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K. Thirusangu, K. Srinivasan, and P. Elumalai

On Product Root Sum Mean Labeling of Some New Class of Graphs

K.Thirusangu[1,a) ,K.Srinivasan[2] and P.Elumalai[3]

[1]Department of Mathematics,S.I.V.E.T. College,Gowrivakkam,Chennai,India.
[2] Department of Mathematics, Vels Institute of Science ,Technology & Advanced Studies. Pallavaram, Chennai,India.
[3]Department of Mathematics, Government Arts College, Thiruvannamalai,India.

[a)]Corresponding author : kthirusangu@gmail.com

Abstract. In 2019, a new labeling called Product Root Sum Mean Labeling was introduced in the literature . It is defined in a graph G=(p,q) as an injective function f: V →{1,2,3,...,q+1} such that the induced function f* defined by $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$ yield different values on edges. A paper which admits this labeling is known as Product Root Sum Mean graph. In this paper we prove that the ladder graph , the Square Ladder graph , the cocunut tree CT(m,m), the graph Y_{r+1} , the Star graph $K_{1,n}$, the Shadow graph of Star $K_{1,n}$, the Split graph of Star $K_{1,n}$, the BiStar graph $K_{n,n}$, the Shadow graph of BiStar $K_{n,n}$, the Comb graph $P_n \odot K_1$, the Square graph of Comb $P_n \odot K_1$, the Shadow graph of Comb $D_2(P_n \odot K_1)$ and the splitting graph of Comb are Product Root Sum Mean graphs.

Keywords:
Graphs, shadow graphs, split graphs, square graph, Graph labeling, Product Root Sum Mean labeling

INTRODUCTION

Graph theory is the fast growing area of combinatorics. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for a wide range of applications. After the introduction of graph labeling , various labeling of graphs such as graceful labeling, cordial labeling , prime cordial labeling. Magic labelling , Anti magic labeling etc., have been studied in over 2100 papers[2]. Several researchers refer to Rosa's [2] work. The concept of mean labeling has been introduced by S.Somasundaram and R.Ponraj [4]. In [1], labeling such as mean, odd mean and even mean labeling for the Extended Duplicate Graph of Kite Graph has been investigated. A new of labeling technique called Product Root Sum Mean labeling was introduced by K.Thirusangu , P.Elumalai and K.Srinivasan [5]. They proved that the graphs such as Path ,Middle graph of a Path, Total graph of a Path, Split graph of a Path, Shadow graph of a Path, Cycle, Middle graph of a Cycle ,Triangular Snake , Alternate Triangular Snake, Double Triangular Snake , Alternate Triangular Snake, Kite, Quadrilateral snake graph ,Double Quadrilateral snake graph and Alternative Quadrilateral snake graph are Product Root Sum Mean graphs. In this paper we prove that the ladder graph, the Square graph of Ladder , the cocunut tree CT(m,m), the graph Y_{n+1} , the Star graph $K_{1,n}$, the Shadow graph of Star $K_{1,n}$, the Split graph of Star $K_{1,n}$, the BiStar graph $K_{n,n}$, the Shadow graph of BiStar $K_{n,n}$, the Comb graph $P_n \odot K_1$, the Square graph of Comb $P_n \odot K_1$, the Shadow graph of Comb $D_2(P_n \odot K_1)$ and the splitting graph of Comb are Product Root Sum Mean graphs.

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PRELIMINARIES

In this section we provide some basic definitions and examples which are necessary for this article.

Definition: 2.1

A walk in which $u_1, u_2 \ldots, u_n$ are distinct is called a **path**. A path on n vertices is denoted by P_n. A closed path is called a **cycle**. A cycle on n vertices is denoted by C_n.

Example: 2.2

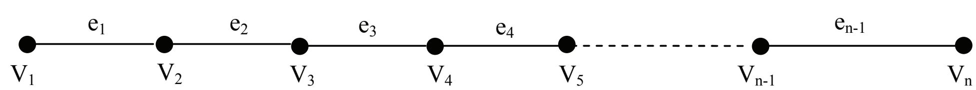


Figure: 2.1 A path graph P_n

Definition : 2.3

A Y- tree Y_n is obtanined from a path P_{n-1} by adding an pendant edge at (n-2) nd node.

Example: 2.4

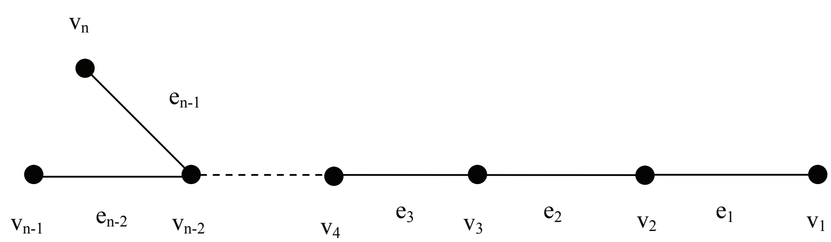


Figure: 2.2 A Y-tree Y_n

Definition: 2.5

The graph obtained by joining a single pendent edge to each vertex of a path is called as **Comb**.

Example: 2.6

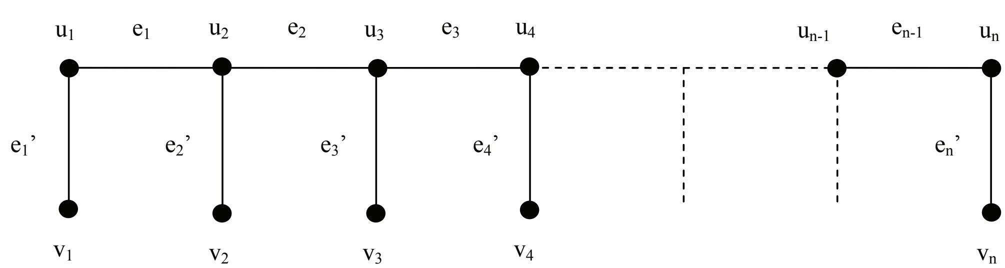


Figure: 2.3 A Comb $P_n \ \Theta \ K_1$

Definition: 2.7

The product graph $P_2 \times P_n$ is called a ladder and it is denoted by L_n. This graph has 2n vertices and 3n – 2 edges.

Example: 2.8

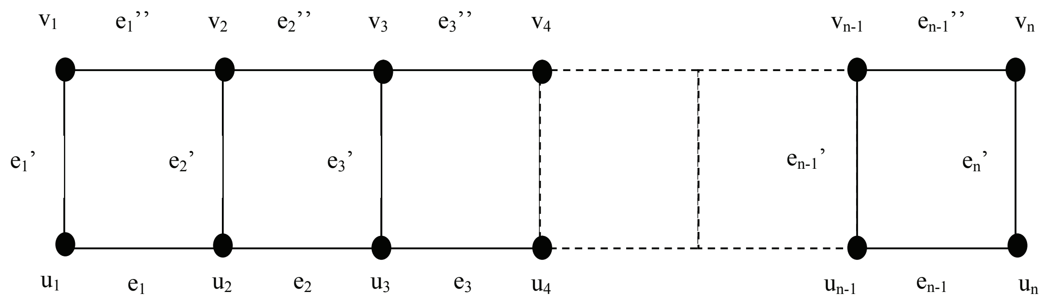


Figure: 2.4 A Ladder L_n

Definition: 2.9

A Star graph S_n is a complete bipartite graph $K_{1,n}$.

Example: 2.10

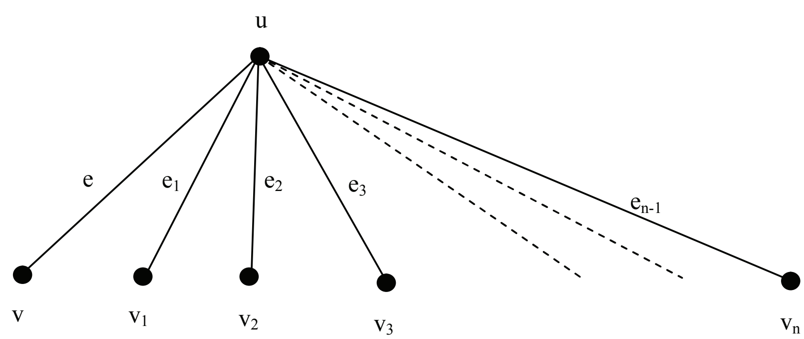


Figure: 2.5 A Star graph S_n

Definition: 2.11

A Bi-star graph is a graph obtained by joining the apex vertices of two star graphs S_n by an edge. It is denoted as B(n,n).

Example: 2.12

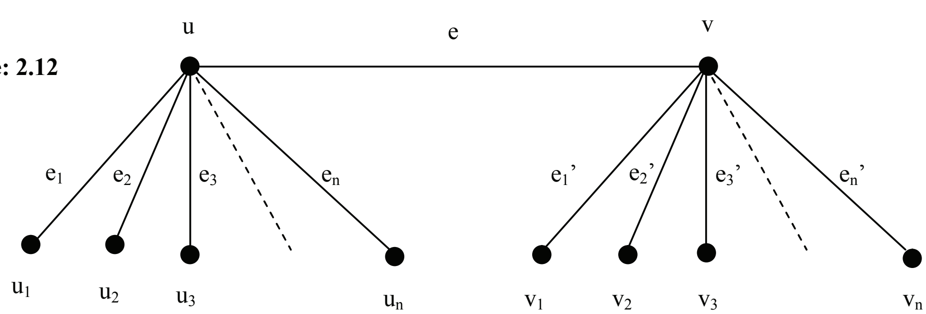


Figure: 2.6 A Bi-Star graph B(n,n)

Definition: 2.13

A coconut tree CT(m,n) is a graph obtained by joining the apex vertex of a star graph $K_{1,n}$ to a path graph P_m. **Example: 2.14**

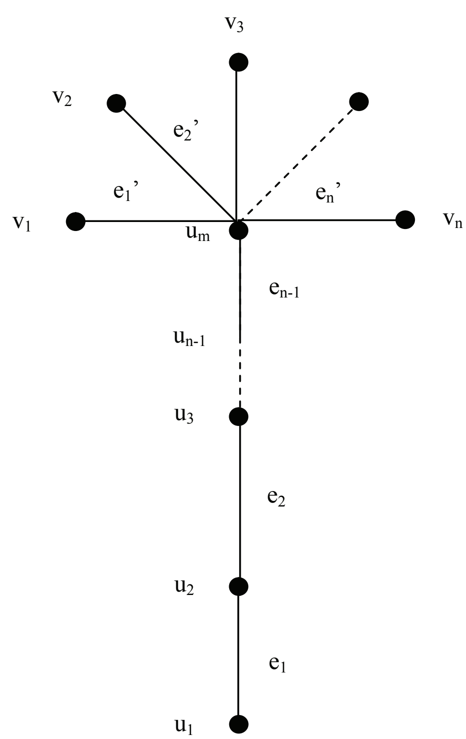


Figure: 2.7 A Cocunut Tree CT(m,n)

Definition: 2.15

For a graph G the **split graph** is obtained by adding to each vertex v a new vertex such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as spl(G).

