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Quantification of tolerance limits of engineering system using uncertainty modeling for sustainable energy

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ABSTRACT

Measurements always associate a certain degree of uncertainty. In order to achieve high precision measurement in presence of uncertainty an efficient computation is desired. Statistical definition of precision of any measurement is defined as one standard deviation divided by the square root of the sample size taken for measurements. Accordingly, tolerance limits are statistical in nature. Therefore, measurements are required to repeat large number of times to obtain better precision. Hence, the target is to establish the tolerance limits in presence of uncertainty in computer and communication systems. Nonparametric method is applied to establish the tolerance limits when uncertainty is present in measurements. The basic aim of the present paper is to explore order statistics based nonparametric method to estimate the appropriate number of samples required to generate the realizations of the uncertain random parameters which further will facilitate user to establish the tolerance limits. A case study of solute transport model is experimented where tolerance limits of solute concentration at any spatial location at any temporal moment is shown. Results obtained based on the nonparametric simulation are compared with the results obtained by executing traditional method of setting tolerance limits using Monte Carlo simulations using computer and communication systems.

1. Introduction

The usage of tolerance limit to characterize uncertainty of power plant engineering system, for example nuclear power plant and corresponding allied systems gains popularity due to presence of complex models and computational cost. Uncertainty modeling of engineering system such as peak clad temperature of nuclear fuel system, to test the dynamic load of wing of an aircraft, material processing, etc. is very important to formulate a regulatory protocol or to design a standard technical specification of the system. There are two types of uncertainty generally we come across, viz., (1) Aleatory uncertainty, where uncertain variables are simulated using their random distribution and aleatory uncertainty is irreducible and (2) Epistemic uncertainty, where uncertain variables are subjective in nature due to their lack of measurements. Hence uncertain variables in case of modeling epistemic uncertainty are characterized by their fuzziness. The objective of this article is to propose a nonparametric technique to estimate tolerance limits to achieve high precision of engineering measurements where uncertainty is addressed due to randomness of the model of interest. Sample size of any such random variable required to characterize its actual random behavior is generally large in number. Uncertainty being random is categorized as aleatory uncertainty which is not possible to reduce by increasing the sample size of measurements. So, obvious question here is that how one should select the appropriate sample size. This is because if one considers a very large sample size, say 10^6 for example, computational load will unnecessarily be a burden of the total problem. On the contrary, if sample size is too small then important information one may miss due to ignorance of actual sampling zone. Moreover measured parameters may be correlated and therefore existence of correlation will increase sample size. Representative uncertainty of the measured parameters may not provide any sort of statistical distribution other than they are a continuous function. Uncertainty due to randomness is basically termed as aleatory uncertainty which is not reducible [1].

A substantial huge number of samples (~> 10000) are required to execute traditional Monte Carlo simulation for quantification of aleatory uncertainty. However, traditional Monte Carlo simulation is not at all practical simulation method for achieving high precision in any engineering measurement. Moreover, confidence interval of the

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measurements for quoted percentiles of the target may not be possible to locate exactly due to presence of local and global variations of the parameters of the model. With a view to this issue, a non-parametric method for deciding the appropriate sample size proves to be a practical approach. Tolerance limits here provides the sample size or experimental runs and when experiments are performed for obtaining that many values of the uncertain parameters, model uncertainties can be easily calculated or estimated using these results of uncertainties of the parameters. Tolerance limits are the extreme bounds of the model value, so that any value within those bounds can be acceptable with specified level of confidence with specified percentiles [2].

Proposed method of setting tolerance limits is demonstrated with a case study of migration of contaminants through water bodies (subsurface or groundwater). Contaminant as solute is discharged or poured into a river and the samples are collected at any downstream location at a certain instant of time. Concentrations of contaminant present in the collected samples are measured and decisions of accepting these samples are made based on tolerance limits.

Paper is presented in the following ways. Section II describes the mathematical details of tolerance limits. Section III presents the application of the concept of tolerance limits in high precision measurement via uncertainty modeling. Section IV presents some example problems for handling uncertainty issues in measurements. Section V presents a case study and section VI concludes the impact of setting tolerance limits for achieving high precision measurements in any engineering problems.

2. Derivation of tolerance limits

Lower and upper bound of tolerance [LB, UB] is expressed as an interval that contains probability (or confidence) β at a fraction γ of the system under observation [3]. On the basis of desired confidence, the probability beta (β) and percentile (fraction) gamma (γ) are the selected preference of an analyst towards setting tolerance limits of the system [3]. The problem of fixing tolerance limits can be formulated as follows [3]: For some given positive values $\beta < 1$ and $\gamma < 1$ one can construct two functions $LB(x_1, x_2, .., x_n)$ and $UB(x_1, x_2, .., x_n)$ called tolerance limits for n independent observations $x_1, x_2, .., x_n$ on x such that the probability (Eq. (1))

$$\int_{LB}^{UB} f(t)dt \geq \gamma \tag{1}$$

holds good and is equal to β. According to Wilks [3,4] following solution of the problem is proposed.

