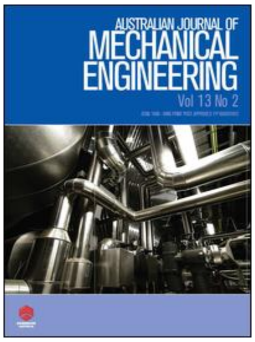

Taylor & Francis
Taylor & Francis Group

Australian Journal of Mechanical Engineering

ISSN: 1448-4846 (Print) 2204-2253 (Online) Journal homepage: http://www.tandfonline.com/loi/tmec20

Comparison of DEA frontier and ranking in performance assessment and optimisation of jatropha and fish oil biodiesel blend in CI engine

Venkat Pranesh, Balaji Rakesh, S. Mahalingam, S. Senthil & Rittik Kashyap

To cite this article: Venkat Pranesh, Balaji Rakesh, S. Mahalingam, S. Senthil & Rittik Kashyap (2018): Comparison of DEA frontier and ranking in performance assessment and optimisation of jatropha and fish oil biodiesel blend in CI engine, Australian Journal of Mechanical Engineering, DOI: 10.1080/14484846.2018.1429039

To link to this article: https://doi.org/10.1080/14484846.2018.1429039

AUSTRALIAN JOURNAL OF MECHANICAL ENGINEERING, 2018
https://doi.org/10.1080/14484846.2018.1429039

Comparison of DEA frontier and ranking in performance assessment and optimisation of jatropha and fish oil biodiesel blend in CI engine

Venkat Pranesh[a], Balaji Rakesh[b], S. Mahalingam[c], S. Senthil[d] and Rittik Kashyap[e]

[a]Department of Petroleum Engineering, Vels University, Chennai, India; [b]Department of Aerospace Engineering, University of North Dakota, Grand Forks, ND, USA; [c]Department of Mechanical Engineering, Sona College of Technology, Salem, India; [d]Department of Mechanical Engineering, Kamaraj College of Engineering and Technology, Virudhunagar, India; [e]Department of Sociology, Indira Gandhi National Open University, New Delhi, India

ABSTRACT
This paper presents a novel integrated statistical model to evaluate the diesel engine performance that is operating with duel fuel Jatropha and fish oil blend. Hence, a single cylinder, four stroke, direct Injection constant speed diesel engine was used. The experiments were taken from each category of the injection timing such as retardation, standard and advanced along with single proportion 100% biodiesel blend while other parameters' injection pressure was maintained constant at 220 bar, respectively. The injection timing was changed during testing of the engine and the performance parameters were held for three different load conditions between 1, 50 and 100% at the rated power of 4.4 kW and a speed of 1500 rpm. From the experimental results, it was observed that BSFC tends to decrease from no load to full load. The maximum BTE for JF100 was found in retardation injection timing of 18° at full load. DEA a multi-response linear programming statistical optimisation tool was used to measure the performance and efficiency of a Biodiesel blend in the DI diesel engine. It was formulated by taking fuel blends relative performance efficiency in relation to input and output variables. Both Frontier and Ranking methods in DEA indicated nearly a good convergence with experimental results.

ARTICLE HISTORY
Received 8 May 2017
Accepted 11 January 2018

KEYWORDS
Dual Fuel; injection timing; diesel engine; performance; DEAF; DEAR

Nomenclature

DI	direct injection
DEA	data envelopment analysis
CI	compression ignition
DEAF	data envelopment analysis frontier
CIE	compression ignition engine
DMU	decision-making unit
D100	pure diesel fuel
DEAR	data envelopment analysis ranking
JF100	50% jatropha Oil + 50% fish oil
CRS	constant return to scale
IT	injection timing
Ac	actual
IP	injection pressure
Tr	target
MFR	mass flow rate
AEDS	all experimental data samples
Cc	cubic capacity
MRPI	multi-response performance index
deg	degree
W	response weight
CA	crank angle
AOWM	assignment of weight method
MGP	maximum gas cylinder pressure
S/N	noise to sound ratio
BSFC	brake-specific fuel consumption
DEA	data envelopment analysis
BTE	brake thermal efficiency
DMU	decision-making unit
bTDC	before top dead centre
DEA	data envelopment analysis

1. Introduction

Fossil Fuels have been recorded as the highest global consumption, due to this there is a rapid depletion of fossil fuels, especially petroleum products. The surplus exhausts of fuel have bolstered in search of bio-fuels and the top petroleum companies are showing concern in alternative energy. British Petroleum is a British multi-national oil and gas company has invested eight billion dollars in alternative energy (Dudley 2013). So, bio-fuels have gathered great attention among the peoples all around the world. Compression Ignition Engines are more effective to operate than gasoline-powered engines,

CONTACT Venkat Pranesh ✉ venkpran2092@gmail.com

partly because diesel fuel costs less and consume less fuel. A disadvantage of the diesel engine is the production of sooty, smelly smoke and high exhaust emission which causes contamination and damage to the environment also diesel engine performance is also noisy and subject to a higher vibration. Diesel Engines are suitable mostly for the automotive, agricultural, shipping and locomotive industries. The renewable liquid or gaseous transport fuels made from organic waste have come forth as potential alternative fuels to satisfy the global energy needs. In this research work, we have used dual fuel half percent jatropha oil and fish oil blend (JF100) as an alternative bio-dual fuel for the DI diesel engine.

Jatropha curcas belongs to the Euphorbiaceae family. Jatropha is a multipurpose plant that originated in Central America, but can now be found throughout the tropics, including Africa and Asia. As a second-generation (non-food supply) bio-fuel crop, it can afford and sustainable help to provide a portion of the current fuel supply with minimal environmental impact. The goal of second-generation bio-fuel is to increase the bio-fuel supply with crops such as Jatropha, castor and Camelina. Jatropha yields a considerable amount of inedible oil that can be converted to biodiesel. The oil can be used as a direct replacement for fuel in engines and machines, and it has other industrial and commercial uses as well (Nahar and Hampton 2011). Fish oil is oil derived from the tissues of oily fish. Fish oils contain the omega-3 fatty acids eicosapentaenoic acid and docosahexaenoic acid, precursors of certain eicosanoids that are known to reduce inflammation in the body, and have other health benefits (Moghadasian and Mohammed 2008). The fish used as sources do not actually produce omega-3 fatty acids, but instead accumulate them by consuming either microalgae or prey fish that have accumulated omega-3 fatty acids, together with a high quantity of antioxidants such as iodide and selenium, from microalgae, where these antioxidants are able to protect the fragile polyunsaturated lipids from peroxidation (Venturi and Venturi 2007; Venturi et al., 2000; 2007).

How et al. (2018) analysed the injection timing influence and examined the split injection strategies on the performance and combustion characteristics of diesel engine, which is operated with biodiesel blended fuels. Medium duty diesel engine was investigated with B20 and B50 blends, and the injection timing was changed during engine testing. Generally, in standard and advanced injection timing modes, the engine will emit high amount of nitrogen oxides (NOx). But in this case, it was observed that NOx level below 100 ppm can be achieved when a CI engine running on retardation injection timing mode. Therefore, the authors suggest that retardation timing is a good parameter for achieving good outcomes such as efficient combustion, high thermal and mechanical efficiency, less fuel consumption and less emissions. Some research suggests that in addition to retardation injection timing, an injection pressure also plays a vital role in the production of higher biodiesel combustion rate in diesel engine (Shameer and Ramesh 2018).

Ganapathy, Gakkhar, and Mutugesan (2011) investigated the compression ignition engine performance and emission characteristics fuelled with jatropha biodiesel. Injection timing was altered during the experiment and it results showed that the optimal injection timing for Jatropha biodiesel with minimum BSFC, and maximum BTE is 340 CAD (crank angle degree). Pradhan, Raheman, and Padhee (2014) analysed the Combustion and performance of a diesel engine with preheated Jatropha curcas oil using waste heat from exhaust gas Experiments were conducted to evaluate the combustion characteristics of a DI (direct injection) diesel engine using PJO (preheated Jatropha oil). It exhibited a marginally higher cylinder gas pressure, rate of pressure rise and heat release rate as compared to HSD (high speed diesel) during the initial stages of combustion for all engine loadings. Ignition delay was shorter for PJO as compared to HSD. The results also indicated that BSFC (brake-specific fuel consumption) and EGT (exhaust gas temperature) increased while BTE (brake thermal efficiency) decreased with PJO as compared to HSD for all engine loadings.

Mahalingam, Suresh Mohan Kumar, and Pranesh (2013) studied the performance and emission characteristics of jatropha and rubber seed oil blend with diesel in compression ignition engine with variation of fuel injection pressures namely 200, 220 and 240 bar. From the results, it was observed that CO_2, HC and CO emissions were reduced about 5–10%, when the fuel injection pressure is increased with less amount of biodiesel blends. However, the smoke value and NOx is increased significantly when the injection pressure is reduced to 200 bar. Jindal et al. (2010) experimentally investigated the effects of compression ratio and injection pressure in a DI diesel engine running on jatropha methyl ester. Results indicated that the compression ratio and injection pressure increases the BTE and reduces BSFC while having lower emissions.

