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Normal Ideals in Generalized Almost Distributive Fuzzy Lattices

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Abstract. In this paper we introduced the concept of Generalized Almost Distributive Fuzzy Lattices (GADFLs) as a Generalization of an Almost distributive Fuzzy Lattices (ADFLs). The concept of Normal ideal is introduced, and some results are proved. In addition, for any two normal ideal N and M of GADFL, then N ∩ M and N ∪ M of normal ideals are also a normal ideal in GADFL. Finally prove the result fuzzy homomorphism preserving normal ideal into normal ideal.

AMS subject classification: 06D72, 06B10, 06D75

Keywords: Generalized almost distributive fuzzy lattices (GADFL), ideal, Normal ideal and Fuzzy homomorphism.

Introduction

The concept of a Generalized Almost distributive lattice (GADL) was bring together by Rao, Ravi Kumar and Rafi [1] as a generalization of an Almost Distributive Lattice (ADL) [2] which was a common abstraction of almost all the existing ring theoretic generalization of a Boolean algebra on one hand and distributive lattice on the other. The class of GADLs inherit almost all the properties of distributive lattice except possibly the commutativity of $\wedge, \vee$, the right distributive of either of the operations $\wedge or \vee$ over the other. The class of GADLs include the class of ADLs properly and retain many important properties of ADLs. On the other hand L.
A. Zadeh [3] in 1965 introduced the notion of fuzzy set to describe vagueness mathematically in its very abstractness and tried to solve such problems by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. Again in 1971, Zadeh [4] defined a fuzzy ordering as a generalization of the concept of ordering, that is, a fuzzy ordering is fuzzy relation that is transitive. In particular, a fuzzy partial ordering is a fuzzy ordering that is reflexive and antisymmetric. In 1994, Ajmal and Thomas [4] defined a fuzzy lattice as a fuzzy algebra and characterized fuzzy sublattices. In 2009, Chon [5], considering the notion of fuzzy order of Zadeh, introduced a new notion of fuzzy lattice and studied the level sets of fuzzy lattices. He also introduced the notions of distributive and modular fuzzy lattices and considered some basic properties of fuzzy lattices.

In this paper we introduced the concept of Generalized Almost Distributive Fuzzy Lattices (GADFLs) as a Generalization of an Almost distributive Fuzzy Lattices (ADFLs). The concept of Normal ideal is introduced, and some results are proved. In addition, for any two normal ideal N and M of GADFL, then N ∩ M and N ∪ M of normal ideals are also a normal ideal in GADFL. Finally prove the result fuzzy homomorphism preserving normal ideal into normal ideal.

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Preliminaries

An algebraic structure $(L, \vee, \wedge)$, consisting of a set L and two binary operations $\vee$, and $\wedge$, on L is a lattice if the following axiomatic identities hold for all elements a, b, c of L.

Commutative Laws

1. $a \vee b = b \vee a$
2. $a \wedge b = b \wedge a$

Associative Laws

1. $a \vee (b \vee c) = (a \vee b) \vee c$
2. $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

Absorption Laws

1. $a \vee (a \wedge b) = a$
2. $a \wedge (a \vee b) = a$

The following two identities are also usually regarded as axioms, even though they follow from the two absorption laws taken together.

Idempotent Laws

1. $a \vee a = a$
2. $a \wedge a = a$

These axioms assert that both $(L, \vee)$ and $(L, \wedge)$ are semilattices. The absorption laws, the only axioms above in which both meet and join appear, distinguish a lattice from an arbitrary pair of semilattice structures and assure that the two semilattices interact appropriately. In particular, each semilattice is the dual of the other.

