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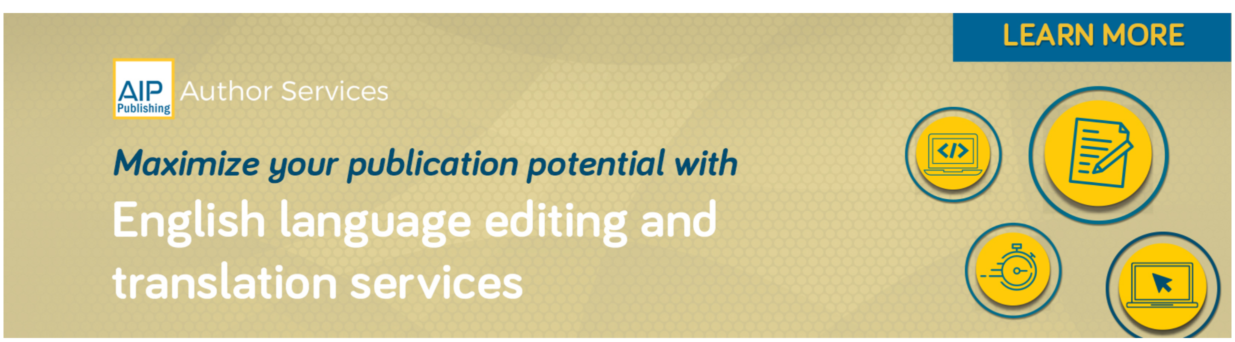

Percolation Theory in the Road Map Using Network Analysis

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Abstract. The study of self-similarity in the characteristic of the fractals and it is implemented in Network Analysis. The percolation theory explains how networks behave when nodes or connections are removed, and fractals for graphs show that it is theoretically connected to graphs.. The Network analysis explains about the finite ramified self-similarity sets, that emphasis on the post-critically finite ones. This shows the network analysis has the complexity, random walk and connectivity. Here considering the Chennai city road map in Tamil Nadu, India indicates that, it is connected to each other between the Chennai fort to the Vandalur zoo. Finally, the road map is determine by the Topological features can be analyzed by Voronoi diagrams and Delaunay triangulation. Percolation theory is the simplest model displaying a phase transition. This model leads to fractals.

INTRODUCTION

Leonhard Euler published the first paper in the history of graph theory in 1736 on the Seven Bridges of Konigsberg [1]. The Konigsberg bridge conundrum was a long-standing puzzle involving the likelihood of finding a way across all seven bridges that span a bifurcate river flowing across an island not more than once. According to Euler, such a path does not exist. With only references to the physical arrangement of the bridges, he proved the first theorem in graph theory [1]. Novel by Frank Harary, published in 1969. The analysis of graphs, which are mathematical constructs used to model pair-wise relationships between objects in mathematics, is known as graph theory. In this case, a graph is inclusive of nodes that are bound by links. Undirected graphs and directed graphs are distinguished [2].

Fractals

Benoit Mandelbrot discovered the fractals (1975). He defined them as geometric shapes in which each part is a smaller representation of the entire shape when divided into parts. As a new scientific term for this mathematical expression, he coined the term Fractal. The word fractals are derived from the Latin word Fractus. Benoit was dubbed the "Father of Fractal Geometry" (fractal geometry) [3]. As a result, fractal objects and processes are said to be self-invariant. Fractal structures and processes do not have a single length, and fractal processes do not have a single time scale. The Euclidean concept of length is regarded as a method in fractal analysis. The fractal dimension [4] is a constant parameter that characterises this operation.

Percolation Theory

Percolation theory is the most basic, but not perfectly solved, model of a phase transition. The understanding of the percolation theory problem also aids the understanding of a variety of other physical processes. Furthermore, the idea of fractals, which is closely linked to the percolation theory problem, is of general interest concepts in fields as diverse as biology, physics, and geophysics, as well as practical applications such as oil recovery. We'll start by building a foundation in percolation theory, which will provide as a natural introduction to the concepts of scaling and renormalization group theory [5].

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METHODS

In this paper, some methods are applicable such as Percolation theory, Voronoi diagram, Delaunay triangulation.