Definition: 2.16

The **shadow graph** $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G" join each vertex v' in G; to the neighbours of the corresponding vertex v" in G".

Definition: 2.17

Square graph of a graph G denoted by G^2 has the same vertex set as of G and two vertices are adjacent in G^2, if they are at a distance of 1 or 2 apart from G.

3. MAIN RESULTS

In this section we prove our main results.

Theorem 3.1 : The graph Y $_{n+1}$ tree admits Product Root Sum Mean Labeling.

Proof:

Let the vertex set be V={ $v_1, v_2, \ldots\ldots v_{n-1}, v_n$}. The edge set is E = { $v_i v_{i+1}$} for all values of n.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(v_i) = i$, $1 \le i \le n-1$;

$f(v_n) = n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(v_i v_{i+1}) = \frac{[i*(i+1)+\sqrt{i+(i+1)}]}{2} = \frac{(i^2+i)+\sqrt{2i+1}}{2}, 1 \le i \le n-2.$

$f^*(v_{n-2} v_n) = \frac{[(n-2)*n+\sqrt{n-2+n}]}{2} = \frac{[n^2-2n+\sqrt{2n-2}]}{2}$

Thus the edges { 2,5,8 ,…..,$\frac{(i^2+i)+\sqrt{2i+1}}{2}$; $\frac{[n^2-2n+\sqrt{2n-2}]}{2}$} are all distinct.

Theorem 3.2 : The graph cocunut tree CT(m,m) is a Product Root Sum Mean graph.

Proof:

Let the vertex set be V={ $u_1, u_2, \ldots\ldots u_{m-1}, u_m, v_1, v_2, \ldots\ldots v_{n-1}, v_n$}.

The edge set is E = { $u_i u_{i+1}$, $1 \le i \le m-1$; $u_m v_i$, $1 \le i \le n$;} for all values of n.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = i$, $1 \le i \le m$;

$f(v_i) = n+i$, $1 \le i \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(u_i u_{i+1}) = \frac{[i*(i+1)+\sqrt{i+(i+1)}]}{2} = \frac{[(i^2+i)+\sqrt{2i+1}]}{2}, 1 \le i \le m-1.$

$f^*(u_m v_i) = \frac{[m*(n+i)+\sqrt{m+(n+i)}]}{2} = \frac{[mn+mi+\sqrt{m+n+i}]}{2}, 1 \le i \le n.$

Thus the edges { 2,5,8 ,…..,$\frac{[(i^2+i)+\sqrt{2i+1}]}{2}$; 5,6,8,9,11,…..,$\frac{[mn+mi+\sqrt{m+n+i}]}{2}$} are all distinct.

Theorem 3.3: The graph P_n^2 , the square graph of a Path is a Product Root Sum Mean graph.

Proof:

Case(i): if n is odd

Let the vertex set be V={ $u_1, u_2, \ldots\ldots, u_n$}.

The edge set is E = { $u_i u_{i+1}$, $1 \le i \le n-1$; $u_{2i-1} u_{2i+1}$, $1 \le i \le \frac{n-1}{2}$} for values of n=3.

E = { $u_i u_{i+1}$, $1 \le i \le n-1$; $u_{2i-1} u_{2i+1}$, $1 \le I \le \frac{n-1}{2}$; $u_{2i} u_{2i+2}$, $1 \le i \le \frac{n-3}{2}$ } for all Odd values of n ; except n=3.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = i$, $1 \le I \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

For n=3

$$f^*(u_i u_{i+1}) = \frac{[i*(i+1)+\sqrt{i+(i+1)}]}{2} = \frac{(i^2+i)+\sqrt{2i+1}}{2}, 1 \le i \le n-1.$$

$$f^*(u_{2i-1}u_{2i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{(2i-1)+(2i+1)}]}{2} = \frac{(4i^2-1)+\sqrt{4i}}{2}, 1 \le i \le \frac{n-1}{2}.$$

For n = Odd numbers; except n=3.

$$f^*(u_i u_{i+1}) = \frac{[i*(i+1)+\sqrt{i+(i+1)}]}{2} = \frac{(i^2+i)+\sqrt{2i+1}}{2}, 1 \le i \le n-1.$$

$$f^*(u_{2i-1}u_{2i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{(2i-1)+(2i+1)}]}{2} = \frac{(4i^2-1)+\sqrt{4i}}{2}, 1 \le i \le \frac{n-1}{2}.$$

$$f^*(u_{2i}u_{2i+2}) = \frac{[(2i)*(2i+2)+\sqrt{(2i)+(2i+2)}]}{2} = \frac{(4i^2+4i)+\sqrt{4i+2}}{2}, 1 \le i \le \frac{n-3}{2}.$$

Thus the edges

For n=3

E = { 1,4,2} are all distinct.

For n = Odd numbers; except n=3.

E = { 1,4,7,….., $\frac{(i^2+i)+\sqrt{2i+1}}{2}$; 2,8,19,….., $\frac{(4i^2-1)+\sqrt{4i}}{2}$; 5,13,25,…., $\frac{(4i^2+4i)+\sqrt{4i+2}}{2}$ } are all distinct.

Case(ii): if n is even

Let the vertex set be V={ $u_1, u_2, \ldots, u_n$ }.

The edge set is E = { $u_i u_{i+1}$, i=1 } for values of n=2.

E = { $u_i u_{i+1}$, $1 \le i \le n-1$; $u_{2i-1}u_{2i+1}$, $1 \le i \le \frac{n-2}{2}$; $u_{2i}u_{2i+2}$, $1 \le i \le \frac{n-2}{2}$ } for all even values of n except n=2

.Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = i$, $1 \le i \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

For n=2

$$f^*(u_i u_{i+1}) = \frac{[i*(i+1)+\sqrt{i+(i+1)}]}{2} = \frac{(i^2+i)+\sqrt{2i+1}}{2}, i = 1.$$

For n = Even numbers; except n=2.

$$f^*(u_i u_{i+1}) = \frac{[i*(i+1)+\sqrt{i+(i+1)}]}{2} = \frac{(i^2+i)+\sqrt{2i+1}}{2}, 1 \le i \le n-1.$$

$$f^*(u_{2i-1}u_{2i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{(2i-1)+(2i+1)}]}{2} = \frac{(4i^2-1)+\sqrt{4i}}{2}, 1 \le i \le \frac{n-2}{2}.$$

$$f^*(u_{2i}u_{2i+2}) = \frac{[(2i)*(2i+2)+\sqrt{(2i)+(2i+2)}]}{2} = \frac{(4i^2+4i)+\sqrt{4i+2}}{2}, 1 \le i \le \frac{n-2}{2}.$$

Thus the edges

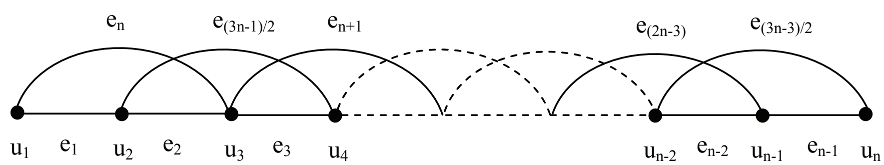
For n=2 E = { 2} .

For n = Even numbers; except n=2.

E = { 1,4,7,…..,$\frac{(i^2+i)+\sqrt{2i+1}}{2}$; 2,8,19,…..,$\frac{(4i^2-1)+\sqrt{4i}}{2}$; 5,13,25,….,$\frac{(4i^2+4i)+\sqrt{4i+2}}{2}$ } are all distinct.

The above theorem is illustrated as follows.

Example: 3.4



Path P_n^2

Theorem 3.5: The graph Comb $P_n \odot K_1$ admits Product Root Sum Mean Labeling.

Proof:

Let the vertex set be V={ $u_1, u_2, \ldots\ldots u_{n-1}, u_n$; $v_1, v_2, \ldots\ldots v_{n-1}, v_n$ }.

The edge set is E = { u_iu_{i+1} , $1 \le i \le n-1$; u_iv_i , $1 \le i \le n$ } for all values of n.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = 2i-1$, $1 \le i \le n$

$f(v_i) = 2i$, $1 \le i \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(u_iu_{i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{2i-1+2i+1}]}{2} = \frac{[(4i^2-1)+\sqrt{4i}]}{2}$, $1 \le i \le n-1$.

$f^*(u_iv_i) = \frac{[(2i-1)*(2i)+\sqrt{(2i-1)+(2i)}]}{2} = \frac{[(4i^2-2i)+\sqrt{4i-1}]}{2}$, $1 \le i \le n$.

Thus the edges { 3,9,20 ,…..,$\frac{[(4i^2-1)+\sqrt{4i}]}{2}$; 2,8,17,….,$\frac{[(4i^2-2i)+\sqrt{4i-1}]}{2}$ } are all distinct.