Let $a \in L(R, A), x, y, s \in X$, R be a L-fuzzy partial order on A. 's' is called L-fuzzy supremum of x, y if the following conditions are true:

1. $A(s) \geq A(x) \wedge A(y)$
2. $R(x, s) \geq R(x, x); R(y, s) \geq R(y, y)$
3. $R(s, z) \geq R(x, z) \wedge R(y, z)$

Let $a \in L(R, A), x, y, t \in X$, R be a L-fuzzy partial order on A. t is called L-fuzzy infimum of x, y if the following conditions are true:

1. $A(t) \geq A(x) \wedge A(y)$
2. $R(t, x) \geq R(x, x); R(t, y) \geq R(y, y)$
3. $R(z, t) \geq R(z, x) \wedge R(z, y)$

An L-fuzzy partial order on (R, A) is called a L-fuzzy lattice on X if for any $x, y \in A_{(0)}$, both L-fuzzy supremum and L-fuzzy infimum of x, y exist.

An Almost Distributive lattice with 0 or simply ADL is an algebra $(R, \wedge, \vee, 0)$ of type $(2, 2, 0)$ satisfying the following conditions

1. $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
2. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
3. $(x \vee y) \wedge y = y$
4. $(x \vee y) \wedge x = x$
5. $x \vee (x \wedge y) = x$

6. $0 \wedge x = 0$
7. $x \vee 0 = x$

Let X be a set, a function $A : X \times X \to [0,1]$ is called fuzzy relation in X. The fuzzy relation A is reflexive if and only if $A(x,x) = 1 \; \forall x \in X$, A is symmetric if and only if $A(x,y) = A(y,x), \; \forall \; x,y \in X,$ A is transitive if and only if $A(x,z) \geq \underset{y \in X}{Sup} \min(A(x,y), A(y,z))$ and A is anti-symmetric if and only if $A(x,y) > 0$ and $A(y,x) > 0$ implies x = y.
A fuzzy relation A is fuzzy partial order relation if A is reflexive, anti-symmetric and transitive.
A fuzzy partial order relation A is a fuzzy total order relation if and only if $A(x,y) > 0$ and $A(y,x) > 0$ for all $x, y \in R$. If A is a fuzzy partial order relation in set X, then (X, A) is called fuzzy partial orderedset or a fuzzy poset. If B is a fuzzy total order relation in a set X, then (X, B) is called a fuzzy totally ordered set.
Let (R, A) be a fuzzy poset. $A : R \times R \to [0,1]$ (R, A) is a fuzzy lattice if and only if ($x \vee y$) and ($x \wedge y$) exist for all $x, y \in R$.
Let (R, A) be a fuzzy lattice. (R, A) is distributive if and only if $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee y) \quad \forall \quad x,y,z \in R.$

Let $(R, \wedge, \vee, 0)$ be algebra of type $(2,2,0)$ and (R, A) be a fuzzy poset. Then we call (R, A) is an almost distributive fuzzy lattices if it satisfies the following axioms.

1. $A(a, a \vee 0) = A(a \vee 0, a) = 1$
2. $A(0, 0 \wedge a) = A(0 \wedge a, 0) = 1$
3. $A((a \vee b) \wedge c, (a \wedge c) \vee (b \wedge c)) = A((a \wedge c) \vee (b \wedge c), (a \vee b) \wedge c) = 1$
4. $A(a \wedge (b \wedge c), (a \wedge c) \vee (a \wedge c)) = A((a \wedge c) \vee (a \wedge c), a \wedge (b \wedge c)) = 1$
5. $A(a \vee (b \wedge c), (a \vee b) \wedge (a \vee c)) = A((a \vee b) \wedge (a \vee c), a \vee (b \wedge c)) = 1$
6. $A((a \vee b) \wedge b, b) = A(b, (a \vee b) \wedge b) = 1 \;\; \forall a,b,c \in R$

Generalized almost distributive fuzzy lattices

In this section the definition of GADFL, normal ideal and fuzzy homomorphism are presented

Definition 3.1 Let $(R, \wedge, \vee, 0)$ be algebra of type $(2,2,0)$ and (R, A) be a fuzzy poset. Then we call (R, A) is a Generalized almost distributive fuzzy lattices if it satisfies the following axioms.