Ushakov, Valland, and Aesoy (2013) tested pure and 50% fish oil blends with conventional low-sulphur marine gas oil in a direct injection heavy-duty diesel engine. Experiments were performed at various operating conditions under standard propulsion and generator mode marine cycles. Fish oil showed fairly good combustion and ignition properties, which were very similar to those of marine gas oil. Bhaskar, Nagarajan, and Sampath (2013) experimentally optimised the FOME (fish oil methyl esters) blend and EGR (exhaust gas recirculation) for simultaneous control of NOx and particulate matter emissions in diesel engines. In this work, performance and emission characteristics of FOME and its blends are evaluated in a direct-injection single-cylinder constant-speed diesel engine primarily used in the agricultural sector. It is seen that 20% FOME blend gives almost the same BTE with lower unburned hydrocarbons, carbon monoxide and soot emissions, but higher NOx emissions compared to diesel fuel.

Sakthivel et al. (2014) analysed the performance, emission and combustion parameters of diesel engine fuelled with ethyl esters of fish oil and its diesel blends. Various properties such as viscosity, density, calorific value, flash point and cetane value of biodiesel and biodiesel–diesel blends of different proportions were investigated. Later, experimental tests were carried out to evaluate the performance, emission and combustion characteristics of a single cylinder, constant speed, DI diesel engine using biodiesel–diesel blends, under variable load conditions. The BTE was found to be higher compared to diesel for the entire load. An analysis of the cylinder pressure rise, heat release and other combustion parameters such as peak pressure, rate of pressure rise, combustion duration and ignition delay was carried out. The ignition delay, maximum heat release rate and combustion duration were lower for biodiesel–diesel blends compared to diesel. Behcet, Yumrutas, and Oktag (2014) studied and compared the effects of fuel produced from fish and cooking oils on performance and emissions of a diesel engine. Two fuels called as FOME and COME (Cooking Oil Methyl Ester) were produced from waste fish and cooking oils using the transesterification method. An experimental study was performed for investigating the performance and exhaust emissions of the Diesel engine using the fuels. According to the test results, it was observed that the fish oil-based fuel indicated better performance and exhaust emission parameters than those of cooking oil. The Result clearly showed that the power and torque values were lower than those of the diesel and the BSFC for the produced fuels increased up to 5.69% compared to diesel fuel. Agarwal et al. (2013), investigated the effects of fuel injection strategies and injection timings on engine combustion, performance and emission characteristics. The experiments were directed at constant speed (2500 rpm) with two FIPs (500 and 1000 Bars, respectively) and different start of injection (SOI) timings. For advanced SOI, BMEP, BSFC and EGT reduced significantly.

Zhou et al. (2018) conducted investigations in the sustainability of applying DEA in applications. The authors suggest that DEA technique is used in the application or resource optimisation since 1980 and almost all industries employed this method to optimise their resources. They reported that an innovative method in the DEA is still undeveloped and no new method has come into practice. The authors pointed out that this technique could be old, but it's very efficient and reliable to find the best parameter, which is affecting an engineering system. But, in our paper, we have employed a new technique in DEA – that is we have combined the frontier and ranking methods to evaluate an engine efficiency. Usually, frontier method is widely applied in all systems and ranking method is used rarely since it

Table 1. Properties of fuels.

Property	Unit	Diesel	Jatropha oil	Fish oil	Biodiesel (JF100)
Density	gm/cc	0.745	0.826	0.854	0.9
Kinematic viscosity at 40 °C	mm^2/s	3.4	4.4	15.2	4.3
Flash point	°C	68	132	154	42
Calorific value	(kJ/kg)	42000	38500	39670	39948
Specific gravity	–	0.74	0.96	0.85	0.91
Carbon residue	%	0.12	0.61	0.18	0.24
Iodine value	–	0.067	120.5	132.42	135.17

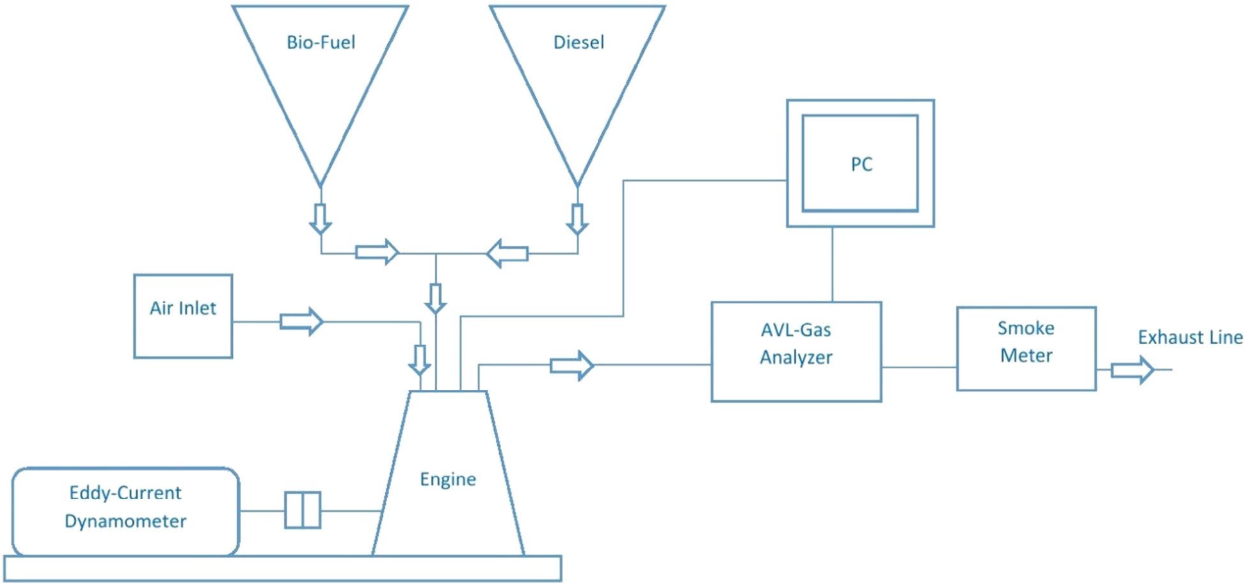

Figure 1. Schematic diagram of tested engine layout.

Table 2. Kirloskar engine specifications.

Specifications	Values
Bore	87.5 mm
Stroke	110.0 mm
Speed	1500 (constant speed)
Compression ratio	17.5:1
Rated power	4.4 KW
Number of cylinder	1
Type of cooling	Air cooled-eddy current dynamometer
Injection opening	24° (BTDC)
Pressure	220 bar
Number of stroke	4 stroke

contains lots of experimental design constrains. Recent reporting indicates that by combining DEA with ranking efficient decision-making units (DMU) can yield good results. Additionally, few algorithms were developed to effectively apply this combined technique in complex applications (Blas, Martin, and Gonzalez 2018). Seiford and Thrall (1990) examined the effect of model orientation on the efficient frontier and the effect of convexity requirements on returns to scale. Transformations between models are provided. Methodological

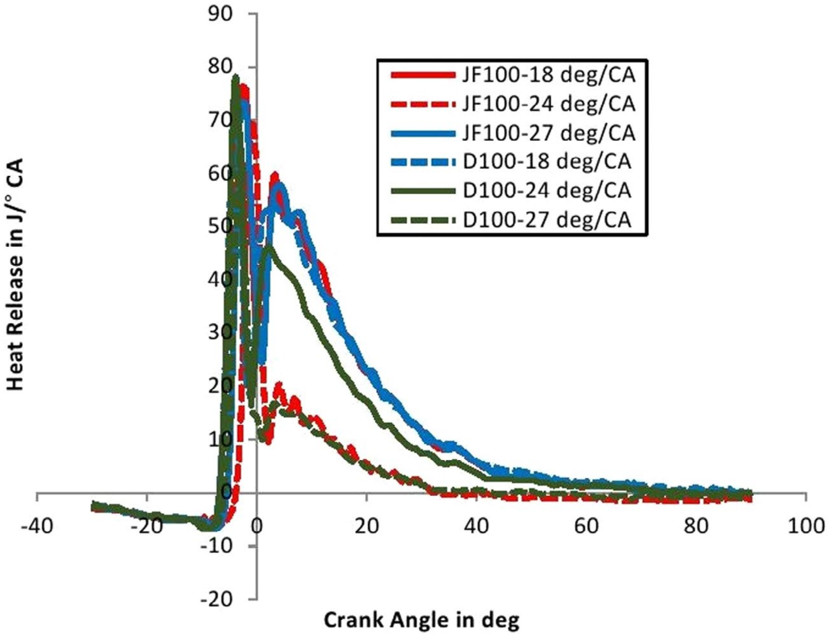

Figure 2. Variation of heat release with respect to crank angle.