Theorem 3.6 : The Square graph of a Comb is a Product Root Sum Mean graph.

Proof:

Case: If n is odd.(n>3)

Let the vertex set be V={ $u_1, u_2, \ldots\ldots u_{n-1}, u_n$; $v_1, v_2, \ldots\ldots v_{n-1}, v_n$ }.

The edge set is E = { u_iu_{i+1} , $1 \le i \le n-1$; u_iv_i , $1 \le i \le n$; u_iv_{i+1} , $1 \le i \le n-1$; $u_{i+1}v_i$, $1 \le i \le n-1$; $u_{2i-1}u_{2i+1}$, $1 \le i \le (n-1)/2$; $u_{2i}u_{2i+2}$, $1 \le i \le (n-3)/2$ } for all values of odd n > 3.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = 3i$, $1 \le i \le n$

$f(v_i) = 3i-2$, $1 \le i \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(u_i u_{i+1}) = \frac{[(3i)*(3i+3)+\sqrt{3i+3i+3}]}{2} = \frac{[(9i^2+9i)+\sqrt{6i+3}]}{2}, 1 \le i \le n-1.$

$f^*(u_i v_i) = \frac{[(3i)*(3i-2)+\sqrt{(3i+3i-2)}]}{2} = \frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}, 1 \le i \le n.$

$f^*(u_i v_{i+1}) = \frac{[(3i)*(3i+1)+\sqrt{(3i+3i+1)}]}{2} = \frac{[(9i^2+3i)+\sqrt{6i+1}]}{2}, 1 \le i \le n-1.$

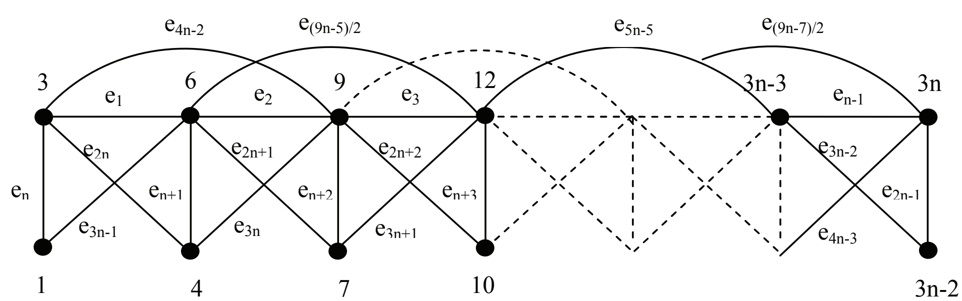
$f^*(u_{i+1} v_i) = \frac{[(3i+3)*(3i-2)+\sqrt{(3i+3+3i-2)}]}{2} = \frac{[(9i^2+3i-6)+\sqrt{6i+1}]}{2}, 1 \le i \le n-1.$

$f^*(u_{2i-1} u_{2i+1}) = \frac{[(6i-3)*(6i+3)+\sqrt{(6i-3+6i+3)}]}{2} = \frac{[(36i^2-9)+\sqrt{12i}]}{2}, 1 \le i \le \frac{n-1}{2}.$

$f^*(u_{2i} u_{2i+2}) = \frac{[(6i)*(6i+6)+\sqrt{(6i+6i+6)}]}{2} = \frac{[(36i^2+36i)+\sqrt{12i+6}]}{2}, 1 \le i \le \frac{n-3}{2}.$

Thus the edges { 10,28,56,….., $\frac{[(9i^2+9i)+\sqrt{6i+3}]}{2}$; 2,13,33,….., $\frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}$; 7,22,47,….., $\frac{[(9i^2+3i)+\sqrt{6i+1}]}{2}$; 4,19,44,……, $\frac{[(9i^2+3i-6)+\sqrt{6i+1}]}{2}$; 15,69,….., $\frac{[(36i^2-9)+\sqrt{12i}]}{2}$; 38,110,…., $\frac{[(36i^2+36i)+\sqrt{12i+6}]}{2}$} are all distinct.

Example:



Square comb (Odd Case)

Case: If n is Even.(n>2)

Let the vertex set be V={ $u_1, u_2, \ldots u_{n-1}, u_n$; $v_1, v_2, \ldots v_{n-1}, v_n$ }.

The edge set is E = { $u_i u_{i+1}$, $1 \le i \le n-1$; $u_i v_i$, $1 \le i \le n$; $u_i v_{i+1}$, $1 \le i \le n-1$; $u_{i+1} v_i$, $1 \le i \le n-1$; $u_{2i-1} u_{2i+1}$, $1 \le i \le (n-2)/2$; $u_{2i} u_{2i+2}$, $1 \le i \le (n-2)/2$ } for all values of even n > 2.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = 3i$, $1 \le i \le n$

$f(v_i) = 3i-2$, $1 \le i \le n$;

The edge labels are obtained as

$f^*(u_i u_{i+1}) = \frac{[(3i)*(3i+3)+\sqrt{3i+3i+3}]}{2} = \frac{[(9i^2+9i)+\sqrt{6i+3}]}{2}, 1 \le i \le n-1.$

$f^*(u_i v_i) = \frac{[(3i)*(3i-2)+\sqrt{(3i+3i-2)}]}{2} = \frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}, 1 \le i \le n.$

$f^*(u_i v_{i+1}) = \frac{[(3i)*(3i+1)+\sqrt{(3i+3i+1)}]}{2} = \frac{[(9i^2+3i)+\sqrt{6i+1}]}{2}, 1 \le i \le n-1.$

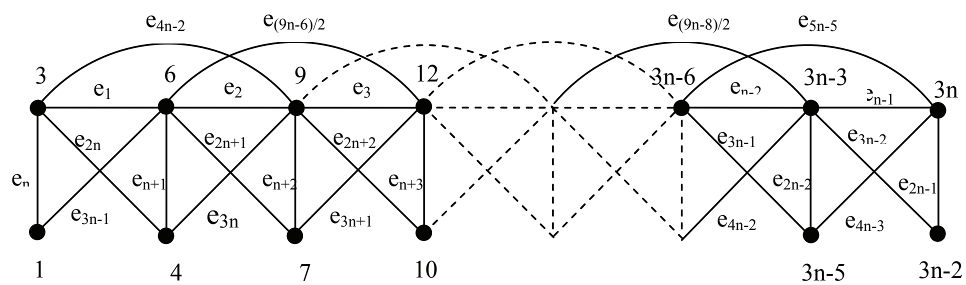
$f^*(u_{i+1} v_i) = \frac{[(3i+3)*(3i-2)+\sqrt{(3i+3+3i-2)}]}{2} = \frac{[(9i^2+3i-6)+\sqrt{6i+1}]}{2}, 1 \le i \le n-1.$

$f^*(u_{2i-1}u_{2i+1}) = \frac{[(6i-3)*(6i+3)+\sqrt{(6i-3+6i+3)}]}{2} = \frac{[(36i^2-9)+\sqrt{12i}]}{2}$, $1 \le i \le \frac{n-2}{2}$.

$f^*(u_{2i}u_{2i+2}) = \frac{[(6i)*(6i+6)+\sqrt{(6i+6i+6)}]}{2} = \frac{[(36i^2+36i)+\sqrt{12i+6}]}{2}$, $1 \le i \le \frac{n-2}{2}$.

Thus the edges { 10,28,56,….., $\frac{[(9i^2+9i)+\sqrt{6i+3}]}{2}$; 2,13,33,….., $\frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}$; 7,22,47,….., $\frac{[(9i^2+3i)+\sqrt{6i+1}]}{2}$; 4,19,44,……, $\frac{[(9i^2+3i)+\sqrt{6i+1}]}{2}$; 15,69,….., $\frac{[(36i^2-9)+\sqrt{12i}]}{2}$; 38,110,…., $\frac{[(36i^2+36i)+\sqrt{12i+6}]}{2}$ } are all distinct.

Example:



Square comb (Even Case)

Theorem 3.7: The graph $D_2(P_n \odot K_1)$, shadow graph of Comb admits Product Root Sum Mean Labeling.

Proof:

Let the vertex set be V={ $u_1,u_2,……u_{n-1}, u_n$; $u_1',u_2',……u_{n-1}', u_n'$; $v_1,v_2,……v_{n-1}, v_n$; $v_1',v_2',……v_{n-1}'$, v_n' }.

The edge set is E = { u_iu_i' , $1 \le i \le n$; $u_i'u_{i+1}'$, $1 \le i \le n-1$; v_iv_{i+1} , $1 \le i \le n-1$; v_iv_i' , $1 \le i \le n$; u_iv_i , $1 \le i \le n$; $u_i'v_i'$, $1 \le i \le n$; $u_i'v_{i+1}$, $1 \le i \le n-1$; v_iu_{i+1}' , $1 \le i \le n-1$ } for all values of n.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = 4i-3$, $1 \le i \le n$;

$f(u_i') = 4i-1$, $1 \le i \le n$;

$f(v_i) = 4i$, $1 \le i \le n$;

$f(v_i') = 4i-2$, $1 \le i \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(u_iu_i') = \frac{[(4i-3)*(4i-1)+\sqrt{4i-3+4i-1}]}{2} = \frac{[(16i^2-16i+3)+\sqrt{8i-4}]}{2}$, $1 \le i \le n$.

$f^*(u_i'u_{i+1}') = \frac{[(4i-1)*(4i+3)+\sqrt{4i-1+4i+3}]}{2} = \frac{[(16i^2+8i-3)+\sqrt{8i+2}]}{2}$, $1 \le i \le n-1$.

$f^*(v_iv_i') = \frac{[(4i)*(4i-2)+\sqrt{(4i+4i-2)}]}{2} = \frac{[(16i^2-8i)+\sqrt{8i-2}]}{2}$, $1 \le i \le n$.