Let $x_1, x_2, ..,.., x_n$ be the observed values of vector **x** arranged in increasing order. Then $LB = x_r$ and $UB = x_{n-r+1}$ where r denotes a positive integer. The exact sampling distribution of the statistic $\int_{LB}^{UB} f(t)dt$ is derived by Wilks and this provides the solution for the problem of setting tolerance limits. The very important feature of Wilks' solution is the fact that the distribution of $\int_{LB}^{UB} f(t)dt$ is entirely independent of the unknown density function f(x), i.e., the distribution of $\int_{LB}^{UB} f(t)dt$ is the same for any arbitrary continuous density function f(x). If we have p random variables, tolerance limits setting for these p variables can be made by extending Wilks' method for multivariate case. Under this umbrella, for a given positive values $\beta < 1$ and $\gamma < 1$ construction of p pairs of functions of the observations $LB_i(x_{11}, \ldots, x_{pn})$ and $UB_i(x_{11}, \ldots., x_{pn})$ $(i = 1, \ldots, p)$ such that the probability that

$$\int_{LB_p}^{UB_p} \ldots \int_{LB_1}^{UB_1} f(t_1, \ldots\ldots, t_p) dt_1, \ldots., dt_p \geq \gamma \tag{2}$$

holds is equal to β. The other symbols (LB_p and UB_p, …) shown in Eq. (2) have the usual significances (lower and upper tolerance limits of x_i). Tolerance limits are of two kinds, nonparametric and parametric. We do not have any information of the probability distribution of random variable for nonparametric tolerance limits. In case of parametric tolerance limit, the probability distribution of the random variable of interest is known but knowledge on the distribution parameters is unknown. Equations (1) and (2) can be further interpreted mathematically as follows:

If γ is tolerance limit for [LB, UB] and if β be the level of the probability for sample size N of a limited sample S_1 then probability β with γ proportion (at least) of the random variable x's in another set of sample S_2 (larger in size compared to sample S_1) will lie between LB and UB can be written as [3–6].

$$p\left(\int_{LB}^{UB} f(x)dx \geq \gamma\right) = \beta \tag{3}$$

where f(x) is the probability density function of the random variable x. Equation (3) implies that the probability that a single experiment will fail to fall in S_1 is γ. Therefore, the probability that number of experiments, N will fail to fall in S_1 is γ^N. Hence the probability that at least one experiment will fail to fall in the domain S_1 can be written as

$$\beta = 1 - \gamma^N \tag{4}$$

Thus by simple algebra, we can rewrite equation (4) as

$$N = \frac{\ln(1-\beta)}{\ln(\gamma)} \tag{5}$$

Sample values of size N are generated using equation (5), sorted in an ascending order and finally the maximum value, X_N th value is quoted as the upper tolerance limit. Extending to this derivation it can be put forward a case in which the probability β_1 of exactly one experiment in S_1 and which can be represented mathematically as

$$\beta_1 = (N_{C_1})(\gamma^{N-1})(1-\gamma) = N(1-\gamma)\gamma^{N-1} \tag{6}$$

Having more conservative estimate one can have a requirement in which the probability β_2 [3] of at least 2 experiments in the same region can be written as

$$\beta_2 = 1 - \gamma^N - N(1-\gamma)\gamma^{N-1} \tag{7}$$

In a similar way, one can evaluate the probability β_3 of obtaining at least 3 experiments in the domain of interest as (1 – probability that none of the experiment will fell in the domain of interest + exactly one experiment was in the domain of interest + exactly two experiments were in the domain of interests). Therefore, β_3 [3] can be mathematically written as

$$\beta_3 = 1 - \gamma^N - N(1-\gamma)\gamma^{(N-1)} - C_2\left(1-\gamma^2\right)\gamma^{(N-2)} \tag{8}$$

So, in general, the probability β_m for obtaining 'm' values in the domain of interest [7–9] is given by

$$\beta_m = 1 - \sum_{i=N-m+1}^{N} \frac{N!}{i!(N-i)!} \gamma^i \, (1-\gamma)^{N-i} \tag{9}$$

For our present study, lower tolerance limits is obtained by quoting the minimum of $\{x_i, i = 1, \ldots N\}$ and the upper tolerance limit is quoted by the maximum of $\{x_i, i = 1, \ldots N\}$.