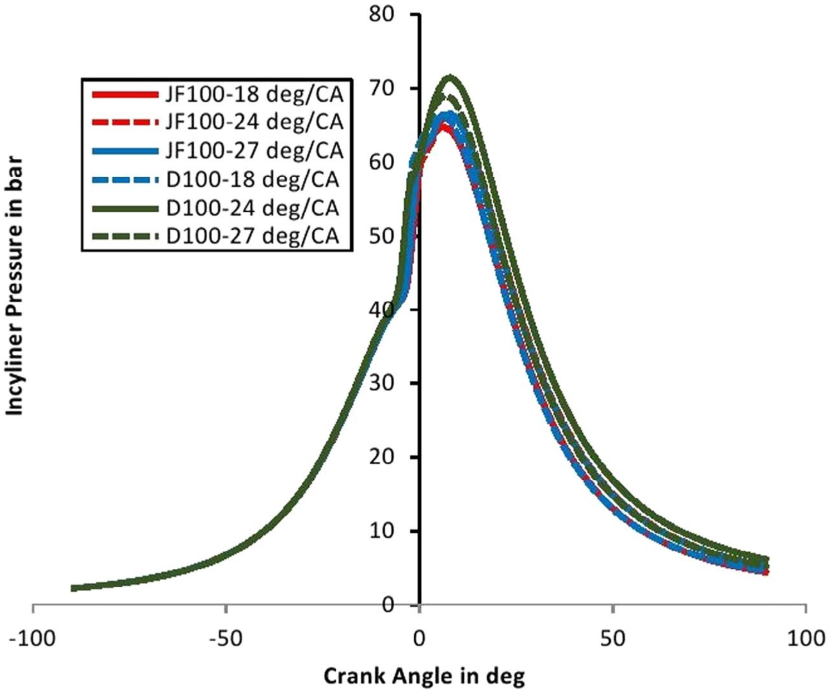

Figure 3. Variation of in-cylinder pressure with respect to crank angle.

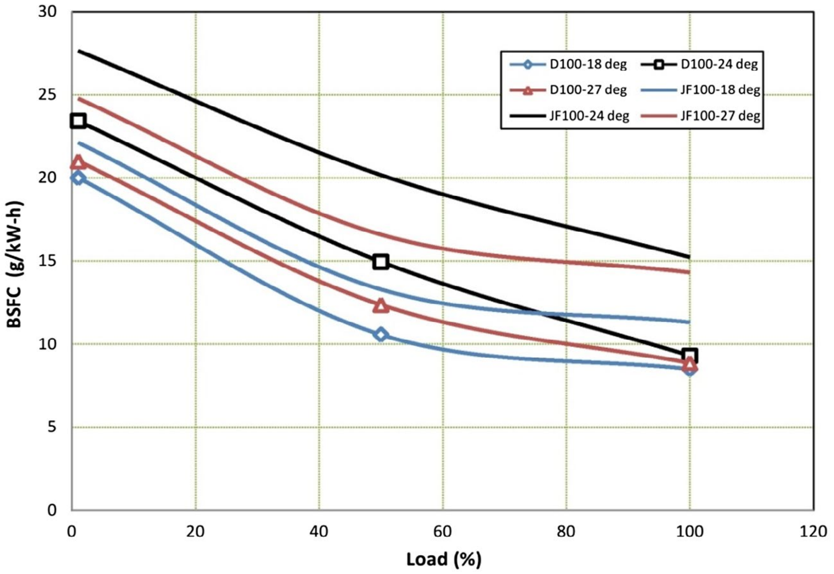

Figure 4. BSFC for various engine load conditions at 18°, 24° and 27°.

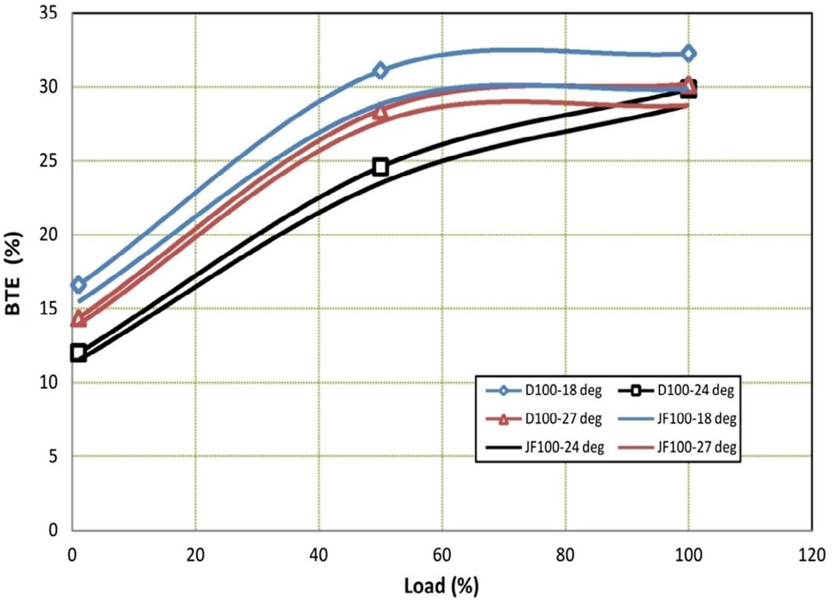

Figure 5. BTE for various engine load conditions at 18°, 24° and 27°.

extensions and alternate models that have proposed are reviewed and the advantages and limitations of a DEA approach are quantified by developing a mathematical programming approach to frontier estimation. Vessal (2007) evaluated the comparative efficiencies of several universities in two different time periods. Using this technique, it is potential to identify which schools are relatively inefficient compared to the composite school efficiency ratings change could be attributed to alterations in their inputs and outputs. Yu and Wang (2006) evaluated the relative performance efficiency in peoples' hospital of Perking University, China. Data Envelopment Analysis (DEA is a linear programming-based technique for assessing the relative performance efficiency of organisational units where the presence of multiple inputs and outputs) was applied. They set up and compared the relative performance efficiency of various departments based on source data from the hospital. From the report, it is clearly inferred that DEA can be utilised to assist in resource allocation decision, such as beds relocation, staff appointment and medication process improvement [20].

Lin, Lo, and Chein (2003) evaluated the overall efficiency, technical efficiency and Scale efficiency in Taiwan Power Company service centre in China by using DEA. The sources studied the number of Staff, General equipment as input parameters and number of Customer, Distribution network transformer capacity as output parameters. Khodabakhsi and Aryavash (2012) studied an equitable method for ranking DMUs based on the DEA concept. Author used the minimum and maximum efficiency values of each DMU to compute under the assumptions that the sum of efficiency values of all DMU's is equal to unity, the rank of each DMU is determined in proportion to a combination of its minimum and maximum efficiency values. Washio and Yamada (2013) proposed a model called rank-based measure (RBM) to evaluate DMU from a different standpoint. The author suggested a method to obtain a weight which gives the best rank, and estimated a weight between maximising the efficiency score and continuing the best rank; this can be possible if user continuously solves the linear programming problems. Sadjadi et al. (2011) presented a new method which incorporates the robust counterpart of super-efficiency DEA. The perturbation and uncertainty in data are assumed an ellipsoidal set and the robust super-efficiency DEA model is extended and the author implemented this method to rank different gas companies in Iran. Shanling, Jahanshahloo, and Khodabakhshi (2007) analysed the ranking DEA efficient units has become the pursuit of many DEA researchers and a variety of models were developed to rank DEA efficient units. The authors studied the traditional DEA ranking methods and found that some models have the disadvantages of being impossible, for that author researched to develop a super-efficient model to overcome some deficiencies in the earlier models.

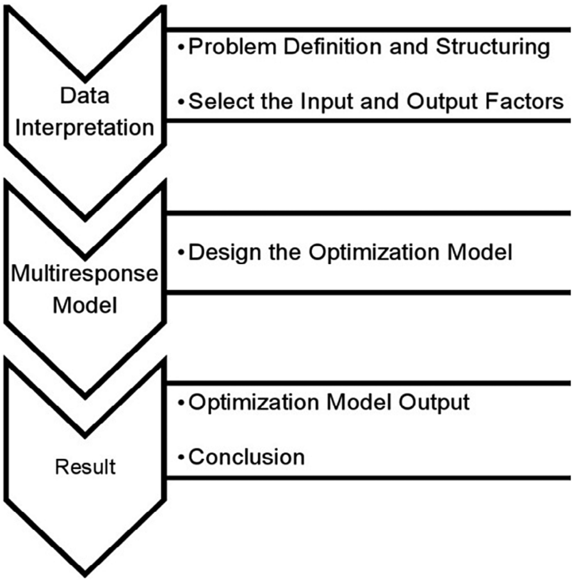

Figure 6. Methodology for CI Engine performance evaluation process.

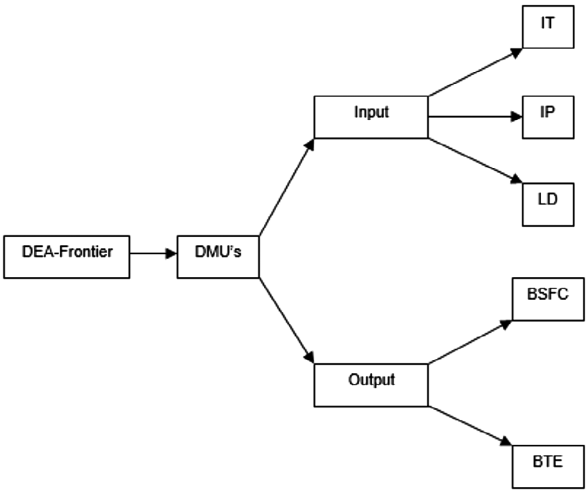

Figure 7. Proposed DEAF network model for CIE in relation to input and output.