$f^*(v_iv_{i+1}) = \frac{[(4i)*(4i+4)+\sqrt{(4i+4i+4)}]}{2} = \frac{[(16i^2+16i)+\sqrt{8i+4}]}{2}$, $1 \le i \le n-1$.

$f^*(u_i'v_{i+1}) = \frac{[(4i-1)*(4i+4)+\sqrt{(4i-1+4i+4)}]}{2} = \frac{[(16i^2+12i-4)+\sqrt{8i+3}]}{2}, 1 \le i \le n-1.$

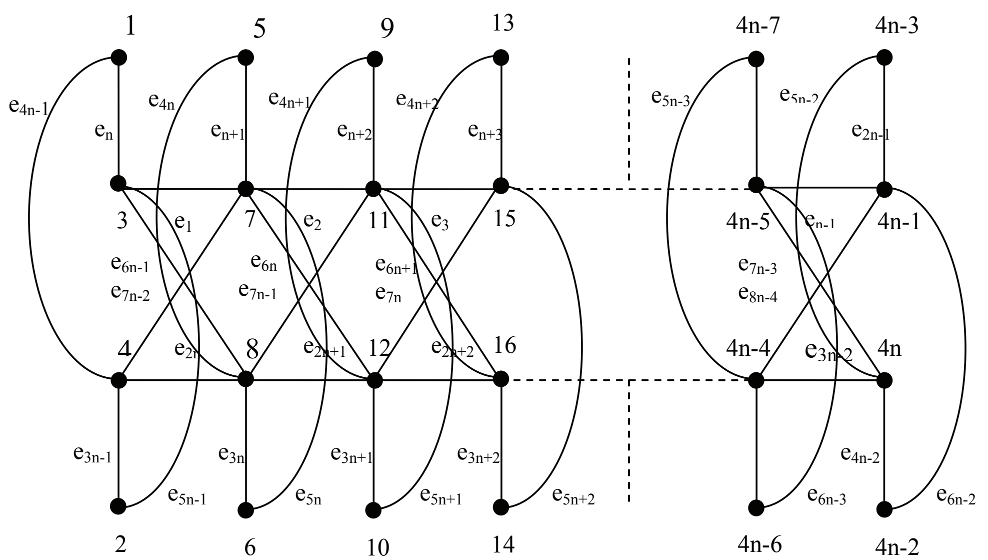
$f^*(u_{i+1}'v_i) = \frac{[(4i+3)*(4i)+\sqrt{(4i+3+4i)}]}{2} = \frac{[(16i^2+12i)+\sqrt{8i+3}]}{2}, 1 \le i \le n-1.$

$f^*(u_i v_i) = \frac{[(4i-3)*(4i)+\sqrt{4i-3+4i}]}{2} = \frac{[(16i^2-12i)+\sqrt{8i-3}]}{2}, 1 \le i \le n.$

$f^*(u_i'v_i') = \frac{[(4i-1)*(4i-2)+\sqrt{4i-1+4i-2}]}{2} = \frac{[(16i^2-12i+2)+\sqrt{8i-3}]}{2}, 1 \le i \le n.$

Thus the edges $\{ 2,19,51,\ldots, \frac{[(16i^2-16i+3)+\sqrt{8i-4}]}{2}$; $12,40,85,\ldots, \frac{[(16i^2+8i-3)+\sqrt{8i+2}]}{2}$; $5,25,62,\ldots, \frac{[(16i^2-8i)+\sqrt{8i-2}]}{2}$; $17,50,98,\ldots, \frac{[(16i^2+16i)+\sqrt{8i+4}]}{2}$; $13,44,90,\ldots, \frac{[(16i^2+12i-4)+\sqrt{8i+3}]}{2}$; $15,46,92,\ldots, \frac{[(16i^2+12i)+\sqrt{8i+3}]}{2}$; $3,21,56,\ldots, \frac{[(16i^2-12i)+\sqrt{8i-3}]}{2}$; $4,22,57,\ldots, \frac{[(16i^2-12i+2)+\sqrt{8i-3}]}{2} \}$ are all distinct.

Example:



Shadow Comb $D_2(P_n \odot K_1)$

Theorem 3.8: The splitting graph of comb, Spl($P_n \odot K_1$) is a Product Root Sum Mean graph.

Proof:

Let the vertex set be $V=\{ u_1,u_2,\ldots u_{n-1}, u_n ; u_1',u_2',\ldots u_{n-1}', u_n' ; v_1,v_2,\ldots v_{n-1}, v_n ; v_1',v_2',\ldots v_{n-1}', v_n' \}$.

The edge set is $E =\{ u_iu_{i+1}' , 1 \le i \le n-1 ; u_{i+1}u_i' , 1 \le i \le n-1 ; u_i'u_{i+1}' , 1 \le i \le n-1 ; u_i'v_i , 1 \le i \le n ; u_i'v_i' , 1 \le i \le n ; u_iv_i , 1 \le i \le n \}$ for all values of n.

Define a map $f : V \rightarrow \{1,2,3,\ldots q+1\}$ Such that

$f(u_i) = 4i , 1 \le i \le n$;

$f(u_i') = 4i-1$, $1 \leq i \leq n$;

$f(v_i) = 4i-2$, $1 \leq i \leq n$;

$f(v_i') = 4i-3$, $1 \leq i \leq n$;

Define the induced function $f^*: E \rightarrow N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$$f^*(u_i u_{i+1}') = \frac{[(4i)*(4i+3)+\sqrt{4i+4i+3}]}{2} = \frac{[(16i^2+12i)+\sqrt{8i+3}]}{2}, 1 \leq i \leq n-1.$$

$$f^*(u_{i+1} u_i') = \frac{[(4i+4)*(4i-1)+\sqrt{4i+4i+4i-1}]}{2} = \frac{[(16i^2+12i-4)+\sqrt{12i-1}]}{2}, 1 \leq i \leq n-1.$$

$$f^*(u_i' u_{i+1}') = \frac{[(4i-1)*(4i+3)+\sqrt{4i-1+4i+3}]}{2} = \frac{[(16i^2+8i-3)+\sqrt{8i+2}]}{2}, 1 \leq i \leq n-1.$$

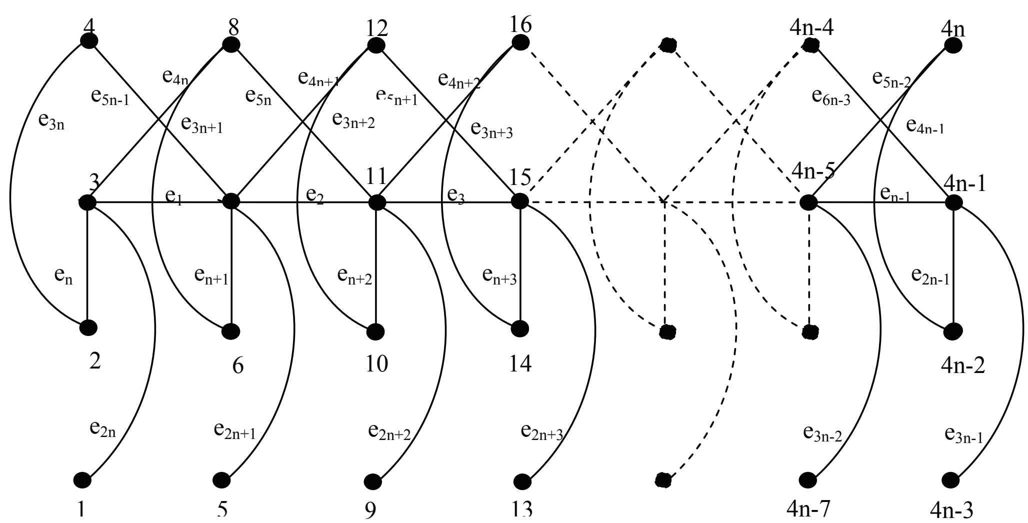
$$f^*(u_i' v_i) = \frac{[(4i-1)*(4i-2)+\sqrt{(4i-1+4i-2)}]}{2} = \frac{[(16i^2-12i+2)+\sqrt{8i-3}]}{2}, 1 \leq i \leq n.$$

$$f^*(u_i' v_i') = \frac{[(4i-1)*(4i-3)+\sqrt{(4i-1+4i-3)}]}{2} = \frac{[(16i^2-16i+3)+\sqrt{8i-4}]}{2}, 1 \leq i \leq n.$$

$$f^*(u_i v_i) = \frac{[(4i)*(4i-2)+\sqrt{(4i+4i-2)}]}{2} = \frac{[(16i^2-8i)+\sqrt{8i-2}]}{2}, 1 \leq i \leq n.$$

Thus the edges { 15,46,92,…..,$\frac{[(16i^2+12i)+\sqrt{8i+3}]}{2}$; 13,44,90,…..,$\frac{[(16i^2+12i-4)+\sqrt{12i-1}]}{2}$; 12,40,85,…..,$\frac{[(16i^2+8i-3)+\sqrt{8i+2}]}{2}$; 4,22,57,……,$\frac{[(16i^2-12i+2)+\sqrt{8i-3}]}{2}$; 2,19,51,……,$\frac{[(16i^2-16i+3)+\sqrt{8i-4}]}{2}$; 5,25,62,……,$\frac{[(16i^2-8i)+\sqrt{8i-2}]}{2}$ } are all distinct.

Example:



Splitting graph of comb $S(P_n \odot K_1)$

Theorem 3.9: The Ladder graph P_n x P_2 is a Product Root Sum Mean Labeling.

Proof:

Let the vertex set be $V=\{u_1,u_2,\ldots\ldots u_{n-1}, u_n , v_1,v_2,\ldots\ldots v_{n-1}, v_n\}$.

The edge set is $E = \{ u_iu_{i+1} , 1 \le i \le n-1 \cup v_iv_{i+1} , 1 \le i \le n-1 \cup u_iv_i , 1 \le i \le n \}$ for all values of n >= 2.

