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The Implementation of the Trapezoidal Fuzzy Number toward the Solution of the A Fuzzy Inventory Model with Shortages

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The Implementation of the Trapezoidal Fuzzy Number toward the Solution of the A Fuzzy Inventory Model with Shortages

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Abstract--In this paper, fuzzy inventory model with shortages has been considered. Trapezoidal fuzzy number with Weibull distribution is used to fuzzify the data of an inventory model. All the parameters used in the inventory model is converted to trapezoidal fuzzy numbers using function principle. We find the estimate of the total inventory cost by defuzzifing the trapezoidal fuzzy numbers using the the graded mean integration. The optimum total cost for the inventory model is found. **Keywords--**Inventory · Trapezoidal fuzzy numbers· Defuzzification

1. Introduction

Inventory models are the models which is used to determine the optimum levels of the inventories which has to be maintained during the various stages of production, storing, ordering and managing the goods.

Inventory models deals with certainty and uncertainty of demand which occurs before and after the production of goods. The inventories during manufacturing can be classified as raw materials, work-in-progress and finished goods. After finishing the product, we have two costs that deals with holding the inventories. They are named as ordering costs and carrying costs.

There are two major models in the world of inventory. One is the deterministic model which deals with no uncertainty of demand and replenishment of inventories. The Probabilistic model which deal with uncertainty with demand pattern and lead time of the inventories.

Inventory models with and without Back orders are considered by Yao et al[24] in fuzzy environment. Trapezoidal number is used to fuzzify the order quantity q. With the help of centroid strategy along with the extension principle, they found the membership functions of fuzzy total cost.

2. Preliminaries

"**Definition 2.1:** A Weibull distribution is defined as a random variable X is said to have a *Weibull distribution* with parameters α and β (α > 0; β > 0) if the probability density function of X is $f(x;\alpha,\beta) = \begin{cases} \frac{\alpha}{\beta\alpha} x^{\alpha-1} e^{\left(\frac{x}{\beta}\right)^{\alpha}} & x \geq 0 \\ 0 & x < 0 \end{cases}$ where α is a shape parameter and β is a scale parameter."

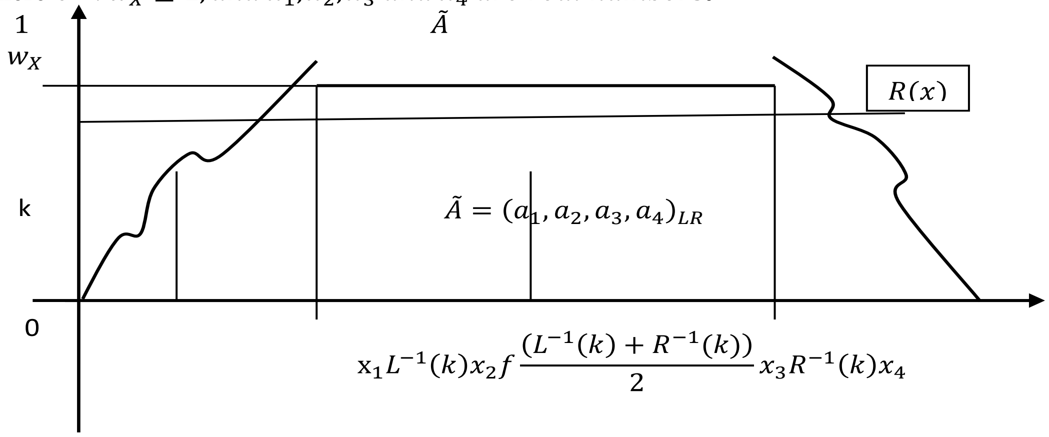
3. Methodology

Graded Mean Integration by Chen and Hsieh[6] is used to defuzzify the fuzzy number which is based on the mean integral value with h-level. The generalised fuzzy number is described as follows.