Network

A network is nothing but a set of interconnected objects. It considers the objects nodes or vertices and depicts them as points. Graph theory is a branch of mathematics concerned with the study of graphs [2]. In this paper, The roadside map displays the route from Chennai Fort to Vandalur Zoo. The following network shows that there is connectivity and that it is restricted. The street is connected to each other in the road-side map, while each area is called a vertices and the connection between them is considered an edge. There is complexity and a random walk in this roadside map.

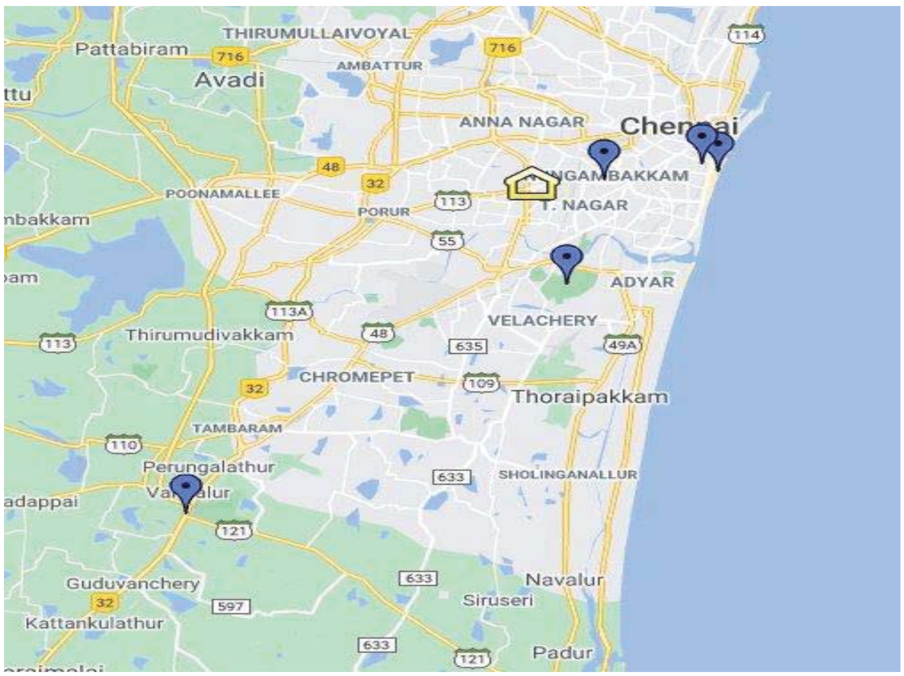

FIGURE 1. The network of the road map-City Chennai

Complexity

A complex network is a graph (network) with non-trivial features vector which does not appear in simple networks such as lattices or random graphs but usually exist in networks representing real structure, according to network theory [6]. The network analysis determines the number of vertices, edges, and proper path in a graph [1]. On the road map from Chennai fort to Vandalur zoo, there are several paths to reach the destination. Complexity is implemented in the road map that shows the connection between the different places. The figure represents that there are various connections or a proper path between them (Fig. 2).