Define a map $f : V \rightarrow \{1,2,3,\ldots.q+1\}$ Such that

$f(u_i) =2i-1 , 1 \le i \le n$;

$f(v_i) =2i , 1 \le i \le n$;

Define the induced function $f^*:E\rightarrow N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(u_iu_{i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{(2i-1)+(2i+1)}]}{2} = \frac{[(4i-1)+\sqrt{4i}]}{2}, 1 \le i \le n-1.$

$f^*(v_iv_{i+1}) = \frac{[(2i)*(2i+2)+\sqrt{(2i)+(2i+2)}]}{2} = \frac{[(4i^2+4i)+\sqrt{4i+2}]}{2}, 1 \le i \le n-1.$

$f^*(u_iv_i) = \frac{[(2i-1)*(2i)+\sqrt{(2i-1)+(2i)}]}{2} = \frac{[(4i^2-2i)+\sqrt{4i-1}]}{2}, 1 \le i \le n.$

Thus the edges $\{ 3,9,20 ,\ldots.., \frac{[(4i-1)+\sqrt{4i}]}{2} ; 6,14,26, \ldots. , \frac{[(4i^2+4i)+\sqrt{4i+2}]}{2} ;2,8,17,\ldots.., \frac{[(4i^2-2i)+\sqrt{4i-1}]}{2}\}$ are all distinct.

Theorem 3.10: The Square graph of Ladder $P_n \times P_2$ is a Product Root Sum Mean graph.

Proof:

Case(i): If n is odd

Let the vertex set be $V=\{u,u_1,u_2,\ldots\ldots u_{n-1}, u_n , v_1,v_2,\ldots\ldots v_{n-1}, v_n\}$.

The edge set is $E =\{ uu_1 ; u_iu_{i+1} , 1 \le i \le n-2 ; uu_2 ; u_{2i}u_{2i+2} , 1 \le i \le \frac{n-3}{2} ; u_{2i-1}u_{2i+1} , 1 \le i \le \frac{n-3}{2} ; v_iv_{i+1} , 1 \le i \le n-1 ; v_{2i-1}v_{2i+1} , 1 \le i \le \frac{n-1}{2} ; v_{2i}v_{2i+2} , 1 \le i \le \frac{n-3}{2} ; uv_2 ; u_iv_{i+2} , 1 \le i \le n-2 ; u_iv_i , 1 \le i \le n-1 ; uv_1 ; u_iv_{i+1} , 1 \le i \le n-1 \}$ for all values of n >= 2.

Define a map $f : V \rightarrow \{1,2,3,\ldots.q+1\}$ Such that

$f(u) =2$;

$f(u_i) =4i+1 , 1 \le i \le n-1$;

$f(v_i) =4i , 1 \le i \le n$;

Define the induced function $f^*:E\rightarrow N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(uu_1) = \frac{[2*5+\sqrt{2+5}]}{2} = 6.\ f^*(uu_2) = \frac{[2*9+\sqrt{2+9}]}{2} = 10.\ f^*(uv_2) = \frac{[2*8+\sqrt{2+8}]}{2} = 9.$

$f^*(u_iu_{i+1}) = \frac{[(4i+1)*(4i+5)+\sqrt{(4i+1)+(4i+5)}]}{2} = \frac{[(16i^2+24i+5)+\sqrt{8i+6}]}{2}, 1 \le i \le n-2.$

$f^*(u_{2i}u_{2i+2}) = \frac{[(8i+1)*(8i+9)+\sqrt{(8i+1)+(8i+9)}]}{2} = \frac{[(64i^2+80i+9)+\sqrt{16i+10}]}{2}, 1 \le i \le \frac{n-3}{2}.$

$f^*(u_{2i-1}u_{2i+1}) = \frac{[(8i-3)*(8i+5)+\sqrt{(8i-3)+(8i+5)}]}{2} = \frac{[(64i^2+16i-15)+\sqrt{16i+2}]}{2}, 1 \le i \le \frac{n-3}{2}.$

$f^*(v_iv_{i+1}) = \frac{[(4i)*(4i+4)+\sqrt{4i+(4i+4)}]}{2} = \frac{[(16i^2+16i)+\sqrt{8i+4}]}{2}, 1 \le i \le n-1.$

$f^*(v_{2i-1}v_{2i+1}) = \frac{[(8i-4)*(8i+4)+\sqrt{(8i-4)+(8i+4)}]}{2} = \frac{[(64i^2-16)+\sqrt{16i}]}{2}, 1 \le i \le \frac{n-1}{2}.$

$f^*(v_{2i}v_{2i+2}) = \frac{[(8i)*(8i+8)+\sqrt{(8i)+(8i+8)}]}{2} = \frac{[(64i^2+64i)+\sqrt{16i+8}]}{2}, 1 \le i \le \frac{n-3}{2}.$

$f^*(u_iv_{i+2}) = \frac{[(4i+1)*(4i+8)+\sqrt{(4i+1)+(4i+8)}]}{2} = \frac{[(16i^2+36i+8)+\sqrt{8i+9}]}{2}, 1 \le i \le n-2.$

$f^*(u_iv_i) = \frac{[(4i+1)*(4i)+\sqrt{(4i+1)+(4i)}]}{2} = \frac{[(16i^2+4i)+\sqrt{8i+1}]}{2}, 1 \le i \le n-1.$

$f^*(uv_1) = \frac{[(2)*(4)+\sqrt{(2+4)}]}{2} = 5.$

$f^*(u_iv_{i+1}) = \frac{[(4i+1)*(4i+4)+\sqrt{(4i+1)+(4i+4)}]}{2} = \frac{[(16i^2+20i+4)+\sqrt{8i+5}]}{2}, 1 \le i \le n-1.$

Thus the edges { 6 ; 10 ; 9 ; 24,60,113,……., $\frac{[(16i^2+24i+5)+\sqrt{8i+6}]}{2}$; 79,215,……, $\frac{[(64i^2+80i+9)+\sqrt{16i+10}]}{2}$; 34,139,……, $\frac{[(64i^2+16i-15)+\sqrt{16i+2}]}{2}$; 17,50,98,……, $\frac{[(16i^2+16i)+\sqrt{8i+4}]}{2}$; 26,122,283,……, $\frac{[(64i^2-16)+\sqrt{16i}]}{2}$; 66,195,….., $\frac{[(64i^2+64i)+\sqrt{16i+8}]}{2}$; 32,74,132,……., $\frac{[(16i^2+36i+8)+\sqrt{8i+9}]}{2}$; 11,38,80,……., $\frac{[(16i^2+4i)+\sqrt{8i+1}]}{2}$; 5 ; 21,56,106,……, $\frac{[(16i^2+20i+4)+\sqrt{8i+5}]}{2}$} are all distinct.

Case(ii): If n is Even

Let the vertex set be V={u,u_1,u_2,……u_{n-1}, u_n , v_1,v_2,……v_{n-1}, v_n}.

The edge set is E ={ uu_1 ; u_iu_{i+1} , $1 \le i \le$ n-2 ; uu_2 ; $u_{2i}u_{2i+2}$, $1 \le i \le \frac{n-4}{2}$; $u_{2i-1}u_{2i+1}$, $1 \le i \le \frac{n-2}{2}$; v_iv_{i+1} , $1 \le i \le$ n-1 ; $v_{2i-1}v_{2i+1}$, $1 \le i \le \frac{n-2}{2}$; $v_{2i}v_{2i+2}$, $1 \le i \le \frac{n-2}{2}$; uv_2 ; u_iv_{i+2} , $1 \le i \le$ n-2 ; u_iv_i , $1 \le i \le$ n-1 ; uv_1 ; u_iv_{i+1} , $1 \le i \le$ n-1 } for all values of n >= 2.

Define a map f : V→ {1,2,3,….q+1} Such that

f(u) =2 ;

$f(u_i)$ =4i+1 , $1 \le i \le$ n-1 ;

$f(v_i)$ =4i , $1 \le i \le$ n ;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(uu_1) = \frac{[2*5+\sqrt{2+5}]}{2} = 6.$

$f^*(uu_2) = \frac{[2*9+\sqrt{2+9}]}{2} = 10.$

$f^*(uv_2) = \frac{[2*8+\sqrt{2+8}]}{2} = 9.$

$f^*(u_iu_{i+1}) = \frac{[(4i+1)*(4i+5)+\sqrt{(4i+1)+(4i+5)}]}{2} = \frac{[(16i^2+24i+5)+\sqrt{8i+6}]}{2}, 1 \le i \le n-2.$

$f^*(u_{2i}u_{2i+2}) = \frac{[(8i+1)*(8i+9)+\sqrt{(8i+1)+(8i+9)}]}{2} = \frac{[(64i^2+80i+9)+\sqrt{16i+10}]}{2}, 1 \le i \le \frac{n-4}{2}.$

$f^*(u_{2i-1}u_{2i+1}) = \frac{[(8i-3)*(8i+5)+\sqrt{(8i-3)+(8i+5)}]}{2} = \frac{[(64i^2+16i-15)+\sqrt{16i+2}]}{2}, 1 \le i \le \frac{n-2}{2}.$

$f^*(v_i v_{i+1}) = \frac{[(4i)*(4i+4)+\sqrt{4i+(4i+4)}]}{2} = \frac{[(16i^2+16i)+\sqrt{8i+4}]}{2}, 1 \le i \le n-1.$

$f^*(v_{2i-1} v_{2i+1}) = \frac{[(8i-4)*(8i+4)+\sqrt{(8i-4)+(8i+4)}]}{2} = \frac{[(64i^2-16)+\sqrt{16i}]}{2}, 1 \le i \le \frac{n-2}{2}.$

$f^*(v_{2i} v_{2i+2}) = \frac{[(8i)*(8i+8)+\sqrt{(8i)+(8i+8)}]}{2} = \frac{[(64i^2+64i)+\sqrt{16i+8}]}{2}, 1 \le i \le \frac{n-2}{2}.$

