipolar fuzzy soft Γ-hyperideal is bipolar valued fuzzy soft $\Gamma -$ hyper intrior-ideal but converse is not true.

For the example 3.19, define the bipolar fuzzy soft set (F, A) as $(F, A) = \{F(w), F(x), F(y)\}$, where

$F(w) = \{(a, 0.3, -0.2), (b, 0.6, -0.5), (c, 0.9, -0.8), (d, 0.2, -0.1), (e, 0.8, -0.7)\}$

$F(x) = \{(a, 0.4, -0.3), (b, 0.5, -0.4), (c, 0.8, -0.7), (d, 0.3, -0.1), (e, 0.6, -0.5)\}$

$F(y) = \{(a, 0.3, -0.2), (b, 0.4, -0.5), (c, 0.7, -0.9), (d, 0.2, -0.1), (e, 0.5, -0.8)\}$

Hence (F, A) is a bipolar fuzzy soft Γ-hyperinteriorideal but not bipolar valued fuzzy soft Γ-hyperideal, as $\inf_{a \in b\alpha d} \mu^{+}_{F(a)}(a) \geq \max\{\mu^{+}_{F(a)}(b), \mu^{+}_{F(a)}(d)\} = 0.2 \ngeq 0.6$.

Theorem 3.21

Let (F, A) be a bipolar fuzzy soft set over S. (F, A) is a bipolar fuzzy soft Γ-hypersemigroup if and only if $(F, A)^{(t,s)}$ is a soft Γ-hypersemigroup of S for each $t \in [0,1]$ and $s \in [-1,0]$.

Proof. Assume that $(F, A)^{(t,s)}$ is a bipolar soft Γ-hypersemigroup over S for each $t \in [0,1]$ and $s \in [-1,0]$.For each $x_1, x_2 \in S$ and $a \in A$, let $t = \min\{\mu^{+}_{F(a)}(x_1), \mu^{+}_{F(a)}(x_2$ and $s = \max\{\mu^{-}_{F(a)}(x_1), \mu^{-}_{F(a)}(x_2)\}$, then $x_1, x_2 \in \mu^{(t,s)}_{F(a)}$. Since $\mu^{(t,s)}_{F(a)}$ is a Γ- hypersubsemigroup of S, then $x_1, x_2 \in \mu^{(t,s)}_{F(a)}$. That is $\mu^{+}_{F(a)}(x_1 \gamma x_2) \geq t = \min\{\mu^{+}_{F(a)}(x_1), \mu^{+}_{F(a)}(x_2)\}$ and $\mu^{-}_{F(a)}(x_1 \gamma x_2) \leq s = \max\{\mu^{-}_{F(a)}(x_1), \mu^{-}_{F(a)}(x_2)\}$. This shows that $\mu_{F(a)}$ is bipolar fuzzy Γ-hypersubsemigroup over S. Thus (F, A) is a bipolar fuzzy soft Γ-hypersemigroup over S.

Conversely, assume that (F, A) is a bipolar fuzzy soft Γ-hypersemigroup. For each $a \in A, t \in [0,1]$ and $s \in [-1,0]$ and $x_1, x_2 \in \mu^{(t,s)}_{F(a)}$.

we have $\mu^{+}_{F(a)}(x_1) \geq t, \mu^{+}_{F(a)}(x_2) \geq t$ and $\mu^{-}_{F(a)}(x_1) \leq s$, $\mu^{-}_{F(a)}(x_2) \leq s$. Therefore $\mu_{F(a)}$ is a bipolar fuzzy Γ-hypersubsemigroup of S. Thus $\gamma \in \Gamma$ there exists $z \in x_1 \gamma x_2$ such that

$\inf_{z \in x_1 \gamma x_2} (z) \geq \min\{\mu^{+}_{F(a)}(x_1), \mu^{+}_{F(a)}(x_2)\} \geq t$ and $\sup_{z \in x_1 \gamma x_2} (z) \leq \max\{\mu^{-}_{F(a)}(x_1), \mu^{-}_{F(a)}(x_2)\} \leq s$. Therefore for all $z \in x_1 \gamma x_2$ we have $z \in \mu^{(t,s)}_{F(a)}$, this implies that $x_1 \gamma x_2 \in \mu^{(t,s)}_{F(a)}$, that is $\mu^{(t,s)}_{F(a)}$ is hyper Γ-subsemigroup of S. Therefore $(F, A)^{(t,s)}$ is a soft Γ-hypersemigroup of S for each $t \in [0,1]$ and $s \in [-1,0]$.