3. Uncertainty quantification using tolerance limits

Measurements are always associated with a specific uncertainty in the sense that measured values are expressed always mean value ± error (uncertainty). In this paper we have addressed error as mean square error or root mean square error (standard deviation). Therefore, high precision measurement means mean value of the item of interest ± accuracy. This accuracy is basically defined as sum of the bias, i.e. error and precision. Tolerance limits addresses this precision because sample size is possible to determine accurately using these tolerance limits. In order to reduce the number of samples from large (few ten thousands) to substantially small (100 or further) is challenging which can be achieved by Wilks' method (an efficient method for setting tolerance limits) [4,5]. It is known that sample size is independent of the number of uncertain parameters of the model is [5]. The problem of setting tolerance limits in parametric and nonparametric domain is to compute a closed interval [LB, UB] for a random variable $X = \{ x_1, ..., x_n \}$.

Equations (5) and (9) can be used for estimation of sample size required for high precision measurement during an experiment. In order to implement methodology based on Wilks' tolerance limits [5,6], let us consider (i) one sided and (ii) two sided tolerance limits.

Tolerance limits (one sided): Consider a model in which we are interested in either shear stress or temperature at any spatial point of the structure as outcome. Our target is say, estimation of tolerance limits of this outcome surface of a structure. In this case, suppose we are interested in setting tolerance limit T_U with the 95% confidence (probability, β) and 95th percentile (fraction γ) of the temperatures from an infinite population. Sample size N based for the desired values of β and γ is shown in Table 1 [7–9]. For example, if β = 0.95, γ = 0.95, then N = 59 (see, equation (5)) samples taken from the model or 59 experiments are required to perform to have a high precision measurements of the temperatures and will alert that the maximum temperature $T_{Highest}$ in the sample space represents 95th percentile of all the possible temperature falls with 95% upper confidence limit.

Tolerance limits (two sided): If we consider the two-sided tolerance limits not much popular in the domain of interest [10], sample size will be different. Table 2 shows the sample size for a combination of (probability β and percentile, γ).

With γ and β both equal to 95%, we will get sample size N = 93. That means if 93 experiments of the model under study are carried out, we can say that T_L and T_U of this experiment represent the 95the percentile of all possible temperature will fall within that bound with 95% confidence. So, uncertainty of the model output will be expressed in terms of the bounds T_L and T_U. Degree of uncertainty (Dou) with a 95% confidence will be calculated using

$$DOU = \frac{T_U - T_L}{T_U + T_L} \tag{10}$$

4. Examples (numerical) for illustration

A few examples are demonstrated to convince the potential capability or efficiency of Wilks' tolerance limits for high precision measurements.

Example 1. Cartoons for shipping the steel bars are required to order by the manufacturer of steel bars. In that order, company wants to have length of the cartoon with 90% confidence so that at least 95% of the cartoon will not exceed the specified length. It is exactly a high precision measurement. Question is the selection of number of samples by the manufacturer. Accurate measure of the length of cartoon is categorized as high precision measurement.

Solution: On the basis of the order format, Table 1 shows that the value of sample size, N = 29 for γ = 95% and β = 90%. That means manufacturer should order length of the cartoon as 29th outcome of the population of 29 samples as length of the box. One can also tag this sort of high precision as best estimate [10].

Table 1
Minimum Sample Size (One-Sided) [3,4].

γ	β		
	0.90	0.95	0.99
0.90	22	45	239
0.95	29	59	299
0.99	44	90	459

Table 2
Minimum Sample Size (Two-Sided) [3,4].

γ	β		
	0.90	0.95	0.99
0.50	17	34	163
0.80	29	59	299
0.90	38	77	388
0.95	46	93	473
0.99	64	130	663

Example 2. A metallic cantilever is loaded with a specific load. It is required to perform an uncertainty analysis of the deflection of cantilever based on Wilks tolerance limits and two-sided with γ = 95% and β = 95%. Uncertainties of parameters of interest of the model are specified as normal distribution. Specification of the uncertainty of the parameters of interest for estimating or measuring the deflection of the cantilever is as shown in Table 3. The deflection of the cantilever is given by $\delta = \frac{4PL^3}{EBt^3}$.

Solution: Same range and distribution (Table 3) is assumed for parameters and with two-sided (95%, 95%) criteria, we design simple sampling with N = 93. A lognormal distribution is fitted to generated data of deflection model for more illustration and shown in Fig. 1. Results of propagation and (95%, 95%) tolerance limits of the deflection of cantilever are shown in Fig. 2 and Fig. 3.

Lower and upper bounds of tolerance limits of deflection of the cantilever (best estimate) [10] from Figs. 2 and 3 can be easily notified as LB = 5.45 and UB = 31.11. Therefore, degree of uncertainty from equation (10) is computed as (31.11–5.45)/(31.11 + 5.45) = 0.7.