$f^*(u_i v_{i+2}) = \frac{[(4i+1)*(4i+8)+\sqrt{(4i+1)+(4i+8)}]}{2} = \frac{[(16i^2+36i+8)+\sqrt{8i+9}]}{2}, 1 \le i \le n-2.$

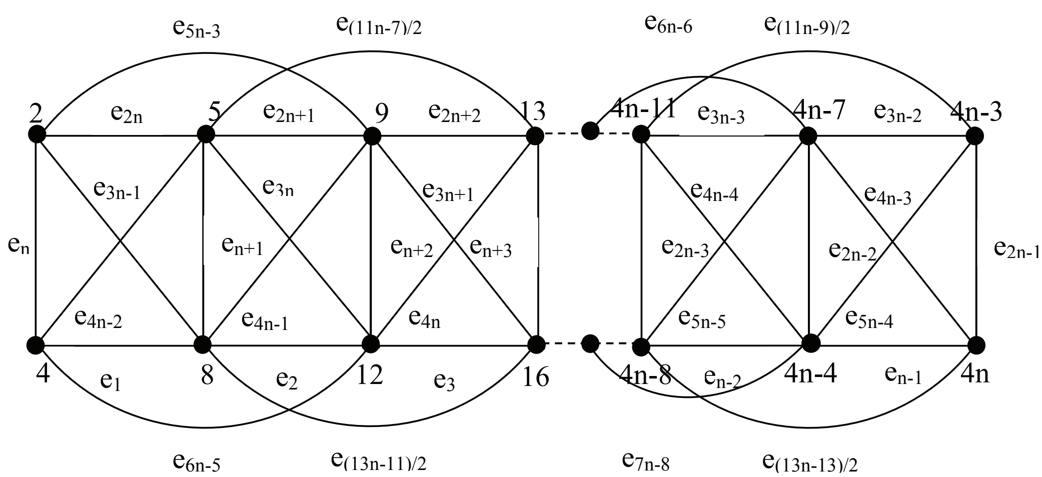
$f^*(u_i v_i) = \frac{[(4i+1)*(4i)+\sqrt{(4i+1)+(4i)}]}{2} = \frac{[(16i^2+4i)+\sqrt{8i+1}]}{2}, 1 \le i \le n-1.$

$f^*(uv_1) = \frac{[(2)*(4)+\sqrt{(2+4)}]}{2} = 5.$

$f^*(u_i v_{i+1}) = \frac{[(4i+1)*(4i+4)+\sqrt{(4i+1)+(4i+4)}]}{2} = \frac{[(16i^2+20i+4)+\sqrt{8i+5}]}{2}, 1 \le i \le n-1.$

Thus the edges { 6 ; 10 ; 9 ; 24,60,113,……., $\frac{[(16i^2+24i+5)+\sqrt{8i+6}]}{2}$; 79,215,……, $\frac{[(64i^2+80i+9)+\sqrt{16i+10}]}{2}$; 34,139,……, $\frac{[(64i^2+16i-15)+\sqrt{16i+2}]}{2}$; 17,50,98,……, $\frac{[(16i^2+16i)+\sqrt{8i+4}]}{2}$; 26,122,283,……, $\frac{[(64i^2-16)+\sqrt{16i}]}{2}$; 66,195,….., $\frac{[(64i^2+64i)+\sqrt{16i+8}]}{2}$; 32,74,132,……., $\frac{[(16i^2+36i+8)+\sqrt{8i+9}]}{2}$; 11,38,80,……., $\frac{[(16i^2+4i)+\sqrt{8i+1}]}{2}$; 5 ; 21,56,106,……, $\frac{[(16i^2+20i+4)+\sqrt{8i+5}]}{2}$} are all distinct.

Example:



Square Ladder (Odd Case)

Theorem 3.11: The graph $P_n \ \Theta \ K_{1,2}$ admits Product Root Sum Mean Labeling.

Proof:

Let the vertex set be $V=\{u_1,u_2,\ldots\ldots u_{n-1}, u_n, v_1,v_2,\ldots\ldots v_{n-1}, v_n, w_1,w_2,\ldots\ldots w_{n-1}, w_n\}$.

The edge set is $E =\{ u_iu_{i+1}, 1 \le i \le n-1 \cup u_iv_i, 1 \le i \le n \cup u_iw_i, 1 \le i \le n \}$ for all values of n >= 2.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = 3i-2$, $1 \le i \le n$;

$f(v_i) = 3i-1$, $1 \le i \le n$;

$f(w_i) = 3i$, $1 \le i \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(u_i u_{i+1}) = \frac{[(3i-2)*(3i+1)+\sqrt{(3i-2)+(3i+1)}]}{2} = \frac{[(9i^2-6i-2)+\sqrt{6i-1}]}{2}, 1 \le i \le n-1.$

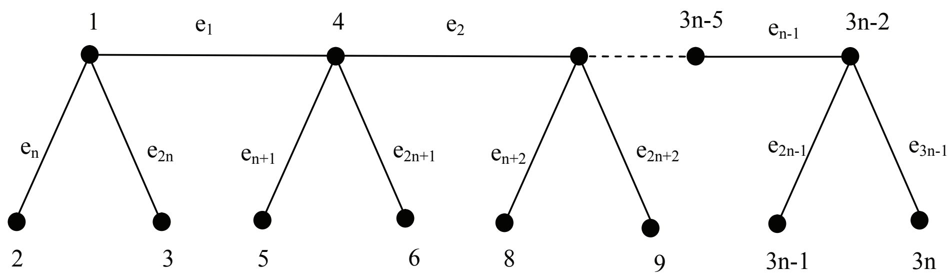
$f^*(u_i v_i) = \frac{[(3i-2)*(3i-1)+\sqrt{(3i-2)+(3i-1)}]}{2} = \frac{[(9i^2-9i+2)+\sqrt{6i-3}]}{2}$ $1 \le i \le n.$

$f^*(u_i w_i) = \frac{[(3i-2)*(3i)+\sqrt{(3i-2)+(3i)}]}{2} = \frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}$ $1 \le i \le n.$

Thus the edges { 4,16,38 ,…..,$\frac{[(9i^2-6i-2)+\sqrt{6i-1}]}{2}$; 2,12,30,…..,$\frac{[(9i^2-9i+2)+\sqrt{6i-5}]}{2}$;

3,14,34,….,$\frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}$} are all distinct.

Example:



$P_n \odot K_2$

Theorem 3.12 : The graph $P_n \odot K_{1,3}$ is a Product Root Sum Mean graph.

Proof:

Let the vertex set be $V=\{u_1,u_2,\ldots u_{n-1}, u_n, v_1,v_2,\ldots v_{n-1}, v_n, w_1,w_2,\ldots w_{n-1}, w_n, x_1,x_2,\ldots x_{n-1}, x_n\}$.

The edge set is $E = \{ u_iu_{i+1}, 1 \le i \le n-1 \cup u_iv_i, 1 \le i \le n \cup u_iw_i, 1 \le i \le n \cup u_ix_i, 1 \le i \le n \}$ for all values of $n >= 2$.

Define a map f : V→ {1,2,3,….q+1} Such that

$f(u_i) = 3i-2$, $1 \le i \le n$;

$f(v_i) = 3i-1$, $1 \le i \le n$;

$f(w_i) = 3i$, $1 \le i \le n$;

$f(x_i) = 3n+i$, $1 \le i \le n$;

Define the induced function f*:E→N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(u_iu_{i+1}) = \frac{[(3i-2)*(3i+1)+\sqrt{(3i-2)+(3i+1)}]}{2} = \frac{[(9i^2-6i-2)+\sqrt{6i-1}]}{2}, 1 \le i \le n-1.$

$f^*(u_iv_i) = \frac{[(3i-2)*(3i-1)+\sqrt{(3i-2)+(3i-1)}]}{2} = \frac{[(9i^2-9i+2)+\sqrt{6i-3}]}{2}, 1 \le i \le n.$

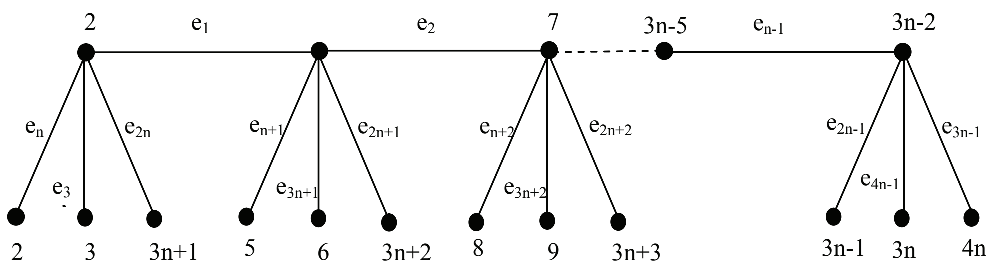
$f^*(u_iw_i) = \frac{[(3i-2)*(3i)+\sqrt{(3i-2)+(3i)}]}{2} = \frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}, 1 \le i \le n.$

$f^*(u_ix_i) = \frac{[(3i-2)*(3n+i)+\sqrt{(3i-2)+(3n+i)}]}{2} = \frac{[(9in+3i^2-6n-2i)+\sqrt{4i+3n-2}]}{2}, 1 \le i \le n.$

Thus the edges { 4,16,38 ,….., $\frac{[(9i^2-6i-2)+\sqrt{6i-1}]}{2}$; 2,12,30,……, $\frac{[(9i^2-9i+2)+\sqrt{6i-3}]}{2}$;

3,14,34,….., $\frac{[(9i^2-6i)+\sqrt{6i-2}]}{2}$; 5,18,….., $\frac{[(9in+3i^2-6n-2i)+\sqrt{4i+3n-2}]}{2}$} are all distinct.

Example:



$P_n \odot K_3$

Theorem 3.13 : The Star graph $K_{1,n}$ admits Product Root Sum Mean Labeling.