Suppose $\tilde{X}$ is the fuzzy number considered then $\tilde{X} \subseteq \mathbb{R}$. The following conditions are met by the membership function $\mu_{\tilde{X}}$ of $\tilde{X}$

i) $\mu_{\tilde{X}}: \mathbb{R} \xrightarrow{continuous} [0,1]$.
ii) $\mu_{\tilde{X}} = 0, -\infty < x \leq x_1$,
iii) $\mu_{\tilde{X}} = L(x)$ is strictly increasing on $[x_1, x_2]$,
iv) $\mu_{\tilde{X}} = w_X, x_2 \leq x \leq x_3$,
v) $\mu_{\tilde{X}} = R(x)$ is strictly decreasing on $[x_3, x_4]$,
vi) $\mu_{\tilde{X}} = 0, x_4 \leq x < \infty$,

Where $0 < w_X \leq 1, and\ x_1, x_2, x_3\ and\ x_4\ are\ real\ numbers.$



Also, the generalised fuzzy number is denoted by $\tilde{X} = (x_1, x_2, x_3, x_4; w_X)_{LR}$ .When $w_X = 1$, it can be simplified as $\tilde{X} = (x_1, x_2, x_3, x_4)_{LR}$ also, upon referring to and employing the Graded Mean Integration Representation approach, we have L^{-1} and R^{-1} are as the inversed functions linked to L and R in that order. For the generalized fuzzy number, its associated graded mean h-level value of in

$$\tilde{X} = (x_1, x_2, x_3, x_4; w_X)_{LR} \text{ is } \frac{h\left(L^{-1}(k) + R^{-1}(k)\right)}{2}.$$

Then the Graded Mean Integration Representation of $\tilde{X}$ is $P(\tilde{X})$ with graded w_X, where

$$P(\tilde{X}) = \int_0^{w_X} \frac{k\left(L^{-1}(k) + R^{-1}(k)\right)}{2} df \Big/ \int_0^{w_X} f df \quad \text{With } 0 < k \leq w_X \text{ and } 0 < w_X \leq 1. \text{ -----(3.1)}$$

In the proposed fuzzy inventory model, we will now represent the fuzzified number as the parameter type. Let us assume that $\tilde{Y}$ as a trapezoidal fuzzy number which is represent as $\tilde{Y} = (y_1, y_2, y_3, y_4)$. Calculation of the Graded Mean Integration of $\tilde{Y}$ is givenby the formula (3.1) as

$$P(\tilde{Y}) = \int_0^1 k\left(\frac{y_1 + y_4 + (y_2 - y_1 - y_4 + y_3)k}{2}\right) dk \Big/ \int_0^1 k dk$$

$$P(\tilde{Y}) = \frac{y_1+2y_2+2y_3+y_4}{6} \qquad \text{------(3.2)}$$

4. Function principle's arithmetical operations

Arithmetic operations on trapezoidal fuzzy number is done usingFunction. The following are the arithmetical operations on fuzzy numbers done under the function principle.Suppose $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ and $\widetilde{\Theta} = (\Theta_1, m_2, m_3, \mathrm{m}_4)$ are the two trapezoidal fuzzy numbers. Then,