In brief summary of literature study, the important findings were critically reviewed; it can be understood that the various ranges of injection timing have an effect on fuels. DEAF and DEAR techniques are widely applied in many applications and to our author's knowledge this is the first paper to implement this both techniques in a diesel engine to evaluate its performance and efficiency. Generally, all analytical modelling and optimisation tool does not exactly emphasise the application and it only signifies the modelling tool that is the main reason why an author's unable to find relevant literature studies on the DEAF and DEAR technique in engine applications. The modelling methodology is first carried out for the frontier and then followed by ranking; by doing this, it is easy to identify specifically the best DMU in the frontier. This theory of DEA comparing was executed successfully in the application. Subsequently, this technique will promote the DEA researcher's a novel approach for evaluating the performance and efficiency of any system. Please note that the emissions were not considered. This paper only focuses on the sections of combustion, performance characteristics and their optimisations.

2. Experimental methods

In this work, the transesterified dual biodiesel was used for obtaining the performance and emission characteristics of a single cylinder, constant speed DI diesel engine running at 1500 rpm and at a constant injection pressure of 220 bar as per engine's designed pressure. The injection timing 18°, 24° and 27° were changed. The diesel engine was performed the change of rated power to measure the carrying out performance and emission characteristics with some engine modification, the injection pressure was varied by changing the spring tension

Table 3. SIPOC diagram for diesel engine.

Suppliers	Inputs	Process	Output	Customer
Bio-fuel agencies	IP	Running the engine with biodiesel at different IT and LD	Engine performance	Universities
Laboratory	IT		Analytical modelling	Industries
	LD			Other organisations

of the injector needle with setting screw. One lower and one higher injection pressure were selected to identify the trend. Table 1 gives the properties of the fuels considered for the experimental work and Figure 1 shows the engine setup line diagram.

Table 4. DMU's inputs and outputs factors.

Fuel name	DMU name	IP (bar)	IT (Deg)	LD (%)	BSFC (g/kW-h)	BTE (%)
JF100	1	220	18	1	22.12	15.46
JF100	2	220	24	50	13.29	28.82
JF100	3	220	27	100	11.31	29.87
JF100	4	220	18	1	27.64	11.5
JF100	5	220	24	50	20.17	23.48
JF100	6	220	27	100	15.22	28.77
JF100	7	220	18	1	24.77	13.91
JF100	8	220	24	50	16.58	27.62
JF100	9	220	27	100	14.3	28.74

Table 5. CRS efficiency.

Fuel name	DMU name	Input-oriented CRS efficiency	Sum of lambdas	RTS
JF100	1	1.00000	1.000	Constant
JF100	2	1.00000	1.000	Constant
JF100	3	1.00000	1.000	Constant
JF100	4	1.00000	1.000	Constant
JF100	5	1.00000	1.000	Constant
JF100	6	1.00000	1.000	Constant
JF100	7	1.00000	1.000	Constant
JF100	8	1.00000	1.000	Constant
JF100	**9**	**0.99126**	**0.991**	**Increasing**

Note: The bold values indicates a change in the input and output DEA system.

Table 6. DMU's target inputs.

Fuel name	DMU name	IP (bar)	IT (Deg)	LD (%)
JF100	1	220.00000	18.00000	1.00000
JF100	2	220.00000	24.00000	50.00000
JF100	3	220.00000	27.00000	100.00000
JF100	4	220.00000	18.00000	1.00000
JF100	5	220.00000	24.00000	50.00000
JF100	6	220.00000	27.00000	100.00000
JF100	7	220.00000	18.00000	1.00000
JF100	8	220.00000	24.00000	50.00000
JF100	**9**	**218.07753**	**22.58260**	**99.12615**

Note: The bold values indicates a change in the input and output DEA system.

A single cylinder, constant speed DI engine was employed to evaluate the engine operation and emission characteristics with biodiesel. The diesel engine operates under different load conditions at a constant speed of 1500 rpm with the different biodiesel proportions. The diesel engine of Kirloskar model was instantly attached to an eddy current dynamometer for changing the different loads (1, 50 and 100%). The different types of measuring devices were mounted in the test engine such as an orifice meter with U tube manometer for measuring air consumption, the one litre burette for fuel consumption and the separate bio-fuel tank. A Hartridge smoke meter was provided for measuring the smoke opacity and exhaust temperatures. The test rig was set up with AVL software for obtaining various curves and effects during operation. A delta 1600S exhaust gas analyzer was used to quantify the emission characteristics such as CO_2, CO, UHC, NOx and O_2 values from the exhaust gas; however, the emissions are not discussed in this paper. The performance and emission test was taken for the compression ratio of 17.5 at a constant injection pressure 220 bar at rated power of 4.4 kW. The injection timing was changed during the engine operation and the trial was held out at full proportion 100% biodiesel. The performance analysis of the engine at different rated power was measured in terms of BSFC and BTE. The combustion analysis was measured in terms of heat release and in-cylinder pressure. The specifications of the test engine are described in Table 2. The uncertainty analysis associated with this research project is presented in Appendix 1.

3. Combustion analysis

This section analyses the combustion outcomes of a diesel engine that ran with duel fuel of different species.

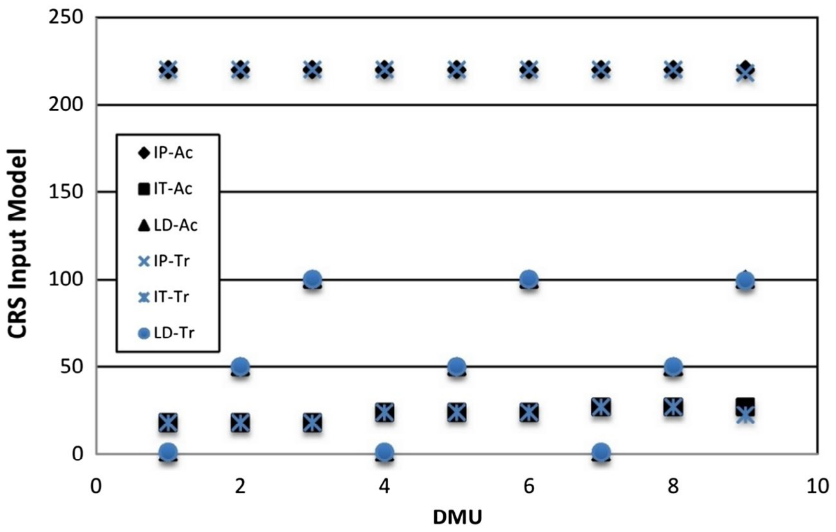

Figure 8. Actual and target inputs in frontier analysis.

3.1. *Heat release rate*

The heat release rate for JF100 and D100 at fuel injection timings of 18°, 24° and 27° are shown in Figure 2. It can be found that the heat release rate for JF100 at almost all three injection timings is closer to that of standard diesel. When the injection timing is altered from standard timing the ignition delay period reduces and thereby producing higher heat release subsequently this enhances the combustion process. The ignition delay period is more sensitive to injection timing than fuel injection pressure. The heat release rate (HRR) for JF100 at 18°, 24° and 27° are 78, 70 and 74/°CA while the diesel gave 79, 75 and 77 J/°CA at the same levels of injection timings. In 18° BTDC only the best HRR can be obtained this may be due to the energy content per unit mass is closer to diesel fuel. This signifies the higher burning rate owing to improvements in diffusion burning phase on account of fuel-bound oxygen content in biodiesel (Velmurugan, Loganathan, and Gunasekaran 2014). Normally, higher ignition delay reduced the heat release rate (Geo et al. 2017), but in this case a moderate result is obtained.

3.2. *Peak pressure*

Figure 3 shows the variation of in-cylinder pressure with respect to engine load. From the graph it is obvious that the maximum cylinder pressure was recorded at the advanced injection timing of 27° bTDC for D100 and JF100 it gave 69 and 70 bar. This may be due to the longer ignition delay period since more fuel accumulates resulting in an increase in peak pressure (Puhan et al. 2009). Ignition delay reduces in retardation fuel injection timing as it is evident from HRR curve. At 18° the ICP is recorded as 64 bar for JF100 and 65 bar for diesel fuel. Apart from ignition delay the fuel properties are playing a major role in accessing cylinder pressure. On the other hand the maximum in-cylinder pressure rises with an increase in load as the engine gains more heat (Velmurugan, Loganathan, and Gunasekaran 2014). After pressure rate rises at a highest point and drops heavily. Lakshmanan, Avinash, and Nagarajan (2017) explain that this phenomenon is due to chemical delay. High chemical delay of fuel reduces the peak pressure, which ultimately leads to temperature drop in the engine.

Table 7. DMU's target outputs.

Fuel name	DMU name	BSFC (g/KW-h)	BTE (%)
JF100	1	22.12000	15.46000
JF100	2	13.29000	28.82000
JF100	3	11.31000	29.87000
JF100	4	27.64000	11.50000
JF100	5	20.17000	23.48000
JF100	6	15.22000	28.77000
JF100	7	24.77000	13.91000
JF100	8	16.58000	27.62000
JF100	**9**	**14.300000**	**28.74000**

Note: The bold values indicates a change in the input and output DEA system.