Proof:

Let the vertex set be $V=\{u ; v ; v_1,v_2,\ldots v_{n-1}\}$.

The edge set is $E = \{ uv ; uv_i , 1 \le i \le n-1 \}$ for all values of n.

Define a map $f : V \to \{1,2,3,\ldots q+1\}$ Such that

$f(u) = 2$; $f(v) = 1$;

$f(v_i) = i + 2$, $1 \le i \le n-1$;

Define the induced function $f^*:E \to N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(uv) = \frac{[2*1+\sqrt{2+1}]}{2} = \frac{[2+\sqrt{3}]}{2} = 1.$

$f^*(uv_i) = \frac{[2*(i+2)+\sqrt{2+(i+2)}]}{2} = \frac{[2i+4+\sqrt{4+i}]}{2}, 1 \le i \le n-1.$

Thus the edges { 2 ; 4,5,6 ,….., $\frac{[2i+4+\sqrt{4+i}]}{2}$ } are all distinct.

Theorem 3.14: The graph Shadow Star $K_{1,n}$is a Product Root Sum Mean Labeling.

Proof:

Let the vertex set be $V=\{ u ; v ; u_1,u_2,\ldots u_n ; v_1,v_2,\ldots v_n \}$.

The edge set is E = { uu_i , $1 \le i \le n$; uv_i , $1 \le i \le n$; vv_i , $1 \le i \le n$; vu_i , $1 \le i \le n$ } for all values of n.

Define a map $f : V \rightarrow \{1,2,3,\ldots.q+1\}$ Such that

$f(u) = 2$;

$f(v) = 4n+1$;

$f(u_i) = 3i+1$, $1 \le i \le n$;

$f(v_i) = 3i$, $1 \le i \le n$;

Define the induced function $f^*: E \rightarrow N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(uu_i) = \frac{\lfloor 2*(3i+1)+\sqrt{2+3i+1} \rfloor}{2} = \frac{\lfloor 6i+2+\sqrt{3i+3} \rfloor}{2}$,$1 \le i \le n$.

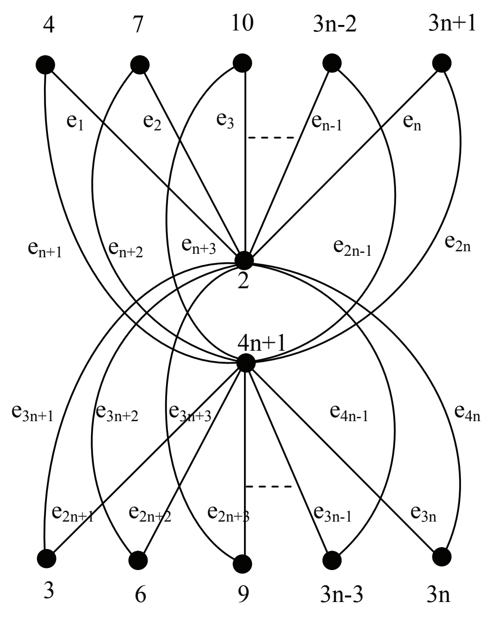
$f^*(uv_i) = \frac{\lfloor 2*(3i)+\sqrt{2+(3i)} \rfloor}{2} = \frac{\lfloor 6i+\sqrt{3i+2} \rfloor}{2}$, $1 \le i \le n$.

$f^*(vv_i) = \frac{\lfloor (4n+1)*(3i)+\sqrt{4n+1+3i} \rfloor}{2} = \frac{\lfloor 12ni+3i+\sqrt{4n+1+3i} \rfloor}{2}$,$1 \le i \le n$.

$f^*(vu_i) = \frac{\lfloor (4n+1)*(3i+1)+\sqrt{4n+1+3i+1} \rfloor}{2} = \frac{\lfloor 12ni+4n+3i+1+\sqrt{4n+3i+2} \rfloor}{2}$,$1 \le i \le n$.

Thus the edges { 5,8,11,……., $\frac{\lfloor 6i+2+\sqrt{3i+3} \rfloor}{2}$; 4,7,10 ,….., $\frac{\lfloor 6i+\sqrt{3i+2} \rfloor}{2}$;
$\frac{\lfloor 12ni+3i+\sqrt{4n+1+3i} \rfloor}{2}$,……., $\frac{\lfloor 12n^2+3n+\sqrt{7n+1} \rfloor}{2}$; $\frac{\lfloor 12ni+4n+3i+1+\sqrt{4n+3i+2} \rfloor}{2}$,…., $\frac{\lfloor 12n^2+7n+1+\sqrt{7n+2} \rfloor}{2}$ } are all distinct.

Example:



Shadow Star $K_{1,n}$

Theorem 3.15 : The Split graph of Star $K_{1,n}$ is a Product Root Sum Mean graph.

Proof:

Let the vertex set be $V=\{ u ; v ; u_1,u_2,\ldots\ldots u_n ; v_1,v_2,\ldots\ldots v_n \}$.

The edge set is $E = \{ uu_i , 1 \le i \le n ; vu_i , 1 \le i \le n ; vv_i , 1 \le i \le n \}$ for all values of n.

Define a map $f : V \rightarrow \{1,2,3,\ldots.q+1\}$ Such that

$f(u) = 1$;

$f(v) = 2(n+1)$;

$f(u_i) = 2i + 1$, $1 \le i \le n$;

$f(v_i) = 2i$, $1 \le i \le n$;

Define the induced function $f^*:E \rightarrow N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

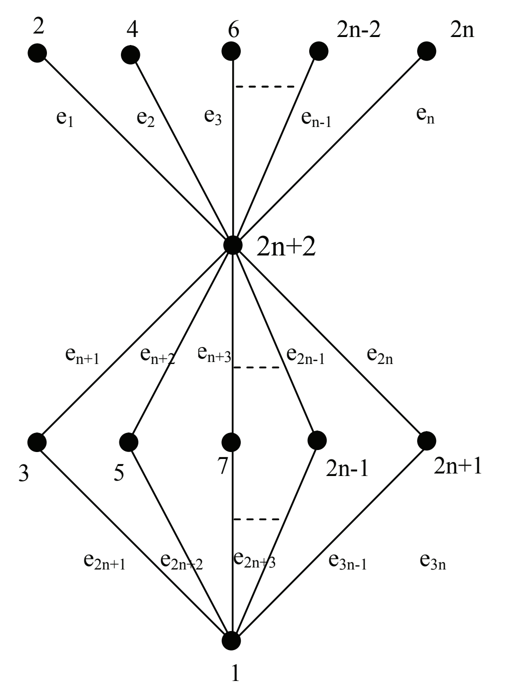
$f^*(uu_i) = \frac{\lceil 1*(2i+1)+\sqrt{1+2i+1} \rceil}{2} = \frac{\lceil 2i+1+\sqrt{2i+2} \rceil}{2}$,$1 \le i \le n$.

$f^*(u_iv) = \frac{\lceil (2i+1)*(2n+2)+\sqrt{(2i+1)+(2n+2)} \rceil}{2} = \frac{\lceil 4ni+4i+2n+2+\sqrt{2n+2i+3} \rceil}{2}$, $1 \le i \le n$.

$f^*(vv_i) = \frac{\lceil (2n+2)*(2i)+\sqrt{2n+2+2i} \rceil}{2} = \frac{\lceil 4ni+4i+\sqrt{2n+2i+2} \rceil}{2}$,$1 \le i \le n$.

Thus the edges $\{ 2,3,4,\ldots\ldots, \frac{\lceil 2i+1+\sqrt{2i+2} \rceil}{2}, \frac{\lceil 4ni+4i+2n+2+\sqrt{2n+2i+3} \rceil}{2}, \ldots.., \frac{\lceil 4n^2+6n+2+\sqrt{4n+3} \rceil}{2}$; $\frac{\lceil 4ni+4i+\sqrt{2n+2i+2} \rceil}{2}, \ldots\ldots, \frac{\lceil 4n^2+4n+\sqrt{4n+2} \rceil}{2} \}$ are all distinct.

Example:



Split Star $K_{1,n}$

Theorem 3.16: The BiStar graph $K_{n,n}$ admits Product Root Sum Mean Labeling.

Proof:

Let the vertex set be $V=\{ u ; v ; u_1,u_2,\ldots\ldots u_n ; v_1,v_2,\ldots\ldots v_n \}$.

The edge set is $E = \{ uv ; uu_i , 1 \le i \le n ; vv_i , 1 \le i \le n \}$ for all values of n.

Define a map $f : V \rightarrow \{1,2,3,\ldots.q+1\}$ Such that

$f(u) = 2n+1$;

$f(v) = 2n+2$;

$f(u_i) =2i - 1$, $1 \le i \le n$;

$f(v_i) =2i$, $1 \le i \le n$;

Define the induced function $f^*:E \rightarrow N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$f^*(uv) = \frac{\lceil (2n+1)*(2n+2)+\sqrt{2n+1+2n+2} \rceil}{2} = \frac{\lceil 4n^2+6n+2+\sqrt{4n+3} \rceil}{2}$.

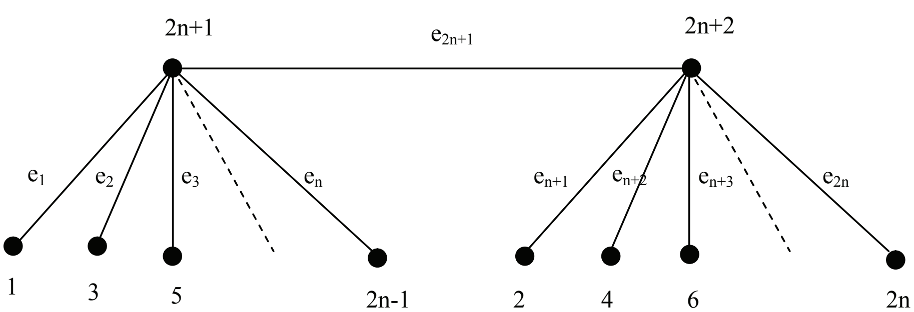
$f^*(uu_i) = \frac{\lceil (2n+1)*(2i-1)+\sqrt{2n+1+2i-1} \rceil}{2} = \frac{\lceil 4ni-2n+2i-1+\sqrt{2n+2i} \rceil}{2}$,$1 \le i \le n$.