5. Case study-contaminant transport model

Transport of contaminants (two dimensions) in surface water body (river) [11] is taken into account to quantify the uncertainty of concentration of contaminant in river water using Wilks' tolerance limit. The model estimates the density variation of contaminants dissolved in water body (river) at any temporal and spatial coordinate from the point of release of the chemical. Velocity of flow of the river water (ν m/day), longitudinal dispersivity (α_L m) and transverse dispersivity (α_T m) of the contaminants are measured parameters of the present model. Since the measurement of the parameters in the field are not possible large in number and always provides only the minimum and maximum value, uncertainty distribution of measured parameters is considered as uniform distribution. The governing equation describing the transport of contaminant in water body (river) due to advection and diffusion can be written as [11].

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} + D_T \frac{\partial^2 C}{\partial y^2} - \nu \frac{\partial C}{\partial x} \quad - \frac{C'W}{\in b} \tag{11}$$

where C(x, y, t) is the concentration of the dissolved contaminant (solute) in mg/l, ν signifies the velocity of flow of the surface water (river) in

Table 3
Uncertainty of input parameters.

Parameter	Symbol	Distribution	Mean Value	Std. Deviation
Concentrated Load	P	Normal	200	10
Length of beam	L	Normal	250	5
Young's Modulus	E	Normal	1 x 10^7	1 x 10^6
Width of beam	B	Normal	4	0.4
Thickness of Beam	T	Normal	3.0	0.3

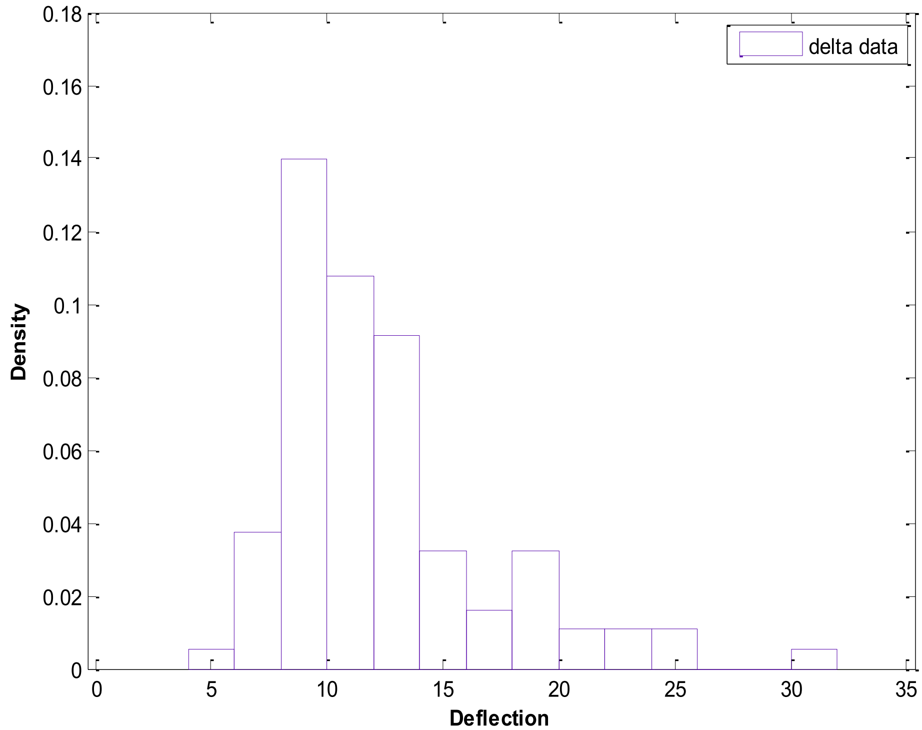

Fig. 1. Probability density of deflection of cantilever.