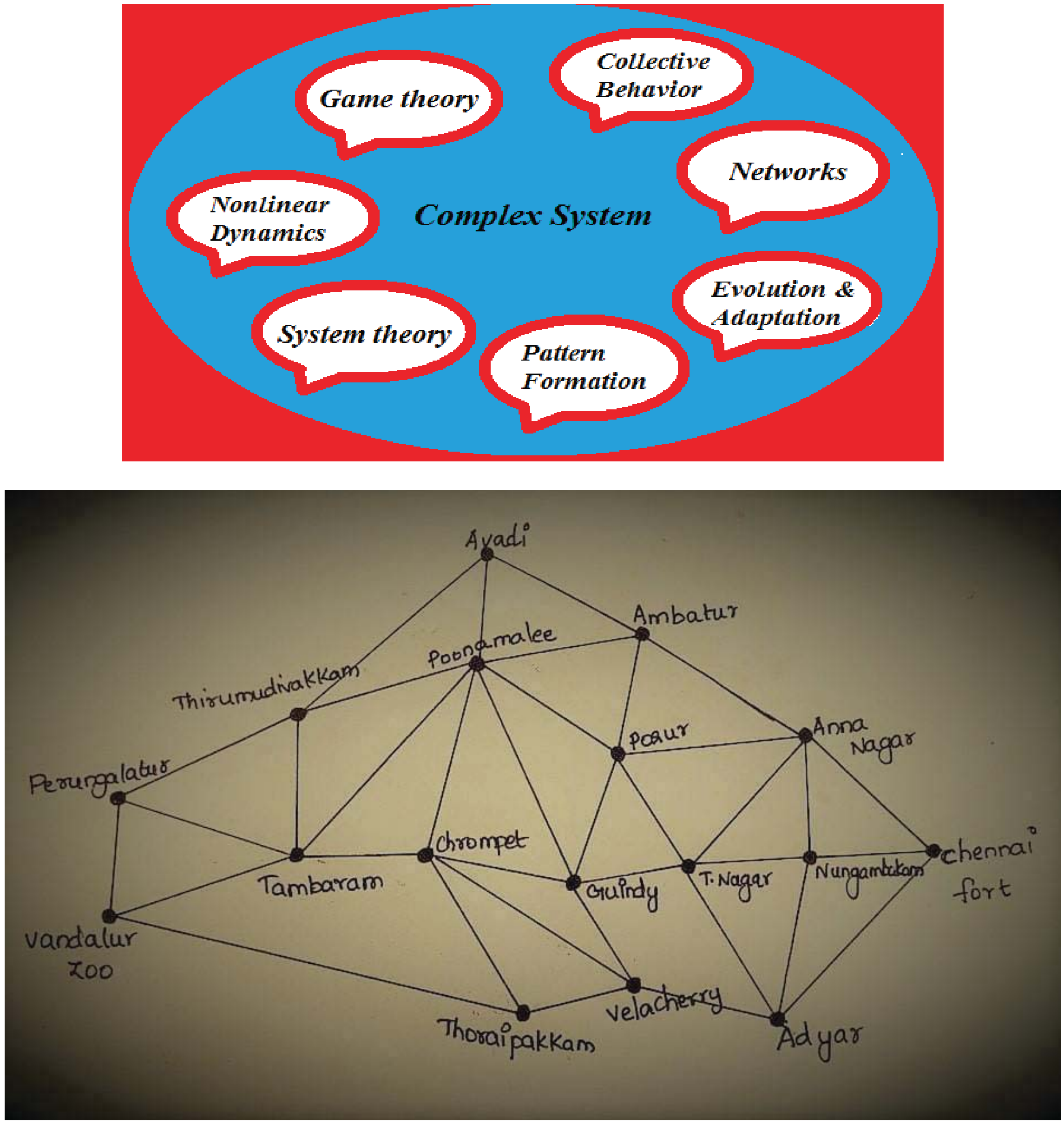

FIGURE 2. The Complexity of network

Random Walk

A graph's vertex and edge sequence is called a walk. A random move in G is a transfer from some node u to a chosen at random neighbour v, given an undirected, connected graph G (V,E) with |V| = n, |E| = m. A random walk is a series of these random steps that starts at one node and ends at another [1]. To get to the destination, users can choose a route at random. A random walk of a network has been represented in Fig. 3 [6].