1. $A((a \wedge b) \wedge c, a \wedge (b \wedge c)) = A(a \wedge (b \wedge c), (a \wedge b) \wedge c) = 1$
2. $A(a \wedge (b \vee c), (a \wedge b) \vee (a \wedge c)) = A((a \wedge b) \vee (a \wedge c), a \wedge (b \vee c)) = 1$
3. $A(a \vee (b \wedge c), (a \vee b) \wedge (a \vee c)) = A((a \vee b) \wedge (a \vee c), a \vee (b \wedge c)) = 1$
4. $A(a \wedge (a \vee b), a) = A(a, a \wedge (a \vee b)) = 1$
5. $A((a \vee b) \wedge a, a) = A(a, (a \vee b) \wedge a) = 1$
6. $A((a \wedge b) \vee b, b) = A(b, (a \wedge b) \vee b) = 1 \;\; \forall a,b,c \in R$

Definition 3.2 Let $L(R, A)$ be a GADFL and I be any non empty subset of R. Then I is said to be an ideal of a GADFL $L(R, A)$, if it satisfies the following conditions.

1. $a, b \in I \Rightarrow a \vee b \in I$

2. $a \in I, b \in R \Rightarrow a \wedge b \in I$

Definition 3.3 Let $L(R, A)$ be a GADFL and N be any non empty subset of an ideal I in R. Then N is said to be normal ideal of a GADFL $L(R, A)$, if it satisfies the following conditions

1. $a, b \in N \Rightarrow a \vee b \in N$
2. $a \in N, b \in I \Rightarrow a \wedge b \in N$

Theorem 3.1 Let I be an ideal of in GADFL $L(R, A)$. Then $H = (I, A)$ is a GADFL.

Proof: Let I be an ideal of $L(R, A)$ and $a, b \in I$. Therefore, we get $a \vee b \in I$ and since $A(a \vee b, b) > 0$, $a \wedge b \in I$.
Clearly, I is a subset of R and $L(R, A)$ is a fuzzy poset. Since I is a subset of R and $L(R, A)$ is a GADFL, all the conditions of a GADFL holds true in (I, A). Hence $H = (I, A)$ is a GADFL. Hence proved.

Theorem 3.2 Let $L(R, A)$ be a GADFL and let S be any non empty subset of R. The set $S^* = \{x \in R | A(x \wedge s, 0) > 0, \forall s \in S\}$ is an ideal of $L(R, A)$.

Proof: Let $L(R, A)$ be a GADFL and let S be any non-empty subset of R. S^* is a non-empty subset of R and it is defined by $S^* = \{x \in R | A(x \wedge s, 0) > 0, \forall s \in S\}$.
Since $A(0 \wedge s, 0) A(0,0) > 0, 0 \in S^*$.
Let $a, b \in S^*$ then $A(a \wedge s, 0) > 0$ and $A(b \wedge s, 0) > 0 \ \forall s \in S$. since $L(R, A)$ is a GADFL. Therefore, we get $A(0, a \wedge s) > 0$ and $A(0, b \wedge s) > 0 \ \forall s \in S$.
This implies $a \wedge s = 0 \, and \, b \wedge s = 0 \forall s \in S$.

$$\begin{aligned} A\big((a \vee b) \wedge s, 0\big) &= A\big((a \wedge s) \vee (b \wedge s), 0\big) \\ &= A(0 \vee 0, 0) \\ &= A(0,0) > 0 \end{aligned}$$

Therefore, we get $(a \wedge b) \in S^*$.
Let $a \in S^* \, and \, x \in R$, therefore we get $A(a \wedge s, 0) > 0$, now we find

$$\begin{aligned} A\big((a \wedge x) \wedge s, 0\big) &= A\big((x \wedge a) \wedge s, 0\big) \\ &= A\big(x \wedge (a \wedge s), 0\big) \\ &\geq \sup_{r \in R} \min\Big(A\big(x \wedge (a \wedge s), r, A(r, 0)\big)\Big) \\ &\geq \min\Big(A\big(x \wedge (a \wedge s), a \wedge s, A(a \wedge s, 0)\big)\Big) > 0 \ \forall s \in S. \end{aligned}$$

Hence $a \wedge s \in S^* \ \forall s \in S$. The S^* is an ideal of GADFL $L(R, A)$.