4. Performance analysis

4.1. *Brake-specific fuel consumption*

The BSFC for various power output of the engine fuelled by JF100 bio-dual fuel blend and diesel is presented in Figure 4.

The compression ignition engine was operating in a synergistic effect on fuel consumption when fuelled with dual fuel (JF100), the blending of two different biomass sources that leads biodiesel to increase in enthalpy and exergy. From the figure, it can be seen that the BSFC starts to deteriorate from a single load to full load. Initially at 1% load the BSFC for both D100 and JF100 are higher this is due to insufficient combustion temperature. The high amount of fuel needs to burn at single load and consequently the emissions will be higher, by gradually varying the load the cylinder temperature and pressure starts to rise and conversely the BSFC tends to decrease. The minimum BSFC was reported in 18° retarded injection timing; it was 22.12 g/kW-h at 1%

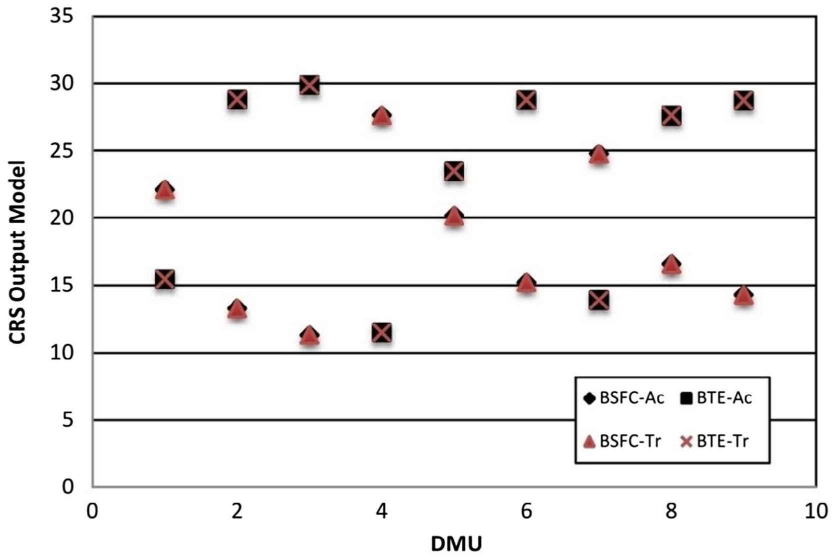

Figure 9. Actual and target outputs in frontier analysis.

load to 11.31 g/kW-h at 100% load. Whereas for diesel in same injection timing, it was at 20 g/kW-h in a single load of 8.5 in full load. More fuel consumption was found in standard IT (24°), it was 27.64 g/kW-h at single load to 15.22 g/kW-h at full load. Typically, it was rendered that the retardation injection timing has a superior effect on BSFC instead of standard and advanced injection timing. Another reason being that at full load, due to maximum gas cylinder pressure (MGP), there will be a less fuel consumption. This is the process by which the lesser amount of fuel gets easily and rapidly vaporised, thereby producing higher thermal efficiency and reduced fuel consumption.

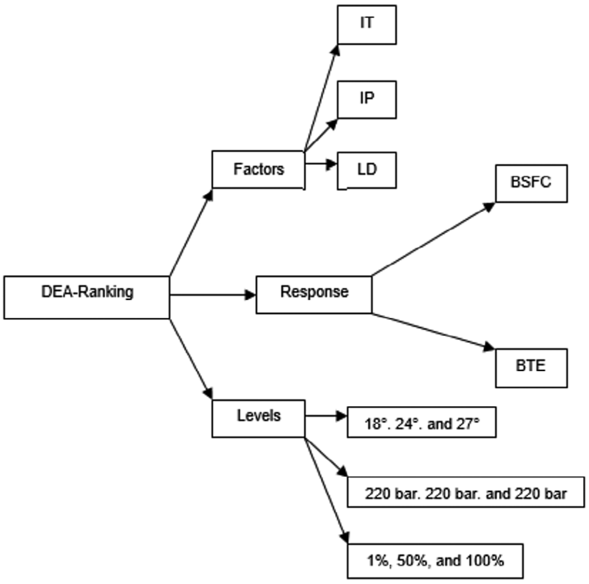

Figure 10. Proposed DEAR network model for CIE in relation to components and response.

Table 8. Factors and levels for CIE combustion process.

	Levels		
Factors	1	2	3
Injection pressure (A)	220	220	220
Injection timing (B)	18	24	27
Load (C)	1	50	100

Table 9. All experimental data for CI engine performance measurement.

	Factors				
Trial No.	A	B	C	BSFC	BTE
1	1	1	1	22.12	15.46
2	1	2	2	13.29	28.82
3	1	3	3	11.31	29.87
4	2	1	2	27.64	11.5
5	2	2	3	20.17	23.48
6	2	3	1	15.22	28.77
7	3	1	3	24.77	13.91
8	3	2	1	16.58	27.62
9	3	3	2	14.3	28.74

4.2. Brake thermal efficiency

The BTE for various power output of the engine fuelled with JF100 and D100 is presented in Figure 5.

The purpose of evaluating the BTE is to find the amount of fuel burned and converted into a mechanical output. The BTE of an engine increases significantly with load up to the rated load (Velmurugan, Loganathan, and Gunasekaran 2014). It can be observed from the figure that at 1% load the BTE for both D100 and JF100 was found to be higher than 10%, this is due to esteemed fuel properties of jatropha and fish oil blend. However, the fuel calorific value and atomisation rate determines the rate of BTE (Geo et al. 2017). The diesel fuel gave the best result at full load for all the three categories of injection timing 18°, 24° and 27° the values are 32.36, 29.84 and 30.19%. The BTE for the JF100 jatropha and fish oil blend at 18° was better than that of other 24° and 27° injection timings; it gave 29.87% providing the finest result for BTE in the case of dual biodiesel, this may be due at full load and high in-cylinder temperature the less quantity of fuel being injected into the engine equals energy input to the engine. Therefore, the blending of two different fuels in equal proportion will stimulate the thermal efficiency.

5. Optimisational modelling: a statistical-based approach

DEAF and DEAR methodology is mentioned in Figure 6.

The foundation and implementation of analytical modelling consists of three major components, they are data interpretation, multi-response model and result. The first component is about understanding and defining the problem and then to the selection of appropriate inputs and outputs. The second component is to design the model, in this stage is to design the optimal model according to the problem structure and the third component consists of designing optimisation model's output assessment analysis and conclusion. This same analytical modelling procedure is followed for DEAF and DEAR methods in this paper.

5.1. DEA frontier

DEA is a non-parametric technique in operation research and economics for estimation of production frontiers. It is used to empirically measure productive efficiency of DMU. Non-parametric approaches have the benefit of not considering a particular functional form/pattern for the frontier; even so they do not offer a general relationship (equation) relating output and input (Ramanathan 2003).

5.1.1. SIPOC diagram

SIPOC Stands for Suppliers Inputs Process Outputs Customer Diagram; it is a boundary condition of DEA

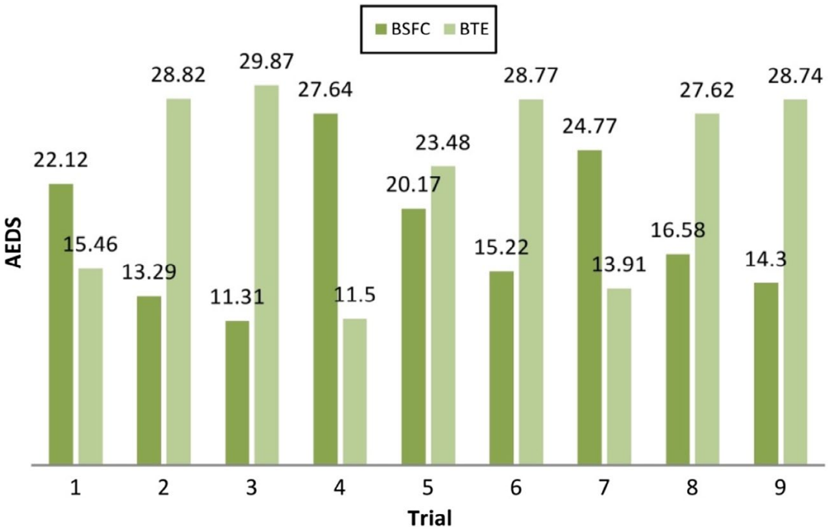

Figure 11. AEDS for various trial conditions.

Table 10. Weights values and response weights.

Trial. no	BSFC	W_{BSFC}	BTE	W_{BTE}
1	22.12	0.1337	15.46	0.0743
2	13.29	0.0804	28.82	0.1384
3	11.31	0.0684	29.87	0.1435
4	27.64	0.1671	11.5	0.0552
5	20.17	0.1219	23.48	0.1128
6	15.22	0.0920	28.77	0.1382
7	24.77	0.1498	13.91	0.0668
8	16.58	0.1002	27.62	0.1327
9	14.3	0.0865	28.74	0.1381

Model. SIPOC diagram is a tool applied by a team to identify all relevant of a process improvement of the project before starting employment. SIPOC (Suppliers, Inputs, Process, Output and Customers) analysis applied to interpret the central components of the process and defined the limit of the process. The accompanying Table 3 shows the SIPOC diagram for diesel engine.