Theorem 3.22

Let (F, A) be a bipolar fuzzy soft set over S. (F, A) is a bipolar fuzzy soft Γ-hyperleft(right)ideal if and only if $(F, A)^{(t,s)}$ is a soft Γ-hyper left(right) ideal of S for each $t \in [0,1]$ and $s \in [-1,0]$.

Proof. Suppose that $(F, A)^{(t,s)}$ is a bipolar soft Γ-hyperleftideal of S for each $t \in [0,1]$, $s \in [-1,0]$ and $a \in A, \gamma \in \Gamma$. For each $x_1 \in S$, let $t = \mu^{+}_{F(a)}(x_1)$, then $x_1 \in \mu^{(t,s)}_{F(a)}$. Since $\mu^{(t,s)}_{F(a)}$ is a Γ-hyper left ideal of S, then $x\gamma x_1 \in \mu^{(t,s)}_{F(a)}$, for each $x \in S$. Hence $\mu^{+}_{F(a)}(x\gamma x_1) \geq t = \mu^{+}_{F(a)}(x_1)$ and $\mu^{-}_{F(a)}(x\gamma x_1) \leq s = \mu^{-}_{F(a)}(x_1)$. This shows that $\mu_{F(a)}$ is bipolar fuzzy Γ-hyperleftideal of S. By definition 3.2,(F, A) is a bipolar fuzzy soft Γ-hyperleftideal of S.

Conversely, assume that (F, A) is a bipolar fuzzy soft Γ-hyper left ideal of S. For each $a \in A, t \in [0,1]$ and $s \in [-1,0]$ and $x_1 \in \mu^{(t,s)}_{F(a)}$ we have $\mu^{+}_{F(a)}(x_1) \geq t$, and $\mu^{-}_{F(a)}(x_1) \leq s$ and by definition 3.2, $\mu^{+}_{F(a)}$ and $\mu^{-}_{F(a)}$ is a bipolar fuzzy Γ-hyper left ideal of S. Thus for $\gamma \in \Gamma$ there exists $z \in x\gamma x_1$ such that $\inf_{z \in x\gamma x_1} (z) \geq \mu^{+}_{F(a)}(x_1) \geq t$ and $\sup_{z \in x\gamma x_1} (z) \leq \mu^{-}_{F(a)}(x_1) \leq s$. Therefore for all $z \in x\gamma x_1$ we have $z \in \mu^{(t,s)}_{F(a)}$, that is $\mu^{(t,s)}_{F(a)}$ is hyper Γ-left ideal of S. Therefore $(F, A)^{(t,s)}$ is a soft Γ-hyper left ideal of S for each $t \in [0,1]$ and $s \in [-1,0]$. Similar proof holds for right ideal also.

Theorem3.23

Let (F, A) be a bipolar fuzzy soft set over S, (F, A) is a bipolar fuzzy soft Γ-hyperideal if and only if $(F, A)^{(t,s)}$ is a soft Γ-hyperideal of S for each $t \in [0,1]$ and $s \in [-1,0]$.

Proof. The proof follows from theroem 3.22

4.Bipolar Fuzzy Soft Image and Inverse Image of Hyper Γ-Semigroups

Definition 4.1

[9] Let $\eta: H_1 \rightarrow H_2$ and $\psi: A \rightarrow B$ be two functions, A and B be two parametric sets from the crisp sets H_1 and H_2,respectively.Then the pair (η, ψ) is called a bipolar fuzzy soft function from H_1 to H_1.

Definition 4.2

Let (F, A) and (G, B) be two bipolar fuzzy soft sets over the sets H_1 and H_2, respectively, and (η, ψ) be a bipolar fuzzy soft map from H_1 to H_2.