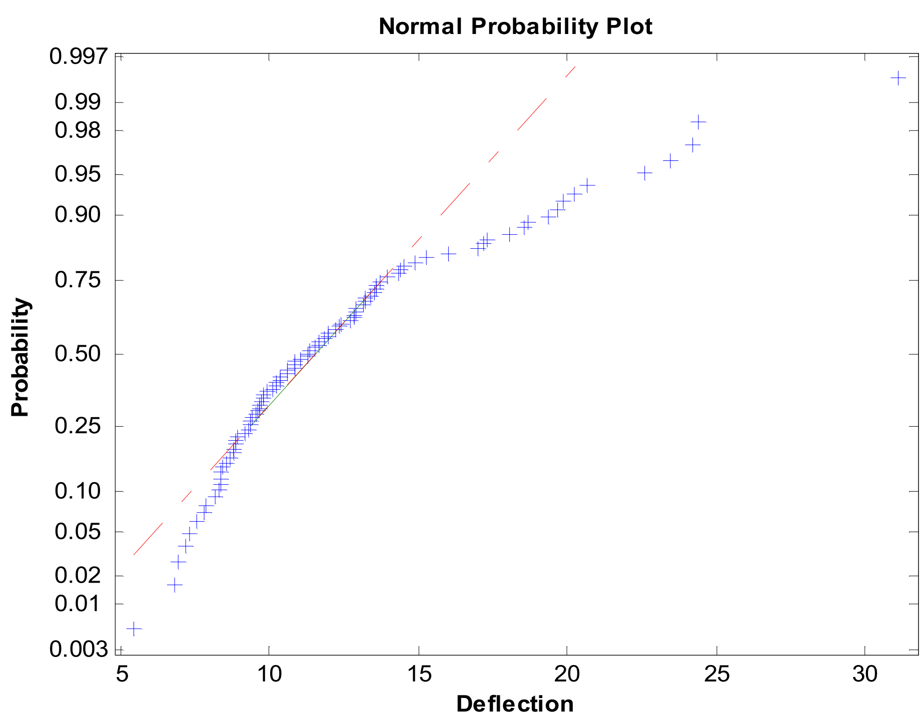

Fig. 2. Probability plot of cantilever deflection.

downstream direction (m/day), D_L and D_T represent the longitudinal and transverse dispersion coefficients (m^2/day) respectively, W is the volume flux per unit area (m/day), $C^{'}$ is the concentration of the dissolved chemical species in a source or sink fluid (mg/l), b is the saturated thickness of the aquifer (m); $\in$ is the effective porosity of the aquifer (dimensionless) and (x, y) is the spatial coordinates that signifies the downstream and cross-stream distance (m) and t is the time of observation (days). Initial and boundary conditions of the solute transport problem are $C(x, y, 0) = 0$, $x \geq 0$, $y \geq 0$, $C(0,t) = C_0$, $t \geq 0$, and $C(\infty, t) = 0$, $t \geq 0$. Longitudinal and transverse dispersion coefficients by definition are given by $D_L = \alpha_L \nu$, $D_T = \alpha_T \nu$ respectively, where α_L and α_T are longitudinal and transverse dispersivity of the aquifer [12]. We have explained the said target using analytical solution of Eq. (11) given by Ref. [13] as.

$C(x,\ y,\ t) = \frac{M}{\in b\nu}\frac{1}{4\sqrt{\pi\alpha_T}}\exp\left(\frac{x-r}{2\alpha_L}\right)\frac{1}{\sqrt{r}}erfc(\zeta)$, where $\zeta = \frac{r-\nu t}{2\sqrt{\alpha_L \nu t}}$ with

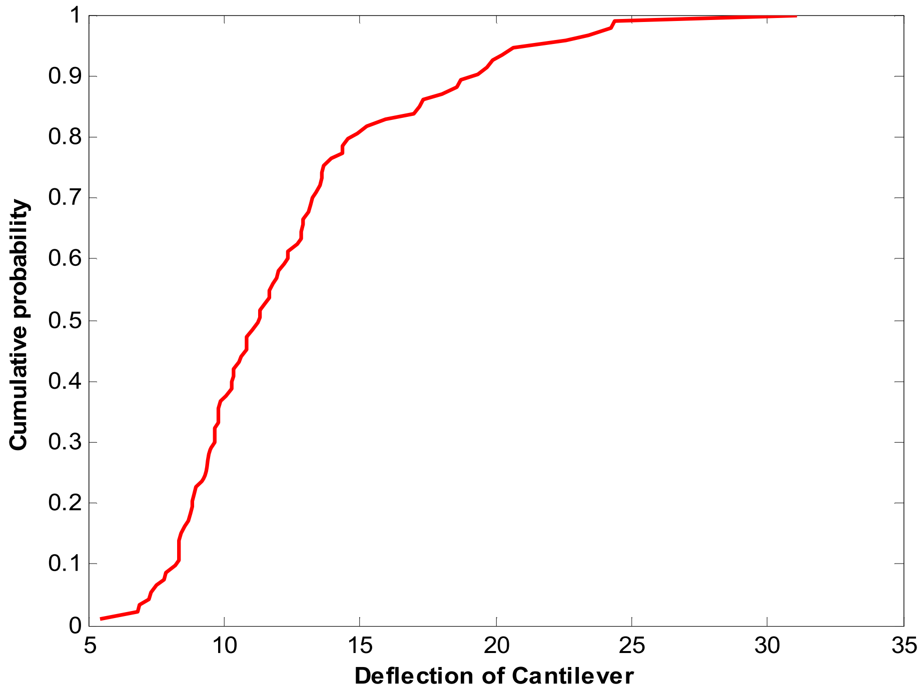

Fig. 3. Cumulative Probability plot of deflection of cantilever.