1. The addition of $\tilde{\lambda}$ and $\widetilde{\Theta}$ is $\tilde{\lambda} \oplus \widetilde{\Theta} = (\lambda_1 + \Theta_1, \lambda_2 + \Theta_2, \lambda_3 + \Theta_3, \lambda_4 + \Theta_4),$
   where $\lambda_1, \Theta_1, \lambda_2, \Theta_2, \lambda_3, \Theta_3, \lambda_4$ *and* Θ_4 are any $\mathbb{R}$.
2. The multiplication of $\tilde{\lambda}$ and $\tilde{\theta}$ is $\tilde{\lambda} \otimes \tilde{\theta} = (\phi_1, \phi_2, \phi_3, \phi_4), where$
   $T = \{\lambda_1\theta_1, \lambda_1\theta_4, \lambda_4\theta_1, \lambda_4\theta_4\}, \quad T_1 = \{\lambda_2\theta_2, \lambda_2\theta_3, \lambda_3\theta_2, \lambda_3\theta_3\}, \phi_1 = minT,$
$\phi_2 = \min T_1, \phi_3 = \max T_1, \phi_4 = \max T_1.$ If $\lambda_1, \Theta_1, \lambda_2, \Theta_2, \lambda_3, \Theta_3, \lambda_4$ *and* $\Theta_4$are$\mathbb{R}$ which are ≥ 0 ,
   then $\tilde{\lambda} \otimes \tilde{\theta} = (\lambda_1\theta_1, \lambda_2\theta_2, \lambda_3\theta_3, \lambda_4\theta_4),$
3. $-\tilde{\theta} = (-\theta_4, -\theta_3, -\theta_2, -\theta_1)$, then the subtraction of $\tilde{\lambda}$and $\tilde{\theta}$ is $\tilde{\lambda} \ominus \tilde{\theta} = (\lambda_1 - \theta_4, \lambda_2 - \theta_3, \lambda_3 - \theta_2, \lambda_4 - \theta_1)$, Where $\lambda_1, \Theta_1, \lambda_2, \Theta_2, \lambda_3, \Theta_3, \lambda_4$ *and* $\Theta_4$are $\mathbb{R}$.
4. $\frac{1}{\tilde{\theta}} = \tilde{\theta}^{-1} = \left({}^{1}/_{\theta_4}, {}^{1}/_{\theta_3}, {}^{1}/_{\theta_2}, {}^{1}/_{\theta_1}\right)$, where$\lambda_1, \Theta_1, \lambda_2, \Theta_2, \lambda_3, \Theta_3, \lambda_4$ *and* $\Theta_4\mathbb{R}$ which are ≥ 0,
   then the quotient of $\tilde{L}$ and $\tilde{\theta}$ is $\frac{\tilde{\lambda}}{\tilde{\theta}} = \left(\frac{\lambda_1}{\theta_4}, \frac{\lambda_2}{\theta_3}, \frac{\lambda_3}{\theta_2}, \frac{\lambda_4}{\theta_1}\right)$
5. Let α ε R. Then
   (i) $\alpha \geq 0, \alpha \otimes \tilde{\lambda} = (\alpha\lambda_1, \alpha\lambda_2, \alpha\lambda_3, \alpha\lambda_4)$
   (ii) $\alpha < 0, \alpha \otimes \tilde{\lambda} = (\alpha\lambda_4, \alpha\lambda_3, \alpha\lambda_2, \alpha\lambda_1)$

5. Symbols used in the model

$\tilde{R}$: Fuzzy demand rate
$\tilde{Q}$: The fuzzy order quantity during a cycle of length T
$\tilde{Q}_1$: The positive inventory fuzzy level at time t, $0 \leq t \leq t_1$
$\tilde{Q}_2$: The negative inventory fuzzy level at time t, $t_1 \leq t \leq T$
$\widetilde{SC}$: Fuzzy shortage cost/ time unit
$\tilde{A}$: Fuzzy ordering cost / order
$\widetilde{TC}\ (t_1, \mathrm{T})$: Fuzzy total cost / time unit
$\tilde{\lambda}, \tilde{\theta}$: Fuzzy numbers

Trapezoidal Fuzzy Numbers:
$\tilde{R} = (r_1, r_2, r_3, r_4)$:Fuzzy Demand rate
$\tilde{l} = (l_1, l_2, l_3, l_4)$: Fuzzy number
$\tilde{m} = (m_1, m_2, m_3, m_4)$:Fuzzy number
$\tilde{A} = (k_1, k_2, k_3, k_4)$:Fuzzy ordering cost / order
The above are the representation of generalized fuzzy numbers in trapezoidal format.
Mathematical Formulation:
During the positive stock interval $[0, t_1]$ and shortage interval $[t_1, \mathrm{T}]$, the rate of change of the inventory is given by the differential equations as follows.