(i) The image of (F, A) under (η, ψ) denoted by $(\eta, \psi)(F, A)$, is a bipolar fuzzy soft set over H_2 defined by $(\eta, \psi)(F, A) = (\eta(F), \psi(A))$, where for all $b \in \psi(A)$ and for all $y \in H_2$,

$$\mu^{+}_{\eta_{F(b)}}(y) = \begin{cases} \sup\limits_{\eta(x)=y} \sup\limits_{\psi(a)=b} \mu^{+}_{F(a)}(x), & \text{if} \eta^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

$$\mu^{-}_{\eta_{F(b)}}(y) = \begin{cases} \inf\limits_{\eta(x)=y} \inf\limits_{\psi(a)=b} \mu^{-}_{F(a)}(x), & \text{if} \eta^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

(ii) The inverse image of (G, B) under (η, ψ) denoted by $(\eta, \psi)^{-1}(G, B)$, is a bipolar fuzzy soft set over H_1 defined by $(\eta, \psi)^{-1}(G, B) = (\eta^{-1}(G), \psi^{-1}(B))$, where for all $a \in \psi^{-1}(B)$ and for all $x \in H_1$,$\mu^{+}_{\eta^{-1}_{G(a)}}(y) = \mu^{+}_{G_{\psi(a)}}(\eta(x))$ and $\mu^{-}_{\eta^{-1}_{G(a)}}(y) = \mu^{-}_{G_{\psi(a)}}(\eta(x))$

Theorem 4.3

Let $\eta: H_1 \rightarrow H_2$ be a homomorphism of S. If (G, B) is a bipolar fuzzy soft Γ-hypersubsemigroup of H_2, then $(\eta, \psi)^{-1}(G, B)$ is a bipolar fuzzy soft Γ-hypersubsemigroup of H_1.

Proof. Let (G, B) is a bipolar fuzzy soft Γ-hypersubsemigroup of H_2. Let $x, y, z \in H_1, \gamma \in \Gamma_1$ then we have

$$\begin{aligned} \inf_{z \in x\gamma y} \left\{\mu^{+}_{\eta^{-1}_{G(a)}}(z)\right\} &= \inf_{z \in x\gamma y} \left\{\mu^{+}_{g_{\psi(a)}}(\eta(z))\right\} \\ &= \inf_{\eta(z) \in \eta(x\gamma y)} \left\{\mu^{+}_{g_{\psi(a)}}(\eta(z))\right\} \\ &= \inf_{\eta(z) \in \eta(x)h(\gamma)\eta(y)} \left\{\mu^{+}_{g_{\psi(a)}}(\eta(z))\right\} \\ &\geq \min\left\{\mu^{+}_{g_{\psi(a)}}\eta(x), \mu^{+}_{g_{\psi(a)}}\eta(y)\right\} \\ &= \min\left\{\mu^{+}_{\eta^{-1}_{G(a)}}(x), \mu^{+}_{\eta^{-1}_{G(a)}}(y)\right\} \end{aligned}$$

and

$$\begin{aligned} \sup_{z \in x\gamma y} \left\{\mu^{-}_{\eta^{-1}_{G(a)}}(z)\right\} &= \sup_{z \in x\gamma y} \left\{\mu^{-}_{g_{\psi(a)}}(\eta(z))\right\} \\ &= \sup_{\eta(z) \in \eta(x\gamma y)} \left\{\mu^{-}_{g_{\psi(a)}}(\eta(z))\right\} \\ &= \sup_{\eta(z) \in \eta(x)h(\gamma)\eta(y)} \left\{\mu^{-}_{g_{\psi(a)}}(\eta(z))\right\} \\ &\leq \max\left\{\mu^{-}_{g_{\psi(a)}}\eta(x), \mu^{-}_{g_{\psi(a)}}\eta(y)\right\} \\ &= \max\left\{\mu^{-}_{\eta^{-1}_{G(a)}}(x), \mu^{-}_{\eta^{-1}_{G(a)}}(y)\right\} \end{aligned}$$

Therefore $(\eta,\psi)^{-1}(G,B)$ is a bipolar fuzzy soft Γ-hypersubsemigroup of H_1.

Theorem 4.4
Let $\eta: H_1 \to H_2$ be a homomorphism of S. If (G,B) is a bipolar fuzzy soft Γ-hyperleft(right, bi-ideal, interior) of H_2, then $(\eta,\psi)^{-1}(G,B)$ is a bipolar fuzzy soft Γ-hyperleft(right, bi-ideal, interior)ideal of H_1.
Proof. Straightforward.