5.1.2. Mathematical formulation

5.1.2.1. Efficiency measure.

Efficiency for the purpose of DEA is defined as the ratio of weighted output to weighted input. Therefore, if $X_{1j,}\ X_{2j,}\ X_{3j}\ldots.\ X_{mi}$ are the m inputs and $Y_{1j,}\ Y_{2j,}\ Y_{3j}\ldots.\ Y_{ni}$ are the n inputs of the unit j then its efficiency θ, are defined $V_1, V_2, V_3 \ldots .V_n \geq 0$ as

$$\theta = \frac{v_1Y_{1j} + v_2Y_{2j} + v_3Y_{3j} + \ldots\ldots\ldots\ldots + v_nY_{ni}}{u_1X_{1j} + u_2X_{2j} + u_3X_{3j} + \ldots\ldots\ldots U_mX_{mi}} \quad (1)$$

PUT Where, $V_1, V_2, V_3 \ldots.\ V_n$ are weights for the outputs and $U_{1,}\ U_{2,}\ U_3 \ldots\ U_m$ are weights for the inputs.

5.1.2.2. DEA model.

$$\text{Maximize.}\theta = \frac{v_1Y_{10} + v_2Y_{20} + v_3Y_{30} + \ldots\ldots\ldots\ldots + v_nY_{n0}}{u_1X_{10} + u_2X_{20} + u_3X_{30} + \ldots\ldots\ldots U_mX_{m0}} \quad (2)$$

$j = 1 \ldots\ldots n$

$$\frac{v_1Y_{1j} + v_2Y_{2j} + v_3Y_{3j} + \ldots\ldots\ldots\ldots + v_nY_{ni}}{u_1X_{1j} + u_2X_{2j} + u_3X_{3j} + \ldots\ldots\ldots U_mX_{mi}} \leq 1 \quad (3)$$

$$V_1, V_2, V_3 \ldots .V_n \geq 0$$

$$U_{1,} U_{2,} U_3 \ldots ..\ U_m \geq 0$$

- Where θ is the designated unit for an optimisation run and n is the total number of the units in the study. That is, in each optimisation run the efficiency of a specific unit is maximised and it is then repeated for all the units.
- Instead of solving the problem as stated above an equivalent model is usually solved since it requires lesser computation and easier to implement. The equivalent representation is obtained by first converting the optimisation problem into a linear programming (LP) problem and then using the duality principle, which gives the following model: Minimise θ Subject to

$$\sum_{j=1}^{n} \lambda_j X_{ij} + S_i^- = \theta X_0$$

$$\sum_{j=1}^{n} \lambda_j Y_{ij} - S_i^+ = Y_0$$

$$i = 1 \ldots \ldots m; j = 1 \ldots \ldots .n$$

X_{ij} – the amount of input i used by DMU j, Y_{rj} – the amount of output r used by DMU j, S_i^- – non zero input slack, S_r^- – non zero output slack, n – number of DMUs, m – number of inputs, DMU is efficient when the following two conditions are satisfied.

(1) $\theta_0 = 1$
(2) $S_i^-, S_r^+ = 0$

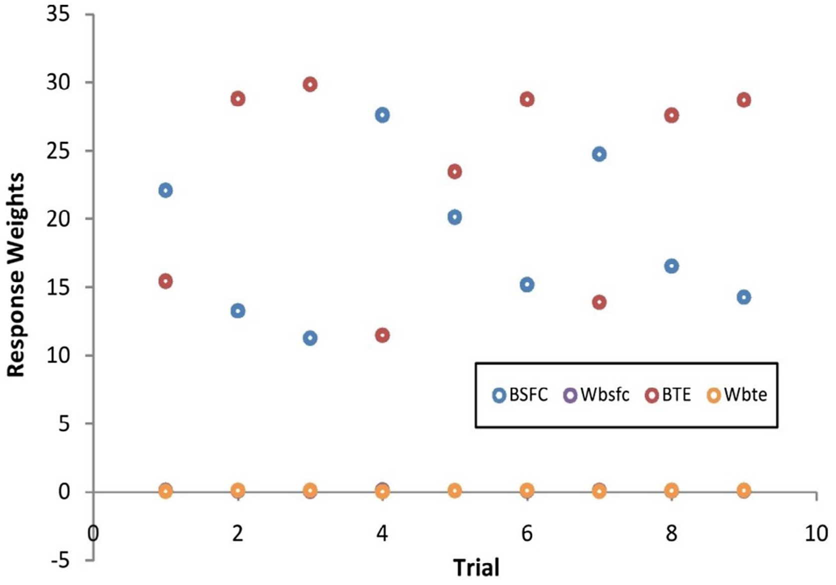

Figure 12. Response Weights for various trial conditions.

Table 11. Weighted responses and MRPI

Trial. no	BSFC × W_{BSFC} (P)	BTE × W_{BTE} (Q)	MRPI = P/Q × 10^3
1	2.9574	1.1487	2574.56
2	1.0685	3.9887	267.88
3	0.7736	4.2863	180.48
4	4.6186	0.6348	7275.68
5	2.4587	2.6485	928.34
6	1.4002	3.9760	352.16
7	3.7105	0.9292	3993.22
8	1.6613	3.6652	453.26
9	1.2370	3.9690	311.67

Note: The bold values indicates a change in the input and output DEA system.

If suppose one DMU is inefficient, the modification of inputs and outputs can be calculated as follows to change and calculate target efficiency.

$$X^*_{i0} = \theta_0 X_{i0} - S^-_i \quad i = 1 \ldots m;$$

$$Y^*_{r0} = Y_{i0} + S^+_r \quad r = 1 \ldots m$$

X^*_{i0}, Y^*_{i0} are target inputs and outputs of an inefficient DMU_0

5.1.3. *Input and output factors*

The diesel engine system is counted only for measurable input parameters like IT, IP and LD and output proposed in this model includes BSFC and BTE. The proposed input and output network model for DEA frontier are mentioned in Figure 7. Table 4 shows the simplified DEA model with overall inputs and outputs.

The above output values are taken from all the experimental data samples (AEDS) recorded at each load (1, 50 and 100%).

5.1.4. *CRS efficiency table*

The below table shows the DEA-Frontier software output for CRS Efficiency: Diesel Engine system progress.

With the help of Linear Programming software DEA FRONTIER the result of each DMU can be well counted. As shown in Table 5, DEA identified DMU1 to DMU8 are technical and scale efficient and reaming DMU9 is technical scale inefficient (non zero input slack and non zero output slack are zero). DMU9 $\lambda_j/\theta = (j = 1 \ldots\ldots n)$ lesser than 1 so, this one DMU is said to be scale inefficient.

Table 6 establishes that the DMU9 Target inputs are very less compared to current input data; it shows the diesel engine with its fuel blends will reduce this much of input parameters. For example, the DMU9's injection timing (IP) is 27°, but Target IT is 22.58°, can be rounded off to 23°. This target input 23 deg is less than 4 deg from current input data of IT (27 deg). This actual input (Ac) and frontier CRS target input (Tr) is diagrammatically presented in Figure 8.

The Table 7 shows that there is no change in target outputs. Even though DMU9 is scale inefficient and no modifications were indicated by frontier analysis. Figure 9 shows the scatter diagram of actual and target outputs, it can be seen from the graph that the outputs (Ac &Tr) are located in the same place without any deviation.

5.2. *DEA ranking*

Data Envelopment Analysis-based Ranking is a multi-response linear programming optimisation tool; this is mostly done to optimise the multi-responses in Taguchi experiments. In this method, a set of original responses is mapped into a ratio (weighted sum of responses with larger-the better is divided by a weighted sum of responses with smaller-the better of nominal-the best) so that the optimal levels can be formed on this ratio (Krishnaiah and Shahabudeen 2012). This ratio can be treated as equivalent to MRPI (multi response

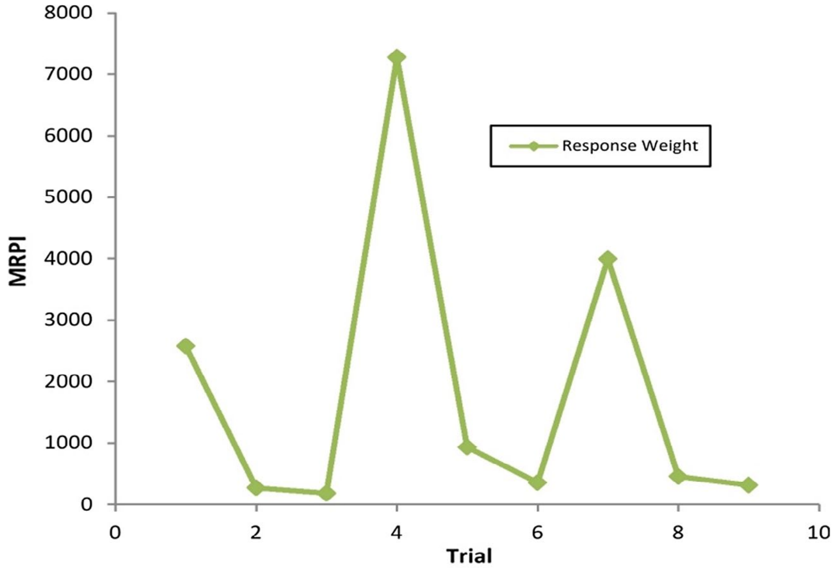

Figure 13. MRPI for various trial conditions.