Remark: For any non-empty set S of GADFL $L(R, A)$. The set $S^* = \{x \in R | A(x \wedge s, 0) > 0, \forall s \in S\}$

This S^* an ideal of GADFL $\mathrm{L(R,A)}$ and is also called the annihilator ideal of A. For any $a \in L,$ we write $[a]^*$ for $\{a\}^*$ and is called an annulet of GADFL $\mathrm{L(R,A)}$.

Definition 3.4 An element a of GADFL $\mathrm{L(R,A)}$ is called dense element if $[a]^* = \{0\}$.

Theorem 3.3 Let $\mathrm{L(R,A)}$ be a GADFL and let I be an ideal of R. the set $N = I^{**}$ is a normal ideal of lattices $\mathrm{L(R,A)}$. $I^* = \{x \in R \,|\, A(x \wedge y, 0) > 0, \forall y \in I\}$.

Proof: Let $\mathrm{L(R,A)}$ be a GADFL and let I be an ideal of R. Therefore, we get

$S^* = \{x \in R \,|\, A(x \wedge y, 0) > 0 \,\forall\, \mathrm{y} \in \mathrm{I}\} - - - - - (1)$.

By the definition of normal ideal N, such that $a,b \in N \Rightarrow a \vee b \in N$ and $a \in N, b \in I \Rightarrow a \wedge b \in N$.

Every normal ideal in $\mathrm{L(R,A)}$ is also an ideal $\mathrm{L(R,A)}$ such that normal ideal satisfies

$S^* = \{x \in R \,|\, A(x \wedge s, 0) > 0 \,\forall s \in S\} - - - - - (2)$

In normally take an element $s \in S$. Therefore, we rewritten as $I^* = \{x \in R \,|\, A((x \wedge s), 0) > 0 \,\forall\, s \in I\}$.

Normal ideal N, for any ideal $I \in R$ satisfies the equation (2),

$N = \{x \in R \,|\, A(x \wedge s, 0) > 0 \,\forall s \in I^*\}$

That is $N = I^{**}$.

Theorem 3.4 Let I be an ideal of GADFL $\mathrm{L(R,A)}$ and N is normal ideal of $\mathrm{L(R,A)}$. Then for any $x, y \in N, x \wedge y \in N$ if and only if $y \wedge x \in N.$

Proof: Assume $x \wedge y \in N$ and $x, y \in I$. Then we get $(x \wedge y) \wedge (y \wedge x) \in N$

$$\begin{aligned} A(y \wedge x, (x \wedge y) \wedge (y \wedge x)) &= A(y \wedge x, (y \wedge x) \wedge (y \wedge x)) \\ &= A((y \wedge x) \wedge (y \wedge x)) > 0 \end{aligned}$$

Hence $y \wedge x \in N.$

Similarly, we assume $y \wedge x \in N$ and $x, y \in I$. Then we get $(y \wedge x) \wedge (x \wedge y) \in N$

$$\begin{aligned} A(x \wedge y, (y \wedge x) \wedge (x \wedge y)) &= A(x \wedge y, (x \wedge y) \wedge (x \wedge y)) \\ &= A((x \wedge y) \wedge (x \wedge y)) > 0 \end{aligned}$$

Thus $x \wedge y \in N.$ Hence proved.

Theorem 3.5

Let N and M be any two normal ideal of GADFL $L(R,A)$. Then $\mathrm{N} \cap \mathrm{M}$ and $\mathrm{N} \cup \mathrm{M}$ are also normal ideals.