$$\frac{d\tilde{Q}_1(\mathrm{t})}{dt} + \theta(\mathrm{t})Q_1(\mathrm{t}) = -\tilde{R} \qquad \text{..... (5.1)}$$

$$\frac{d\tilde{Q}_2(\mathrm{t})}{dt} = -\tilde{R} \quad \text{------ (5.2)}$$

With the boundary conditions
$Q_1(t) = Q_2(t) = 0 \; at \; t = t_1 \; and$ ------ (5.3)
$Q_1(t) = IM \; and \; Q_2(t) = IB \; at \; t = T$ -------(5.4)

6. Mathematical solution for the model

Case I: Inventory Level Without Shortage:
The inventory diminishes due to demand in the interval $[0, t_1]$. Therefore, level of the inventory at any time in the interval $[0, t_1]$ is described by differential equation
From equation (5.1)
$\frac{d\tilde{Q}_1(t)}{dt} + \varphi(t)\tilde{Q}_1(t) = -\tilde{R}$-------(6.1)
With the boundary conditions are $Q_1(0) = IM \; and \; Q_1(t_1) = 0$.
From equation (5.3)

$$\tilde{Q}_1(t) = \tilde{R}\left[t_1 - t + \alpha t^{\beta+1} + \frac{e^{\alpha t_1^{\beta+1}}}{\beta+1} - \frac{e^{\alpha t^{\beta+1}}}{\beta+1}\right] \text{-----(6.2)}$$

Case II: Inventory Level with Shortage:
Differential equation is used to describe the state of inventory during $[t_1, T]$.
From equation (5.2)

$$\frac{d\tilde{Q}_1(t)}{dt} = -\tilde{R}$$

$$\frac{d\tilde{Q}_1(t)}{dt} = -(r_1, r_2, r_3, r_4)$$
$$\frac{d\tilde{Q}_1(t)}{dt} = (r_4, r_3, r_2, r_1)$$

where $t_1 < t < T$

We have the boundary conditions as $Q_2(t_1)$ =0 and Q_2(t) = IB

$$\tilde{Q}_2(t) = -\tilde{R}\,(t - t_1)$$

The solution of differential equation, from equation (5.4) is

$$Q_2(t) = \tilde{R}(t_1 - t)$$
$$Q_2(t) = (r_1, r_2, r_3, r_4)(t_1 - t)$$

Therefore, for one renewal cycle the total cost has the following elements,
Holding cost:

$$\widetilde{HC} = \int_0^{t_1} H(t)Q_1(t)dt$$

From the equation (6.2)

$$\widetilde{HC} = \tilde{l}\tilde{R}\left[\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + \tilde{m}\tilde{R}\left[\frac{t_1^3}{6} + \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}\right] \text{-----(6.3)}$$

$$\widetilde{HC} = \left\{\begin{array}{c} (r_1 l_1, r_2 l_2, r_3 l_3, r_4 l_4)\left[\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] \\ +(r_1 m_1, r_2 m_2, r_3 m_3, r_4 m_4)\left[\frac{t_1^3}{6} + \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}\right] \end{array}\right\} \text{------(6.4)}$$

Shortage Cost during $[t_1, T]$:

$$\widetilde{SC} = \pi \int_{t_1}^{T} \tilde{R}(t_1 - t)dt$$

$\widetilde{SC} = \frac{1}{2}\, \pi\, \tilde{R}(T - t_1)^2$ ------(6.5)

$\widetilde{SC} = \frac{1}{2}\, \pi\, (r_1, r_2, r_3, r_4)(T - t_1)^2$------(6.6)

Ordering Cost per order:

$\tilde{A} = (k_1, k_2, k_3, k_4)$-------(6.7)

Total cost function per time unit:

$$\widetilde{TC}\,(t_1, \mathrm{T}) = \frac{1}{T}[\widetilde{HC} + \widetilde{SC} + \tilde{A}]$$