$r^2 = x^2 + (\alpha_L/\alpha_T)y^2$ where, M signifies a point source of constant rate. To simplify the analysis, density variation of the contaminant along the longitudinal section is only considered. Thus, when y = 0, r = x. Hence the concentration expression becomes,

$C(x,y=0,t) = \frac{M}{\in b\nu}\frac{1}{4\sqrt{\pi\alpha_T}}\frac{1}{\sqrt{x}}erfc(\zeta)$, $\zeta = \frac{x-\nu t}{2\sqrt{\alpha_L \nu t}}$ and the complementary error function is given by $erfc\,(\zeta) = 1\text{-}\sqrt{\frac{2}{\pi}}\int_{\infty}^{\zeta}\exp(-t^2/2)dt$.

Sample values of uncertain parameters are generated on the basis of Wilks' tolerance limits and two-sided with γ = 95% and β = 95%. Simulation of this problem has been carried out in two sets [14]. In one set, we vary downstream distances at a fixed time of observation and in the second case we vary time of observation for a fixed downstream distance. Table 4 presents the uncertainty distribution of input parameters of the contaminant transport model. Input values of the deterministic parameters and uncertainty of the input parameters are quoted from Ref. [14,15].

Table 5 presents the fixed input parameters of the solute transport model.

Number of simulations computed for two sided Wilks' tolerance limit from Table 2 are 93. With these number of simulations the concentration values of solute at locations (downstream distance) x = 50, 100, 150, 200, 250, 300, 350 and 400 m at time, t = 400 days are computed. A plot of cumulative probability of concentration at t = 400 days is as shown in Fig. 4. Two sided tolerance limits of solute concentrations at t = 400 days for each locations are estimated as. Lower sided tolerance limits is the minimum value of the solute concentration for a specified location and at specified time and upper sided tolerance limits is the maximum value of the solute concentration at time t = 400 days and for the specified location. Profile of these two-sided tolerance limits of the solute concentration at time t = 400 days is shown in Fig. 5. It can be stated from Fig. 5 that solute concentration limits (lower and upper) decreases if one collects the sample from farthest downstream distance. Another interpretation from these profiles is that one can have surety with a 95% confidence level that 95th percentile value of the solute concentration at x = 150 m and at time, t = 400 days will not fall beyond either of the values 0.03 kg/day (lower sided tolerance limit) and 0.0979 kg/day (upper sided tolerance limit).

Table 4
Uncertainty distribution of Input parameters.

Parameters	Dist	Lower limit	Upper Limit
Velocity (m/day)	U	0.3	1.0
Longitudinal dispersivity (m)	U	100	300
Transverse dispersivity (m)	U	20	60

Table 5
Fixed Input parameters used in model [15].

Parameters	Value
Thickness of flow, b	50 m
Source strength, M	120 kg/day
Effective porosity, ∈	0.17

Computations of lower and upper sided tolerance limits of solute concentration are further carried out with a specified downstream location, x = 400 m and at different times, t = 100, 200, 400, 600 and 800 days respectively. It is to be noted here that for two-sided tolerance limits 93 simulations (sample size) of input parameters (input parameters have random uncertainty) of the contaminant transport model are generated. Cumulative probability plot for each time of observations at downstream distance, x = 400 m is as shown in Fig. 6. Two sided tolerance limits of the solute concentrations at x = 400 m (fixed) and at different times of observations are computed and the corresponding profiles are shown in Fig. 7. It can be seen from Fig. 7 that for this situation concentration value increases with time of observations and gets saturated at some time. It can be noted from this profile that time of observation for saturated values of the contaminant concentration (minimum and maximum) provide the analyst the knowledge about the time of sampling for analyzing the sample. In practice such type of profiles are called breakthrough curves.

Lower and upper sided limiting value of the solute concentrations will further help in estimating the corresponding limiting values of the longitudinal and transverse dispersion coefficients of the water body through which solute transportation takes place. Measurement of the