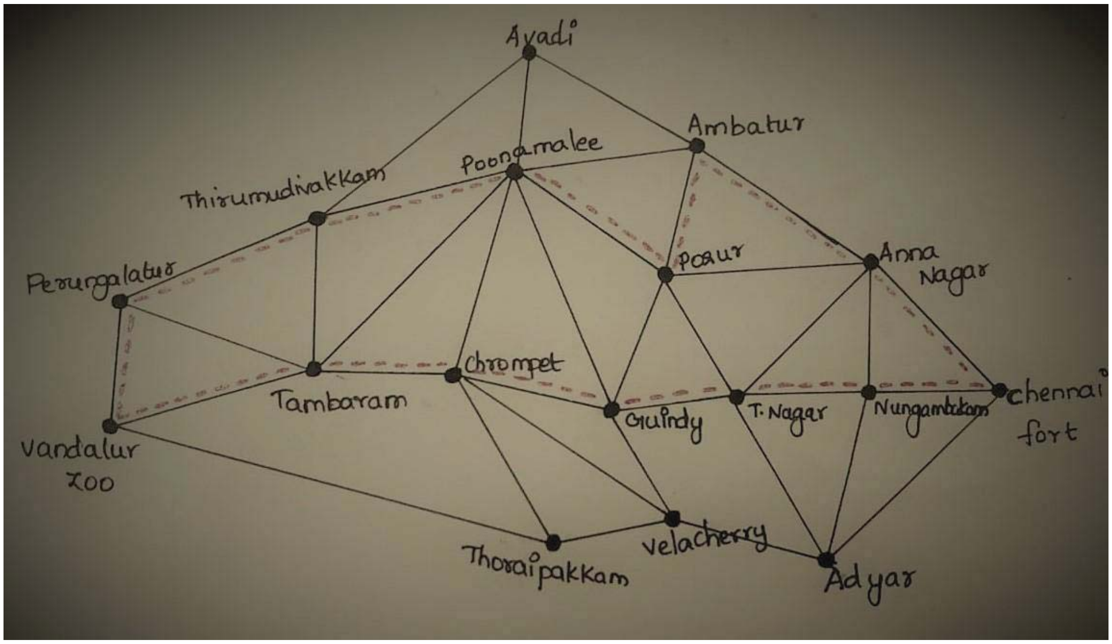

FIGURE 3. The random walk of the road map

A random walk on a graph is begins at one node and travels to the next node at each step. When the graph is unweighted, the node that the current vertex moves to is chosen randomly at random from its neighbours.

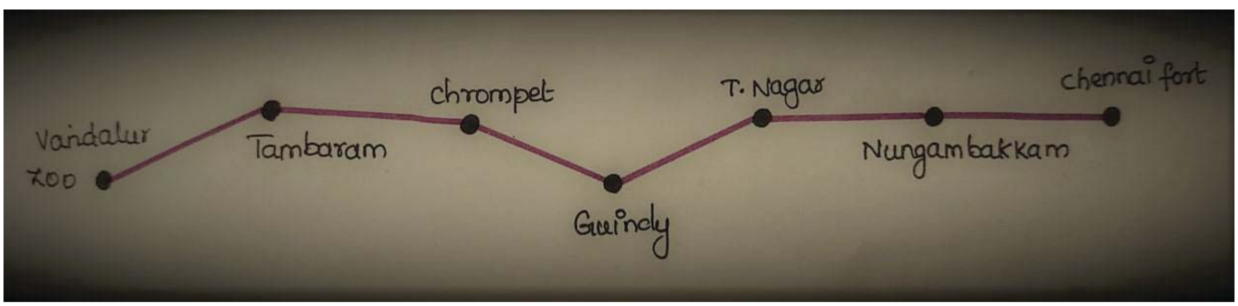

FIGURE 3(A). The voyage of geodesic path-shortest path.

The path can be selected at random, and it can be either the shortest or the longest path. The shortest path has seven vertices, while the longest path has eight vertices. It is obvious that path 3(a) takes less time to get to the destination.