Theorem4.5
Let $\eta: H_1 \to H_2$ be a homomorphism of S. If (F,A) is a bipolar fuzzy soft Γ-hypersubsemigroup of H_1, then $(\eta,\psi)(F,A)$ is a bipolar fuzzy soft Γ-hypersubsemigroup of H_2.
Proof. Let (F,A) is a bipolar fuzzy soft Γ-hypersubsemigroup of H_1. Let $x_1, y_1 z_1 \in H_2, \gamma \in \Gamma_2$ then we have

$$\begin{aligned}
\inf_{z_1\in x_1\gamma y_1}\{\mu^{+}_{\eta_{F(b)}}(z_1)\} &= \inf_{z_1\in x_1\gamma y_1}\left\{\sup_{t\in\eta^{-1}(z_1)}\sup_{\psi(a)=b}\mu^{+}_{F(a)}(t)\right\}\\
&\geq \inf_{z\in x_1\gamma y_1}\left\{\sup_{\psi(a)=b}\mu^{+}_{F(b)}(z_1)\right\}\\
&= \inf_{\eta(z)\in\eta(x)h(\gamma)\eta(y)}\left\{\sup_{\psi(a)=b}\mu^{+}_{F(b)}(z)\right\}\\
&= \inf_{\eta(z)\in\eta(x\gamma y)}\left\{\sup_{\psi(a)=b}\mu^{+}_{F(b)}(z)\right\}\\
&= \inf_{z\in x\gamma y}\left\{\sup_{\psi(a)=b}\mu^{+}_{F(b)}(z)\right\}\\
&\geq \sup_{\psi(a)=b}\min\{\mu^{+}_{F(b)}(x),\mu^{+}_{F(b)}(y)\}\\
&\geq \sup_{x\gamma y\subseteq\eta^{-1}(x_1)h^{-1}(\gamma)\eta^{-1}(y_1)}\left\{\sup_{\psi(a)=b}\min\{\mu^{+}_{F(b)}(x),\mu^{+}_{F(b)}(y)\}\right\}\\
&= \min\left\{\sup_{\eta(x)=y}\sup_{\psi(a)=b}\mu^{+}_{F(a)}(x),\ \sup_{\eta(x)=y}\sup_{\psi(a)=b}\mu^{+}_{F(a)}(y)\right\}\\
&\geq \min\left\{\mu^{+}_{\eta_{F(b)}}(x_1),\mu^{+}_{\eta_{F(b)}}(y_1)\right\}
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z_1\in x_1\gamma y_1}\{\mu^{-}_{\eta_{F(b)}}(z_1)\} &= \sup_{z_1\in x_1\gamma y_1}\left\{\inf_{t\in\eta^{-1}(z_1)}\inf_{\psi(a)=b}\mu^{-}_{F(a)}(t)\right\}\\
&\leq \sup_{z\in x_1\gamma y_1}\left\{\inf_{\psi(a)=b}\mu^{-}_{F(b)}(z_1)\right\}\\
&= \sup_{\eta(z)\in\eta(x)h(\gamma)\eta(y)}\left\{\inf_{\psi(a)=b}\mu^{-}_{F(b)}(z)\right\}\\
&= \sup_{\eta(z)\in\eta(x\gamma y)}\left\{\inf_{\psi(a)=b}\mu^{-}_{F(b)}(z)\right\}\\
&= \sup_{z\in x\gamma y}\left\{\inf_{\psi(a)=b}\mu^{-}_{F(b)}(z)\right\}\\
&\leq \inf_{\psi(a)=b}\max\{\mu^{-}_{F(b)}(x),\mu^{-}_{F(b)}(y)\}\\
&\leq \inf_{x\gamma y\subseteq\eta^{-1}(x_1)h^{-1}(\gamma)\eta^{-1}(y_1)}\left\{\inf_{\psi(a)=b}\max\{\mu^{-}_{F(b)}(x),\mu^{-}_{F(b)}(y)\}\right\}\\
&= \max\left\{\inf_{\eta(x)=y}\inf_{\psi(a)=b}\mu^{-}_{F(a)}(x),\ \inf_{\eta(x)=y}\inf_{\psi(a)=b}\mu^{-}_{F(a)}(y)\right\}\\
&\leq \max\left\{\mu^{-}_{\eta_{F(b)}}(x_1),\mu^{-}_{\eta_{F(b)}}(y_1)\right\}
\end{aligned}$$

Therefore $(\eta,\psi)(F,A)$ is a bipolar fuzzy soft Γ-hyper subsemigroup of H_2.

Theorem 4.6
Let $\eta: H_1 \to H_2$ be a homomorphism of S. If (F,A) is a bipolar fuzzy soft Γ-hyperleft(right, bi-ideal, interior)ideal of H_1, then $(\eta,\psi)(F,A)$ is a bipolar fuzzy soft Γ-hyperleft(right, bi-ideal, interior)ideal of H_2.
Proof. Straighforward.

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