Proof: Let $L(R,A)$. be a GADFL and N and M be any two normal ideal of GADFL $L(R,A)$. Now we prove $\mathrm{N} \cap \mathrm{M}$ is a normal ideal in $L(R,A)$. N and M are non-empty, $x \in N$ and $y \in M$ respectively, this implies $x \wedge y \in N, M$ .Since if $x \in N \; and \; y \in M$

N and M are ideals this implies $x \wedge y \in N$.

Similarly, $x \wedge y \in M$,

$x = x \vee (x \wedge y) \in N \cup M$

$y = (x \wedge y) \wedge \mathrm{y} \in N \cup M$

This implies $x \in N$ and $y \in M$ .Hence is $\mathrm{N} \cup \mathrm{M}$ non-empty.

Let $a, b \in N \cup M$ then there exist $\mathrm{s}, \mathrm{t} \in N$ and $u, v \in M$ such that $a = s \vee u$ and $b = t \vee v$, then we get $(s \vee u) \wedge (a \vee b) \in N$ and $(t \vee v) \wedge (a \vee b) \in M$. This implies $A\big((a \vee b), (s \vee u) \vee (t \vee v)\big) = 1$

Since $s \vee u = a$ *and* $t \vee v = b$.

$$= A\big((a \wedge b), ((s \vee u) \vee (t \vee v)) \wedge (a \vee b)\big)$$

$$= A\big((a \wedge b), ((s \vee u) \wedge (a \vee b)) \vee ((t \vee v) \wedge (a \vee b))\big)$$

$$= A\big((a \wedge b), ((s \wedge (a \vee b)) \vee (u \wedge (a \vee b) \vee ((t \wedge (a \vee b)) \vee (v \wedge (a \vee b))\big)$$

$$= A\big((a \wedge b), ((s \vee u) \wedge (a \vee b)) \vee ((t \vee v) \wedge (a \vee b))\big) > 0$$

Similarly, we find $A\big((s \vee u) \vee (t \vee v), (a \vee b)\big) = 1$

$$= A\big(((s \vee u) \vee (t \vee v)) \wedge (a \vee b), (a \wedge b)\big)$$

$$= A\big(((s \vee u) \wedge (a \vee b)) \vee ((t \vee v) \wedge (a \vee b)), (a \wedge b)\big)$$

$$= A\big(((s \wedge (a \vee b)) \vee (u \wedge (a \vee b) \vee ((t \wedge (a \vee b)) \vee (v \wedge (a \vee b)), (a \wedge b)\big)$$

$$= A\big(((s \vee u) \wedge (a \vee b)) \vee ((t \vee v) \wedge (a \vee b)), (a \wedge b)\big) > 0$$

This implies $\mathrm{a} \vee \mathrm{b} = ((s \vee u) \wedge (a \vee b)) \vee ((t \vee v) \wedge (a \vee b)) \in \mathrm{N} \cup \mathrm{M}$.

Let $a \in N \cup M$ and $c \in I$. Then there exist $s \in N$ *and* $u \in M$ such that $s \vee u = a$.

Since N and M are normal ideals of $L(R, A)$.

Therefore, we get $(a \wedge s) \in \mathrm{N}$ *and* $(a \wedge u) \in M$. This implies $(a \wedge s) \vee (a \wedge u) \in \mathrm{N} \cup \mathrm{M}$.

$$\mathrm{A}\big((a \wedge b), (s \wedge b) \vee (u \wedge b)\big)$$

$$= \mathrm{A}\big((a \wedge b), (s \vee u) \wedge b\big)$$

$$= \mathrm{A}\big((a \wedge b), (\mathrm{a} \wedge b)\big) > 0$$

Hence $(a \wedge b) \in \mathrm{N} \cup \mathrm{M}$. Thus $\mathrm{N} \cup \mathrm{M}$ is normal ideal of $L(R, A)$.