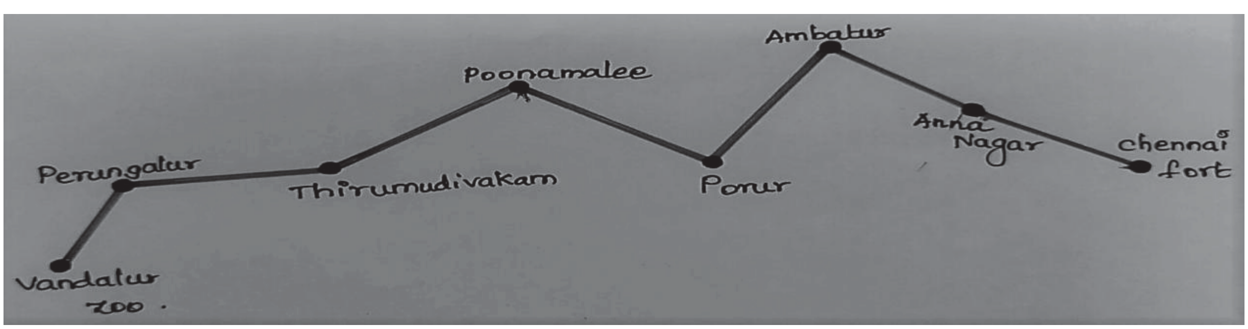

FIGURE 3(B). The voyage of diameter path-longest path.

Consider a object passing to one of its neighbouring points in each phase of a random walk on the integer levels of the real finite interval. It's used to figure out how to get a random walk. We can prove that the path from source x to sink y has unit capacity by applying Menger's theorem, the route from Chennai f ort to Vandalur Zoo is defined by the (x, y)-path. *Let N be a network with source x and sink y in which each arc has unit capacity. Then*

- *the value of a maximum flow in N is equal to the maximum number m of arc-disjoint directed (x, y)-paths in ·N;* [1].

We determine that the network with source x and sink y has system efficiency by estimating that the network's maximum flow (longest path) is equal to the maximum number m of arcs-disjoint guided (x,y)-paths in N, and that the path can be chosen at random. According to the concept of fractal, *A fractal is a non-regular geometric form that is non-regular on all scales to the same degree. Fractals are patterns that never seem to stop*. Each route is fractal and irregular.

Percolation Theory

The behaviour of a network when nodes or links are removed is defined by percolation theory. Since the network splits into substantially smaller linked clusters at a critical fraction of removal, this is a geometric form of phase transition. Percolation theory is the most basic, but abstractly solved, model of a phase transition. The understanding of many other physical systems is assisted by the percolation theory problem. Furthermore, percolation theory explains how networks behave when nodes or links are removed, since at a critical fraction of nodes or links removed, the network breaks down into a substantially smaller linked cluster [5].