From equations (6.3), (6.5) and (6.7)

$$\widetilde{TC}(t_1, T) = \frac{1}{T}\left\{\begin{matrix} \tilde{l}\tilde{R} \otimes \left[\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] \oplus \tilde{m}\tilde{R} \otimes \left[\frac{t_1^3}{6} + \frac{\alpha\beta {t_1}^{\beta+3}}{2(\beta+2)(\beta+3)}\right] \\ \oplus\, \tilde{R} \otimes \frac{1}{2}\, \pi(T - t_1)^2 \oplus \tilde{A} \end{matrix}\right\}$$

From equations (6.4), (6.6) and (6.8)

$$\widetilde{TC}(t_1, T) = \frac{1}{T}\left\{\begin{matrix} (\mathrm{r}_1\lambda_1, \mathrm{r}_2\lambda_2, \mathrm{r}_3\lambda_3, \mathrm{r}_4\lambda_4)\left[\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] \\ +(\mathrm{r}_1\theta_1, \mathrm{r}_2\theta_2, \mathrm{r}_3\theta_3, \mathrm{r}_4\theta_4)\left[\frac{t_1^3}{6} + \frac{\alpha\beta {t_1}^{\beta+3}}{2(\beta+2)(\beta+3)}\right] \\ +\frac{1}{2}\, \pi\, (r_1, r_2, r_3, r_4)(T - t_1)^2 \\ +(k_1, k_2, k_3, k_4) \end{matrix}\right\} \text{------(6.8)}$$

The Necessary condition for the total cost per time unit to be minimized is

$$\frac{\partial \widetilde{TC}}{\partial t_1} = 0 \; and \; \frac{\partial \widetilde{TC}}{\partial T} = 0$$

Given

$$\left(\frac{\partial^2 \widetilde{TC}}{\partial t_1^2}\right)\left(\frac{\partial^2 \widetilde{TC}}{\partial T^2}\right) - \left(\frac{\partial^2 \widetilde{TC}}{\partial t_1 \partial T}\right)^2 > 0 \; and \; \left(\frac{\partial^2 \widetilde{TC}}{\partial t_1^2}\right) > 0$$

7. Numerical example and sensitivity study

In this paper, we consider an inventory problem with variables with correct units as follows:
R=2000, λ=2.0, θ=1.5, C=10, O=50, α=0.15, β=1.5, A=50.
Let us convert the above-mentioned ordinary crispnumbers to trapezoidal fuzzy numbers using, "greater or less than Y" or "about Y".

1. "greater or less than X" = $(0.92Y, 0.94Y, 1.02Y, 1.1Y)$, and
2. "about X" =$(0.92Y, Y, Y, 1.02Y)$

Then by the above injunction, the fuzzy parameters ismodified as follows:

i) Fuzzy Demand rate, R= "greater or less than 2000" R = (0)
ii) Fuzzy ordering cost, A=" about 50" = (46, 50, 50, 51)

iii) Fuzzy numbers, λ= “greater or less than 2” = (1.84, 1.88, 2.04, 2.2)
θ= “greater or less than 1.5” = (1.38, 1.41, 1.53, 1.55)
iv) α = “greater or less than 0.15” = (0.13, 0.14, 0.15, 0.16)
β = “greater or less than 1.5” = (1.3, 1.4, 1.5, 1.6)
Substituting the above values in the equations