$f^*(vv_i) = \frac{\lceil (2n+2)*(2i)+\sqrt{2n+2+2i} \rceil}{2} = \frac{\lceil 4ni+4i+\sqrt{2n+2i+2} \rceil}{2}$,$1 \le i \le n$.

Thus the edges $\{\frac{\lceil 4n^2+6n+2+\sqrt{4n+3} \rceil}{2}, \frac{\lceil 4ni-2n+2i-1+\sqrt{2n+2i} \rceil}{2}, \ldots.., \frac{\lceil 4n^2-1+\sqrt{4n} \rceil}{2}$;

$\frac{\lceil 4ni+4i+\sqrt{2n+2i+2} \rceil}{2}, \ldots., \frac{\lceil 4n^2+4n+\sqrt{4n+2} \rceil}{2} \}$ are all distinct.

Example:



Bistar star $K_{n,n}$

Theorem 3.17: The Shadow graph of BiStar$K_{n,n}$ is a Product Root Sum Mean graph.

Proof:

Let the vertex set be $V=\{ u ; v ; w ; x ; u_1,u_2,\ldots\ldots u_n ; v_1,v_2,\ldots\ldots v_n ; w_1,w_2,\ldots\ldots w_n ; x_1,x_2,\ldots\ldots x_n \}$.

The edge set is $E = \{ uw ; vx ; uu_i , 1 \le i \le n ; vu_i , 1 \le i \le n ; uv_i , 1 \le i \le n ; vv_i , 1 \le i \le n ; ww_i , 1 \le i \le n ;$ $xw_i, 1 \le i \le n ; wx_i , 1 \le i \le n ; xx_i , 1 \le i \le n \}$ for all values of n.

Define a map $f : V \rightarrow \{1,2,3,\ldots q+1\}$ Such that

$f(u) = 2$.

$f(v) = 4n+1$.

$f(w) = 8n$.

$f(x) = 8n+1$.

$f(u_i) = 3i + 1$, $1 \leq i \leq n$;

$f(v_i) = 3i$, $1 \leq i \leq n$;

$f(w_i) = 4n+3i$, $1 \leq i \leq n$;

$f(x_i) = 4n+3i-1$, $1 \leq i \leq n$;

Define the induced function $f^*: E \rightarrow N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$

The edge labels are obtained as

$$f^*(uw) = \frac{\lfloor (2)*(8n)+\sqrt{8n+2} \rfloor}{2} = \frac{\lfloor 16n+\sqrt{8n+2} \rfloor}{2}.$$

$$f^*(vx) = \frac{\lfloor (4n+1)*(8n+1)+\sqrt{4n+1+8n+1} \rfloor}{2} = \frac{\lfloor 32n^2+12n+1+\sqrt{12n+2} \rfloor}{2}.$$

$$f^*(uu_i) = \frac{\lfloor (2)*(3i+1)+\sqrt{2+3i+1} \rfloor}{2} = \frac{\lfloor 6i+2+\sqrt{3i+3} \rfloor}{2}, 1 \leq i \leq n.$$

$$f^*(vv_i) = \frac{\lfloor (4n+1)*(3i)+\sqrt{4n+1+3i} \rfloor}{2} = \frac{\lfloor 12ni+3i+\sqrt{4n+3i+1} \rfloor}{2}, 1 \leq i \leq n.$$

$$f^*(ww_i) = \frac{\lfloor (8n)*(4n+3i)+\sqrt{8n+4n+3i} \rfloor}{2} = \frac{\lfloor 32n^2+24ni+\sqrt{12n+3i} \rfloor}{2}, 1 \leq i \leq n.$$

$$f^*(xx_i) = \frac{\lfloor (8n+1)*(4n+3i)+\sqrt{8n+1+4n+3i} \rfloor}{2} = \frac{\lfloor 32n^2+24ni+4n+3i+\sqrt{12n+3i+1} \rfloor}{2}, 1 \leq i \leq n.$$

$$f^*(uv_i) = \frac{\lfloor (2)*(3i)+\sqrt{2+3i} \rfloor}{2} = \frac{\lfloor 6i+\sqrt{3i+2} \rfloor}{2}, 1 \leq i \leq n.$$

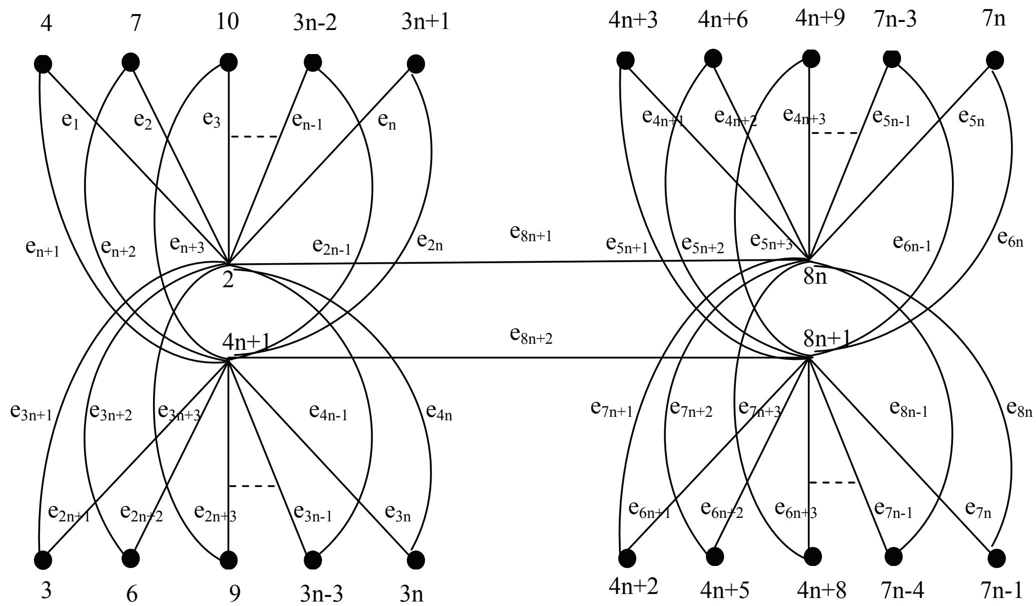
$$f^*(vu_i) = \frac{\lfloor (4n+1)*(3i+1)+\sqrt{4n+1+3i+1} \rfloor}{2} = \frac{\lfloor 12ni+4n+3i+1+\sqrt{4n+3i+2} \rfloor}{2}, 1 \leq i \leq n.$$

$$f^*(xw_i) = \frac{\lfloor (8n+1)*(4n+3i)+\sqrt{8n+1+4n+3i} \rfloor}{2} = \frac{\lfloor 32n^2+24ni+4n+3i+\sqrt{12n+3i+1} \rfloor}{2}, 1 \leq i \leq n.$$

$$f^*(wx_i) = \frac{\lfloor (8n)*(4n+3i-1)+\sqrt{8n+4n+3i-1} \rfloor}{2} = \frac{\lfloor 32n^2+24ni-8n+\sqrt{12n+3i-1} \rfloor}{2}, 1 \leq i \leq n.$$

Thus the edges $\{ \frac{\lfloor 16n+\sqrt{8n+2} \rfloor}{2}; \frac{\lfloor 32n^2+12n+1+\sqrt{12n+2} \rfloor}{2}; 5,8,11,\ldots, \frac{\lfloor 6i+2+\sqrt{3i+3} \rfloor}{2};$ $\frac{\lfloor 12ni+3i+\sqrt{4n+3i+1} \rfloor}{2}, \ldots, \frac{\lfloor 12n^2+3n+\sqrt{7n+1} \rfloor}{2}; \frac{\lfloor 32n^2+24ni+\sqrt{12n+3i} \rfloor}{2}, \ldots, \frac{\lfloor 56n^2+\sqrt{15n} \rfloor}{2}; \frac{\lfloor 32n^2+24ni+4n+3i+\sqrt{12n+3i+1} \rfloor}{2}$ $, \ldots, \frac{\lfloor 56n^2+7n+\sqrt{15n+1} \rfloor}{2}; 4,7,10,\ldots, \frac{\lfloor 6i+\sqrt{3i+2} \rfloor}{2}; \frac{\lfloor 12ni+4n+3i+1+\sqrt{4n+3i+2} \rfloor}{2}, \ldots, \frac{\lfloor 12n^2+7n+1+\sqrt{7n+2} \rfloor}{2};$ $\frac{\lfloor 32n^2+24ni+4n+3i+\sqrt{12n+3i+1} \rfloor}{2}, \ldots, \frac{\lfloor 56n^2+7n+\sqrt{15n+1} \rfloor}{2}; \frac{\lfloor 32n^2+24ni-8n+\sqrt{12n+3i-1} \rfloor}{2}, \ldots, \frac{\lfloor 56n^2-8n+\sqrt{15n-1} \rfloor}{2} \}$ are all distinct.

Example:



Shadow BiStar$K_{n,n}$

Conclusion: In this paper we proved that the ladder graph , the Square Ladder graph , the cocunut tree CT(m,m), the graph Y_{r+1} , the Star graph K_1,n , the Shadow graph of Star $K_{1,n}$, the Split graph of Star $K_{1,n}$, the BiStar graph $K_{n,n}$, the Shadow graph of BiStar $K_{n,n}$, the Comb graph $P_n \odot K_1$, the Square graph of Comb $P_n \odot K_1$, the Shadow graph of Comb $D_2(P_n \odot K_1)$ and the splitting graph of Comb are Product Root Sum Mean graphs.

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