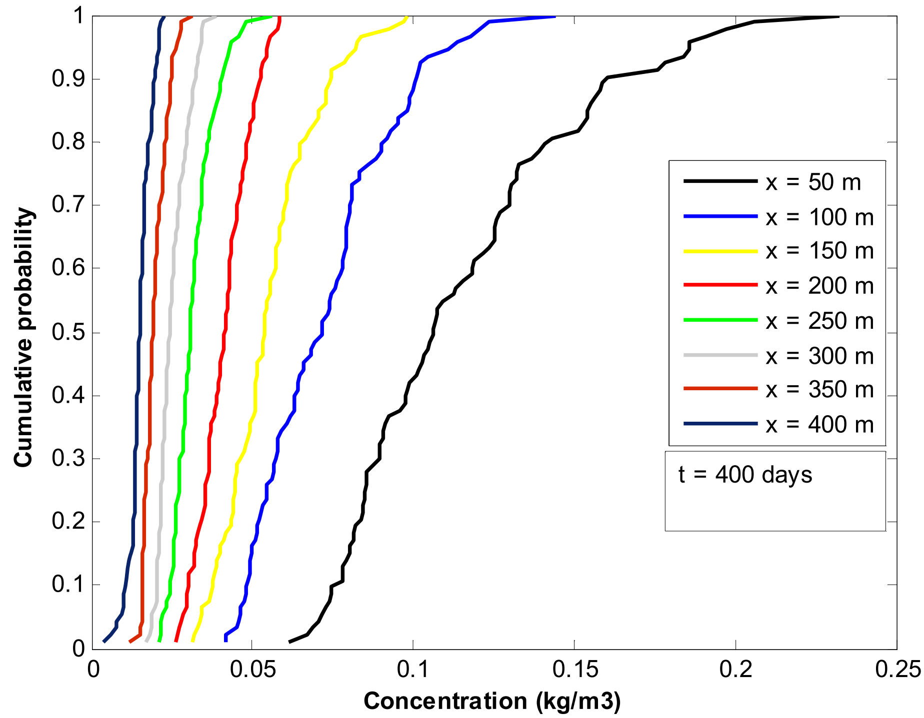

Fig. 4. Cumulative Probability plot of concentration at t = 400 days for various distance.

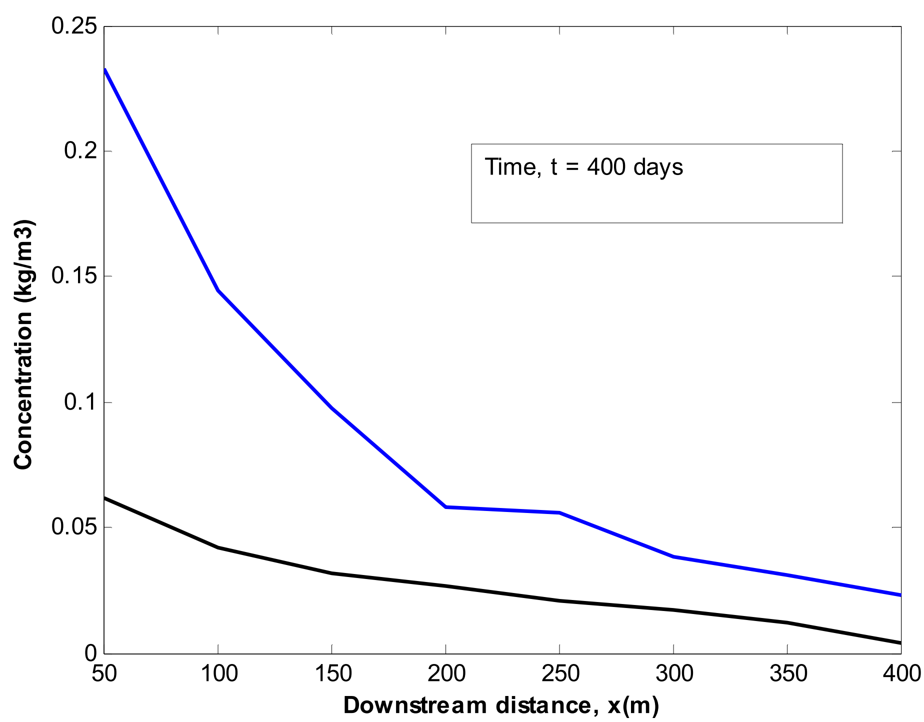

Fig. 5. Lower and Upper tolerance limits of concentration at t = 400 days for various distance.

parameters important for transport of the contaminant after discharged into the surface or groundwater (river, canal, etc.) from any industry will be of high precision with a tag mark that "95th percentile values of the measured parameters will not go beyond the lower and upper sided tolerance limits with a 95% confidence level".

6. Conclusion

Settings of tolerance limits for high precision in measurement uncertainty in engineering problems are discussed. Wilks' agent network method being nonparametric can be applied to any engineering measurements to achieve a specified degree of confidence with a specified percentile value of the measured quantity. Setting of tolerance limits