Table.1: Effect of changes in limitation of the inventory problem

Parameters	%Change	T	t_1	TC
R	+40%	1.419874156	0.011670695	70.41517205
	+20%	1.421459571	0.014699532	70.33800196
	-20%	1.424000822	0.019892018	70.20544017
	-40%	1.427021837	0.026078629	70.04716039
α	+40%	1.421875009	0.015634989	70.31200099
	+20%	1.422006363	0.015861028	70.30726674
	-20%	1.422288646	0.016343423	70.29726113
	-40%	1.422440702	0.016601405	70.29196484
β	+40%	1.422858828	0.017321356	70.27687361
	+20%	1.422670438	0.017019365	70.28255366
	-20%	1.42089431	0.013621192	70.36365589
	-40%	1.418544038	0.008423917	70.50600603
λ	+40%	1.420114479	0.011907429	70.44910654
	+20%	1.420985301	0.013696952	70.37548717
	-20%	1.423751973	0.01946766	70.22971249
	-40%	1.426105064	0.024501661	70.15754767
θ	+40%	1.42212196	0.016063117	70.30295457
	+20%	1.422132987	0.016079982	70.30265333
	-20%	1.422155194	0.016113922	70.30206671
	-40%	1.422166377	0.016130998	70.30178115

Substituting the above values in the equations

$$t_1 = (0.015286761, 0.016010255, 0.017695545, 0.01853819)$$

$$T = (1.2867336, 1.3582188, 1.5011892, 1.5726744)$$

Substituting the values of R, l, m, Aα, βin equation (6.8) the total cost is found as

$$\widetilde{TC}(t_1, T) = (63.268925, 66.774421, 73.635413, 77.541909)$$

From the formula given in equation (3.1) and (3.2) and using the graded mean integration the total cost can be defuzzified. The Defuzzified Total Cost is given by $TC = 70.2716503$

Comparing the above defuzzified value of the total cost with the crisp value of the total cost is negligible

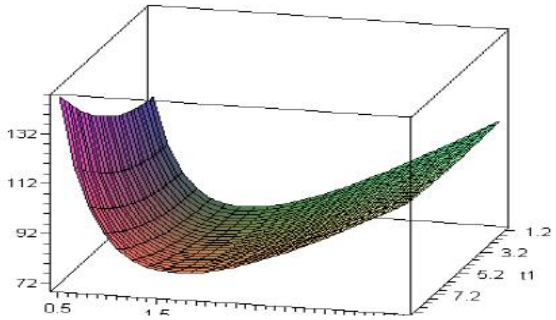
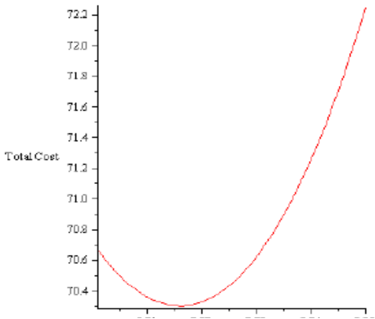


Fig.1:($TC\ vs\ t_1 and\ T$)



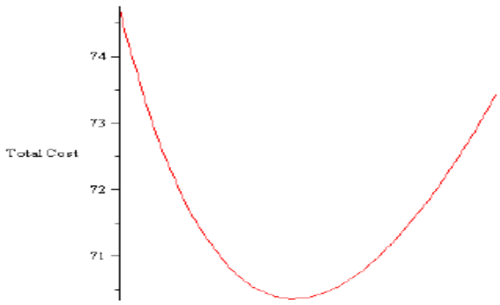


Figure 2. (TC Vs T at $t_1 = 0.01$)Figure-3 (TC Vs T at $t_1 = 0.01$)TC Vs. t_1 at T=1.42)

8. Conclusion

The above example has been defuzzified usingcentroid method in Graded mean Integration and relative changes have been computed. We can see that the total cost maximizes or minimizes with mere change which is considered and within our expectation level.

An analytical solution of an inventory model with holding cost and shortages under Weibull distribution is obtained. Here all the variables are considered as fuzzy. We have used Function Principle to fuzzify the inventory model and Graded Mean Integration to defuzzify the trapezoidal fuzzify numbers and solved the model. Practical problem is illustrated to support the variability of the method given above. The fuzzy inventory model is more real world and accurate than the crisp inventory model.

9. References

[1] Abad, P.L., (1996), Optimal pricing and lot-sizing under conditions of perishability and partial backordering. Management Science, 42, 1093-1104.