Let $a, b \in N \cap M$ .Then $a, b \in N$ *and* $a, b \in M$. Since $a \wedge c \in N, M$ *and* $a, b \in N \cap M$ N and M are normal ideals of $L(R, A)$. Therefore, we get $(a \wedge s) \in \mathrm{N}$ *and* $(a \wedge u) \in M$.

This implies $(a \wedge s) \vee (a \wedge u) \in \mathrm{N} \cup \mathrm{M}$. Hence $\mathrm{N} \cap \mathrm{M}$ is a normal ideal in $L(R, A)$.

Hence proved.

Definition3.5 Let $L(\mathrm{X}, A)$ and $\mathrm{M}(Y, B)$ are bounded fuzzy lattices. A $h : X \to Y$ mapping is a fuzzy homomorphism fromL to M if for all $x, y \in X$

1. $h(x \wedge L(y)) = h(x) \wedge Mh(y)$
2. $h(x \vee L(y)) = h(x) \vee Mh(y)$
3. $h(LB(L)) = LB(M)$
4. $h(UB(L)) = UB(M)$

Theorem 3.6 Let $L(X, A)$ and $L(Y, B)$ be bounded GADFL's, $N \subseteq X$ is a normal ideal and $h : X \to Y$ is a fuzzy homomorphism. Then $h(N)$ is a normal ideal of M.

Proof:

(i). Given $x \in X$ *and* $y \in N$ such that $\mathrm{A}(x, y) > 0$. $h: X \to Y$ is a fuzzy homomorphism therefore we get $\mathrm{B}(h(x), h(y)) > 0$. if $y \in N$ then $h(y) \in h(N)$. by hypothesis $h(N)$ is an ideal of M so we get $h(x) \in h(N)$.

Hence we have that $x \in N$ is normal ideal.

(ii). Given $x, y \in N$, therefore $h(x), h(y) \in \mathrm{h(N)}$. By hypothesis $h(N)$ is a normal ideal of M. Therefore we get $(h(x) \vee \mathrm{M}\, h(y)) \in h(N)$

because $h: X \to Y$ is a fuzzy homomorphism therefore $h(x \vee L(y)) = h(x) \vee Mh(y) \in h(N)$.

Hence we get $(x \vee L(y)) \in \mathrm{N}$.

Hence Proved.

Theorem 3.7 Let $L(X, A)$ and $L(Y, B)$ be bounded GADFL's, $N \subseteq X$ is a normal ideal and $h: X \to Y$ is a fuzzy isomorphism. Then $h(N)$ is a normal ideal of M if and only if N is a normal ideal of $L(X, A)$.

Proof: (By theorem 3.6) $h(N)$ is a normal ideal of M if N is a normal ideal of $L(X, A)$.

Assume N be a normal ideal of $L(X, A)$.

Let $x', y' \in Y$ such that $y' \in h(N)$ *and* $B(x', y') > 0$. Therefore $y' \in h(N)$, then there exist $x \in X$ such that $h(\mathrm{y}) = y'$ because h is on to there exist $x \in X$ such that $h(x) = x'$. Since h is one to one.

By hypothesis, N is a Normal Ideal of $L(X, A), x \in N$ and therefore $x' \in h(x) \in h(N)$.

Let $x' \in h(\mathrm{N})$ and $y' \in h(\mathrm{N})$,there exist atleast one $x, y \in X$ such that $h(x) = x'$ *and* $h(y) = y'$.

Therefore, we get $x, y \in h(h(\mathrm{N}))$ and $x, y \in N$ since h is bijective. By hypothesis, N is a Normal ideal of $L(X, A)$. Therefore $(x \vee L(y)) \in N$, this implies $h(x \vee L(y)) \in h(N)$, we get $x' \vee M(y') = h(x) \vee Mh(y) = h(x \vee L(y)) \in h(N)$.

Hence proved.

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