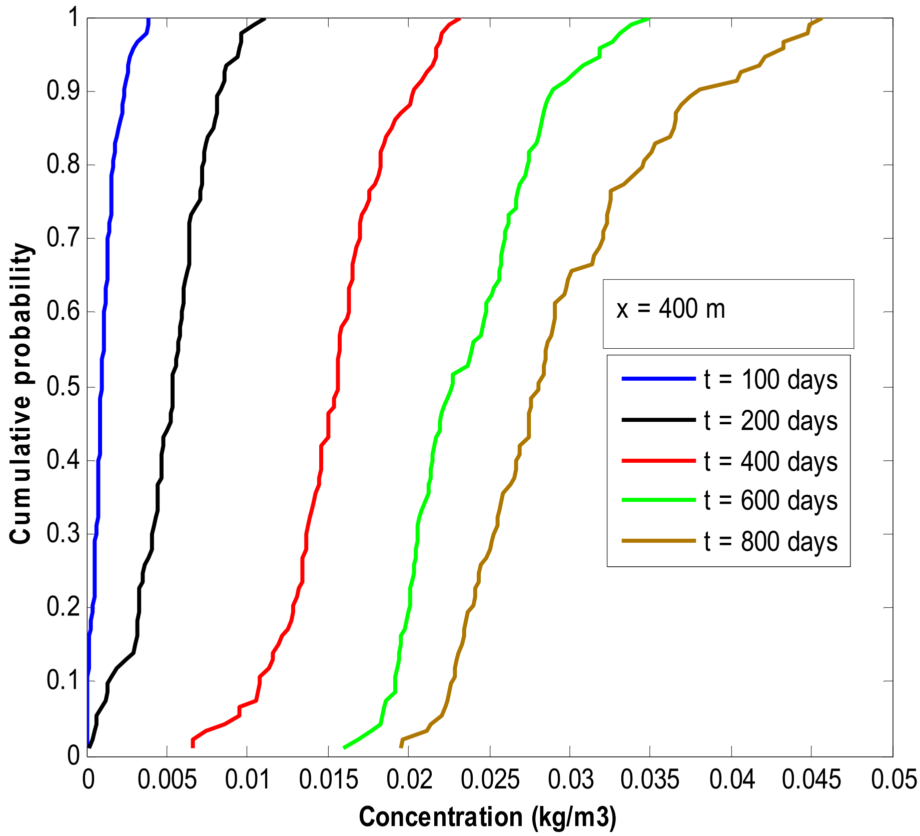

Fig. 6. Cumulative Probability plot of concentration. at x = 400 m for various time.

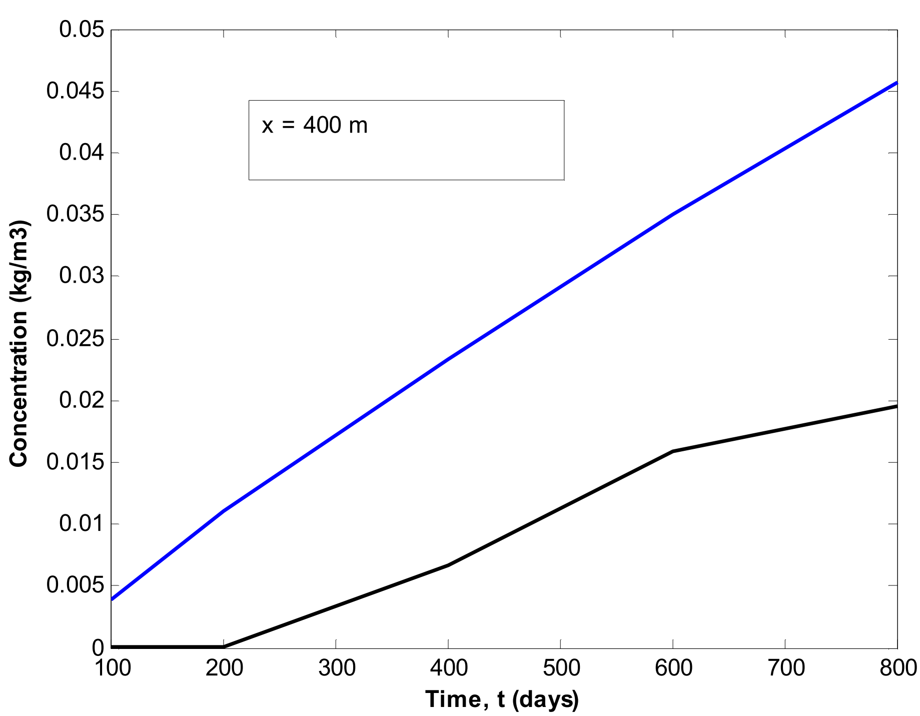

Fig. 7. Lower and Upper tolerance limits of concentration at x = 400 m for various time.

basically provides the knowledge of sample size or experimental runs. Wherever simulation is required for uncertainty quantification with a high precision, sample size can be best estimated by using this concept. Wilks' method also provides the best estimates. Tolerance intervals calculate a confidence interval that contains a fixed percentage (or proportion) of the data. This is related to the confidence interval for the mean. There are two numbers for the tolerance interval: (1) the coverage probability, which represents the fixed percentage of the data to be covered, and (2) the confidence level at which decision is taken. The methodology has been demonstrated with some simple examples in section IV and a case study of setting tolerance limits for concentration of dissolved chemical species is presented in section V. Case study of the similar type can be extended to set the tolerance limits of the radioactive effluent discharge during the routine operation of any nuclear plant. Since tolerance limits can be accepted as regulatory limits, we can say that Wilks' method can be applied as a safety tool for regulators. It can also play a major role of best estimate plus uncertainty methods in major nuclear power plant modifications.

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