[2] Abad, P.L., (2001), Optimal price and order-size for a reseller under partial backlogging. Computers and Operation Research, 28, 53- 65.
[3] Chang, Oct. 1, (1999), Fuzzy production inventory for fuzzy product quantity with triangular fuzzy number, Fuzzy Sets and Systems, v.107 n.1, p.37-57.
[4] Chen, (1985), Fuzzy linear combination of fuzzy linear functions under extension principle and second function principle, Tamsui Oxford Journal of Management Sciences 11-31.
[5] Chen, (1985), Operations on fuzzy numbers with function principle, Tamkang Journal of Management Sciences 6 13-26.
[6] Chen, Hsieh, (1999), Graded mean integration representation of generalized fuzzy number, Journal of Chinese Fuzzy Systems 1-7.
[7] Dubois D, Prade H (1980) Fuzzy sets and systems. Academic Press, New York
[8] Ghare, P. M. and Schrader, G. F., (1983), A model for an exponentially decaying inventory. Journal of Industrial Engineering, 14, 238-243.
[9] Goyal, S. K. and B. C. Giri, (2001) Recent trends in modeling of deteriorating inventory. European Journal of Operational Research, 134, 1-16.
[10] Huang and Hsu, (2008) A note on a buyer-vendor EOQ model with changeable lead-time in supply chain, Journal of Information & Optimization Sciences, 29, 305–310.
[11] Kuo-Chen Hung, (2009) An inventory model with generalized type demand, deterioration and backorder rates, European Journal of Operational Research, 208(3), 239-242.
[12] Lee, W-C., and Wu, J-W. (2004) A Note on EOQ model for items with mixtures of exponential distribution deterioration, Shortages and time-varying demand, Quality and Quantity, 38, 457-473.
[13] Liao, J.J., (2008), An EOQ model with non-instantaneous receipt and exponential deteriorating item under two–level trade credit. International Journal of Production Economics, 113,852-861
[14] Mishra & L. Singh, (2011), deteriorating inventory model for time dependent demand and holding cost with partial backlogging. International Journal of Management Science and Engineering Management, X(X): 1-5
[15] Mishra, V.K. and Singh, L.S., (2010), Deteriorating inventory model with time dependent demand and partial backlogging, Applied Mathematical Sciences, 4(72), 3611-3619.
[16] Pareek,S., Mishra,V.K., and Rani,S., (2009), An Inventory Model for time dependent deteriorating item with salvage value and shortages, Mathematics Today, 25, 31-39.
[17] Park K S (1987) Fuzzy set theoretic interpretation of economic order quantity. IEEE Transactions on Systems, Man & Cybernetics, 17: 1082-1084
[18] Park, Nov./Dec. (1987), Fuzzy set theoretic interpretation of economic order quantity, IEEE Transactions on Systems, Man and Cybernetics, v.17 n.6, p.1082-1084
[19] Roy, Ajanta. An inventory model for deteriorating items with price dependent demand and time varying holding cost. Advanced Modeling and Optimization, 10, 25-37
[20] Skouri, K., I. Konstantaras, S. Papachristos, and I. Ganas, (2009), Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. European Journal of Operational Research. European Journal of Operational Research, 192, 79–92.
[21] Skouri, Papachristos & Goyal, (2008), An EOQ model with trade credit period depending on the ordering quantity, Journal of Information & Optimization Sciences, 29, 947–961.
[22] Syed J K, Aziz L A (2007) Fuzzy inventory models without using signed-distance method. Applied Mathematics and Information Sciences, 1(2): 203-209
[23] Taha. H.A, (1997), in Operations Research, Prentice-Hall, Englewood Cliffs, NJ, USA, pp. 753-777.
[24] Yao J S, Chiang J (2003) Inventory without backorder with fuzzy total cost & fuzzy storing cost defuzzified by centroid and signed-distance. European journal of operations research, 148: 401-409
[25] Zimmermann H J (1991) Fuzzy set theory and its applications. 2nd Edition, Kluwer Academic Publishers, Boston/Dorgrecht/London