Table 12. Level totals of MRPI for weights ranking.

Factors	Levels 1	 2	 3
Injection pressure (A)	3022.92	**8556.18**	4758.15
Injection timing (B)	**13843.46**	1649.48	844.31
Load (C)	3379.98	**7855.23**	5102.04

Note: The bold values indicates a change in the input and output DEA system.

Table 13. Optimum performance parameter value of DEA ranking method.

Factors	DEA ranking of MRPI	Predicted combination	DMU name
A	**8556.18**	220 bar	DMU-2
B	**13843.46**	18 deg/CA	
C	**7855.23**	50%	

Note: The bold values indicates a change in the input and output DEA system.

performance index). Figure 10 presents the proposed network model for DEAR method in relation to its components (factors and levels) to responses (output).

Krishnaiah and Shahabudeen (2012) describe about DEAR process as below:

- Step 1: Determine the weights associated with each response for all experiments using an appropriate weighting technique.
- Step 2: Transform the observed data of each response into weighted data by multiplying the observed data with its own weight.
- Step 3: Divide the weighted data of large-the better type with weighted data on smaller the better type or nominal-the best type.
- Step 4: Treat the value obtained in Step 3 as MRPI and obtain the solution.

As shown in the first step; we need to calculate the weights of each variable in order to proceed further in the data envelopment analysis rank-based method. For that Assignment of Weights method has been used; in this method, the multi-response problem is converted into a single response problem. Let W_1 be the weight assigned to, say the first response R_1 and W_2 be the weight assigned to the second response R_2. The total of the weighted response (W) will be the single response, where

$$W = W_1R_1 + W_2R_2 \tag{4}$$

This (W) is termed Multi Response Performance Index (MRPI). Using this MRPI, the problem is solved as a single response problem. In the multi-response problem, each response can be the original observed data or its transformation such as S/N ratio. In this approach, the major issue is the method of determining the weights. The following factors and levels (in Table 8) were selected for study of the performance assessment.

Where, injection pressure is in bar, injection timing in deg/CA and load in percentage. Other parameters like mass flow rate (10 cc) has been kept constant, so that this one parameter is not counted in this optimisation work. However, the injection pressure is also kept constant, but it was taken into an account since IP is a major factor which affects the CI engine's performance.

The all experimental data samples are presented in Table 9. There are two possible responses to this engine's performance assessment problem. One is the BCFC and the second is the BTE. It is to be noted that mathematically, BSFC and BTE varies alternatively in terms of magnitude, so here it is difficult to determine the larger and the better the type of quality characteristic. Figure 11 diagrammatically represents the all experimental data samples with respect to trial. Theoretically speaking, in optimisation problems the user determines the parameter values of a system for extreme mathematical analysis and obtaining best output.

Totally 9 trials have been conducted by taking 3 factors and 3 levels. An Orthogonal Array L^s

Where L = number of levels: $L = 3$, s = number of factors: $L = 3$, $3^3 = 27$ trials

The factorial design combinations are taken from L9 Orthogonal Array in Taguchi method. The weights are determined as follows. For BSFC and BTE (larger-the better characteristics), the individual response is divided by the total response value $\sum BSFC$ and $\sum BTE$

$$\sum \text{BSFC} = 165.4 \qquad \sum \text{BTE} = 208.17$$

$$W_{\text{BSFC}_1} = \frac{\text{BSFC}_1}{\sum \text{BSFC}} \qquad W_{\text{BTE}_1} = \frac{\text{BTE}_1}{\sum \text{BTE}}$$

For example, in the first test:

$$W_{\text{BSFC}_1} = \frac{22.12}{165.4} = 0.1337 \qquad W_{\text{BTE}_1} = \frac{15.46}{208.17} = 0.0743$$

The weights values and response weights for all the tests are presented in Table 10 (Figure 12).

The above table shows the weights values and response weights which are useful for proceeding next step to DEA ranking. The weighted responses and MRPI are mentioned in Table 11 and figure 12 indicates the response weightages for various trial conditions. Figure 13 represents the scatter diagram for variation of response weights with regard to the trial and the Figure 10 shows the multi response performance index (MRPI) variation with respect to trial. The optimal levels are identified by treating MRPI as a single response as it is mentioned in Table 11. The level totals of MRPI for weights are given in Table 13. So by this process the ranking and optimal level can be determined.

The above Table 12 shows the optimal levels based on maximum MRPI. They are A_2, B_1 and C_2: this specific optimal level rank or rate of trial (experiments) is known as ranking in data envelopment analysis. Therefore, the DEA ranking process has been successfully optimised for dual fuel, biodiesel (JF100) in compression ignition engine.

The above Table 13 indicates the optimum performance parameters and predicted combination of 3 factors, which is obtained from data envelopment analysis rank-based method. As per the DEA ranking the predicted combination of the combustion process is 220 bar (IP), 18° (IT) and 50% (LD). The combination A_2, B_1, C_2 obtained through DEAR method helps to make engineering judgment in frontier to determine the best and optimised DMU. Therefore, this DEAR combination made a solution that the DMU2 is the most excellent fuel parameter of this type of problem.

6. Conclusion

In this research work, it was found that initially the engine was able to run successfully with JF100. The injection pressure was maintained constant and only the injection timing was changed during testing of engine; here the injection timing is the dominant factor in this work. Based on the experimental and statistical optimisation results, the following conclusions can be drawn.

(1) BSFC tends to decrease from a single load to full load and the best result was observed for injection timing of 18°. The maximum BTE is found for JF100 18 deg/CA and its result is 29.87%. It is to be stated that the retardation injection timing is the best IT parameter to obtain maximum performance.

(2) With an assistance of DEA Frontier the performance and emission characteristics of diesel engine has been quantified. It was set up that the DMU 9 is a technical scale inefficient, with respect to its input and output parameters. The objectives of input have been indicated to develop the DMU's to be technical scale efficient. A multiple response problem taken with three independent variables, and it was formulated by the DEA Rank-based method. This method can be used to predict the system's performance within the experimental design. The experimental values have shown good correlation.

(3) Finally, it was concluded that out of 9 DMU's in DEA frontier, the DEAR evaluated the DMU2 is the efficient fuel parameter. Finally, this novel technique will promote the industrial engineers, Economists and R&D (research and development) experts in automotive, renewable and fossil energy industries to optimise the energy resources for efficient production and utilisation.

Acknowledgement

The authors would like to thank Yasar Arafath S. of Bahath Exim Company, Ramanathapuram, Tamil Nadu for having supplied the fish oil needed to conduct the experimental work.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Venkat Pranesh is currently an assistant professor of petroleum engineering at Vels University, Chennai, India. He is a reviewer of many energy-related journals and published several papers in that stream. He received his MSc in Petroleum and Environmental Technology from Coventry University, UK and BE in Mechanical and Production Engineering from Sathyabama University, Chennai, India. His areas of interest are petroleum engineering, thermal engineering, energy and fuels.

Balaji Rakesh is currently a MS student of aerospace engineering at the University of North Dakota, USA. He received his BE in Aeronautical Engineering from Sathyabama University. He has presented many papers in several national and international conferences and participated in global many technical global events. His areas of interest are in thermodynamics, energy, fuels and fluid mechanics.

S. Mahalingam is currently a professor of mechanical engineering at the Sona College Technology, Salem, Tamil Nadu, India. He obtained his PhD in Mechanical Engineering from Sathyabama University, ME in Thermal Engineering from Annamalai University, and all degrees is in Mechanical Engineering. He has published many papers in world class journals and conferences and his major areas of interest are IC Engine, combustion, solar energy and thermal engineering.

S. Senthil is currently a professor of mechanical engineering at Kamaraj College of Engineering and Technology, Virudhunagar, Tamil Nadu, India. He obtained his PhD from National Institute of Technology Calicut, ME from Thiagarajar College of Engineering, and BE from Arulmigu Kalasalingam College of Engineering. All degrees are in Mechanical Engineering. He has published many papers in world class journals and conferences and his major areas of interest are fuels, combustion, design and manufacturing.

Rittik Kashyap is an energy analyst and currently a MS student of sociology at Indira Ghandhi National Open University in New Delhi, India. He obtained his BE in Mechanical and Production Engineering from Sathyabama University, Chennai, India. He has participated in several technical workshops and his areas of interest are energy, petroleum, mechanical engineering, and manufacturing processes.

References

Agarwal, A. K., D. K. Srivastava, A. Dhar, R. K. Maurya, P. C. Shukla, and A. P. Singh. 2013. "Effect of Fuel Timing and Pressure on Combustion, Emission and Performance Characteristics of a Single Cylinder Diesel Engine." *Fuel* 111: 374–383.

Behcet, R., R. Yumrutas, and H. Oktag. 2014. "Effects of Duels Produced from Fish and Cooking Oils on Performance and Emissions of a Diesel Engine." *Energy* 71 (15): 645–655.

Bhaskar, K., G. Nagarajan, and S. Sampath. 2013. "Optimization of FOME (Fish Oil Methyl Esters) Blend and EGR (Exhaust Gas Recirculation) for Simultaneous Control NOx and Particulate Matter Emissions in Diesel Engines." *Energy* 62: 224–234.

Blas, C. S. D., J. S. Martin, and D. G. Gonzalez. 2018. "Combined Social Networks and Data Envelopment Analysis for Ranking." *European Journal of Operational Research* 266: 990–999.

Dudley, B. 2013. "The US, Russia and the World's Energy Journey." *CERA Week*. Moscow.

Ganapathy, T., R. P. Gakkhar, and K. Mutugesan. 2011. "Influence of Injection Timing on Performance, Combustion and Emission Characteristics of Jatropha Biodiesel Engine." *Applied Energy* 88: 4376–4386.

Geo, V. E., A. Sonthalia, G. Nagarajan, and B. Nagalingam. 2017. "Studies Performance, Combustion and Emission of a Single Cylinder Diesel Engine Fueled with Rubber Seed Oil and Its Biodiesel along with Ethanol as Injected Fuel." *Fuel* 209: 733–741.

How, H. G., H. H. Masjuki, M. A. Kalam, and Y. H. Teoh. 2018. "Influence of Injection Timing and Split Strategies on Performance, Emissions, and Combustion Characteristics of Diesel Engine Fueled with Biodiesel Blended Fuels." *Fuel* 213: 106–114.

Jindal, S., B. P. Nandwana, N. S. Rathore, and V. Vashistha. 2010. "Experimental Investigation of the Effect of Compression Ratio and Injection Pressure in a Direct Injection Diesel Engine Running on Jatropha Methyl Ester." *Applied Thermal Engineering* 30: 442–448.

Khodabakhsi, M., and K. Aryavash. 2012. "Ranking All Units in Data Envelopment Analysis." *Applied Mathematics Letters* 25 (12): 2066–2070.

Krishnaiah, K., and P. Shahabudeen. 2012. *Applied Design of Experiments and Taguchi Methods*, 274–278. New Delhi: PHI Learning. ISBN: 978-81-203-4527-0.

Lakshmanan, T., A. Avinash, and G. Nagarajan. 2017. "Experimental Study on DI Diesel Engine with Acetylene in Dual Fuel Mode with DEE an Ignition Enhance." *Transportation Research D* 54: 372–380.

Lin, T., Y. F Lo, and F. C. Chein. 2003. "Using Data Envelopment Analysis to Measure the Relative Efficiency of the Service Center and Improve Operation Efficiency through Reorganization." *IEEE Transaction on Power Systems* 18 (1): 366–374.

Mahalingam, S., P. Suresh Mohan Kumar, and R. V. Pranesh. 2013. "Experimental Study of Performance and Emission Characteristics of a Bio Dual Fuel Blends in Diesel Engine for Varaiation of Injection Pressures." Proceedings of the World Congress on Engineering. Vol 1, London,UK, July 3–5. ISBN: 978-988-19251-0-7,1–4.

Moghadasian, K., and H. Mohammed. 2008. "Advances in Dietry Enrichment with N-3 Fatty Acids." *Critical Reviews in Food Science and Nutrition* 48 (5): 402–410.

Nahar, K., and M. O. Hampton. 2011. *Jatropha: An Alternative Substitute to Fossil Fuel*. (IFAS Publication Number HS1193). Gainesville: University of Florida, Institute of Food and Agricultural Sciences. Accessed December 17, 2011.1–6.

Pradhan, P., H. Raheman, and D. Padhee. 2014. "Combustion and Performance of a Diesel Engine with Preheated Jatropha Curcas Oil Using Waste Heat from Exhaust Gas." *Fuel* 115: 527–533.

Puhan, S., R. Jegan, K. Balasubrmanian, and G. Nagarajan. 2009. "Effect of Injection Pressure on Performance, Emission and Combustion Characteristics of High Linolenic Linseed Oil Methyl Ester in a DI Diesel Engine." *Renewable Energy* 34: 1227–1233.

Ramanathan, R. 2003. *An Introduction to Data Envelopment Analysis*. New Delhi: Sage.

Sadjadi, S. J., H. Omrani, S. Abdollahzadeh, M. Alinaghian, and H. Mohammadi. 2011. "A Robust Super-Efficiency Data Envelopment Analysis Model for Ranking of Provincial Gas Companies in Iran." *Expert Systems with Applications* 38 (9): 10875–10881.

Sakthivel, G., G. Nagarajan, M. Illgkumaran, and A. B. Gaikwad. 2014. "Comparative Analysis of Performance, Emission and Combustion Parameter of Diesel Engine Fulled with Erthyl Ester of Fish Oil and Its Diesel Blends”.” *Fuel* 132 (15): 116–124.

Seiford, L. M., and R. M. Thrall. 1990. “Recent Developments in DEA: The Mathematical Programming Approach to Frontier Analysis.” *Journal of Econometrics* 46: 7–38.

Shameer, P. M., and K. Ramesh. 2018. “Assessment on the Consequences of Injection Timing and Injection Pressure on Combustion Characteristics of Sustainable Biodiesel Fueled Engine.” *Renewable and Sustainable Energy Reviews* 81: 45–61.

Shanling, L., G. R. Jahanshahloo, and M. Khodabakhshi. 2007. “A Super-Efficiency Model for Ranking Efficient Units in Data Envelopment Analysis.” *Applied Mathematics and Computation* 184 (2): 638–648.

Ushakov, S., H. Valland, and V. Aesoy. 2013. “Combustion and Emission Characteristics of Fish Oil Fuel in a Heavy-Duty Diesel Engine.” *Energy Conversion and Management* 65: 228–238.

Velmurugan, A., M. Loganathan, and E. J. Gunasekaran. 2014. “Experimental Investigations on Combustion and Performance and Emission Characteristics of Thermal Cracked Cashew Nut Shell Liquid (TC-CNSL)-Diesel Blends in a Diesel Engine”.” *Fuel* 132: 236–245.

Venturi, S., F. M. Donati, A. Venturi, and M. Venturi. 2000. “Environmental Iodine Deficiency: A Challenge to the Evaluation of Terrestrial Life?” *Thyroid* 10 (8): 727–729.

Venturi, S., and M. Venturi. 2007. “Evaluation of Dietry Antioxidant Defences.” *European Epi-Marker* 11 (3): 1–12.

Vessal, A. 2007. “Evaluating the Performance of Universities by Using Data Envelopment Analysis.” *California Journal of Operation Management* 5 (1):1–14.

Washio, S., and S. Yamada. 2013. “Evaluation Method Based on Ranking in Data Envelopment Analysis.” *Expert Systems with Applications* 40 (1): 257–262.

Yu, M., and R. Wang. 2006. “Evaluating the Efficiency of Hospitals” Department Using Data Envelopment Analysis.” *IEEE Transaction* 1167–1170.

Zhou, H., Y. Yang, Y. Chen, and J. Zhu. 2018. “Data Envelopment Analysis Application in Sustainability: The Origins, Development and Future Directions.” *European Journal of Operational Research* 264: 1–16.

Appendix 1: uncertainty analysis

Uncertainty analysis is nothing but the analysis of errors in the experiment. It is mandatory for a researcher to calculate the percentage of errors in particular experiment. These errors are due to machine calibration, room temperature and surrounding environment, observation, experiment readings, etc. this is unavoidable and there can be no experiment with zero error. An accompanying Table 3 presents the uncertainties and accuracy of measuring instruments used in this experiment.

Table A1: Percentage uncertainties and accuracy of instrument measurements

Measurement	Accuracy	Percentage uncertainties
Load	+0.1 to −0.1 kg	+0.2 to −0.2
Speed	+10 to −10 rpm	+0.1 to −0.1
Smoke	+1 to −1	+1 to −1
Burette fuel measurement	+0.1 to −0.1 cc	+1 to −1
Pressure pickup	+0.1 to −0.1 kg	+0.1 to −0.1
Crank angle encoder	+1° to −1°	+0.2 to −0.2
EGT	+1 to −1 °C	+0.15 to −0.15
HC	+30 to −30 ppm	+2 to −2
CO	+0.2 to −0.2%	+0.2 to −0.2
CO_2	+1 to −1%	+0.15 to −0.15
NO_x	+10 to −10 ppm	+0.1 to −0.1

The percentage uncertainties of various parameters like brake power and BTE were calculated using the percentage uncertainties of Table A1.

Total percentage uncertainty of this experiment is = square root of {(uncertainty of load)^2 + (uncertainty of speed)^2 + (uncertainty of BSFC)^2 + (uncertainty of BTE)^2 + (uncertainty of CO)^2 + (uncertainty of UHC)^2 + (uncertainty of NOx)^2 + (uncertainty of CO_2)^2 + (uncertainty of smoke number)^2 + (uncertainty of EGT)^2 + (uncertainty of pressure pickup)^2 + (burette fuel measurement)^2}

= square root of {$(0.2)^2 + (1)^2 + (1)^2 + (1)^2 + (0.2)^2 + (2)^2 + (0.1)^2 + (0.15)^2 + (0.1)^2 + (0.15)^{2+} (0.1)^2 + (1)^2$}

= ± 2.86%