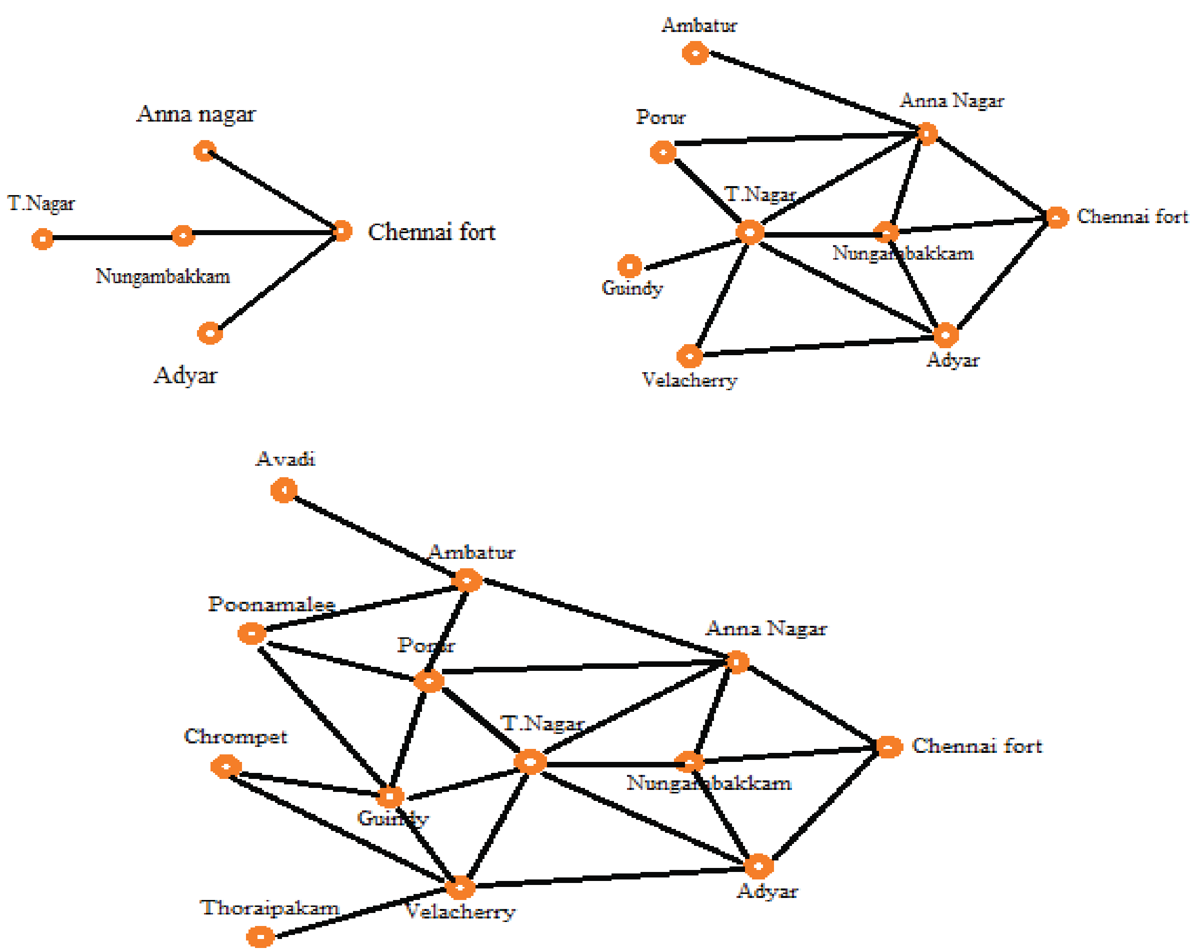

FIGURE 4. Behaviour of routes in rod map

We can find the probability generating functions in two separate paths G_0 (t) and G_1 (t) given a random graph with a degree distribution P(r). Assume that a fraction of the connections (1-p) is omitted at random.
The probability for a randomly chosen node to have k remaining links in the diluted graph after removing a fraction 1-p of the links (bond percolation) is given by,

$$P(r) = \sum_{r_0}^{\infty} p(r) \binom{r_0}{r} p^r (1-p)^{r0-r} \tag{1}$$

Assume that G_0 (t) be a geodesic path and G_1 (t) be a diameter path. The probability generating functions for the related probabilities, $G_0(t) = \sum_{r=0}^{\infty} P(r).t^4$ and $G_1(t) = \sum_{r=1}^{\infty} P_1(r).t^{r-1}$ in the diluted graph is:

$$G_0(t) = \sum_{r=0}^{\infty} P(r).t^4 = \sum_{r=0}^{\infty}\left[\sum_{r_0=r}^{\infty} p(r) \binom{r_0}{r} P^r (1-p)^{r0-r}\right].t^r$$

$$= \sum_{r_0=0}^{\infty} P(r_0) \sum_{r=0}^{r_0} \left(\binom{r_0}{r} (tp)^r\right)(1-p)^{r0-r}$$

$$\widetilde{G_0}(t) = \sum_{r_0=0}^{\infty} P(r_0)(1-p+pt)^{r_o} = G_0(1-p+pt) \tag{2}$$

And, $$G_1(t) = \frac{G_0'(t)}{G_1'(1)} = \frac{pG_0(1-p+pt)}{pG_0'(1)} = \frac{G_0'(1-p+pt)}{G_0'(1)} = G_1(1-p+pt)$$

It can be shown that: $$\widetilde{G_1}(t) = G_1(1-p+pt) \tag{3}$$

$$= \frac{1}{(r)} \sum_{r=1}^{\infty} rp(t).(1-p+pt)$$

Is equal to, $$\widetilde{G_1}(t) = \frac{1}{r_{diluted}} \sum_{r=1}^{\infty} r\,\tilde{P}(r).t^{r-1} \tag{4}$$

Where (r) and P(r) are the original graph's average and distribution of degrees, and (r diluted) = p(r) and P(r) are the diluted graph's average and distribution of degrees. Unless it stated, the generating functions will be referred to. p(k), G_0(t), and G_1(t) of the diluted graph in the following sections when explaining percolation on networks.

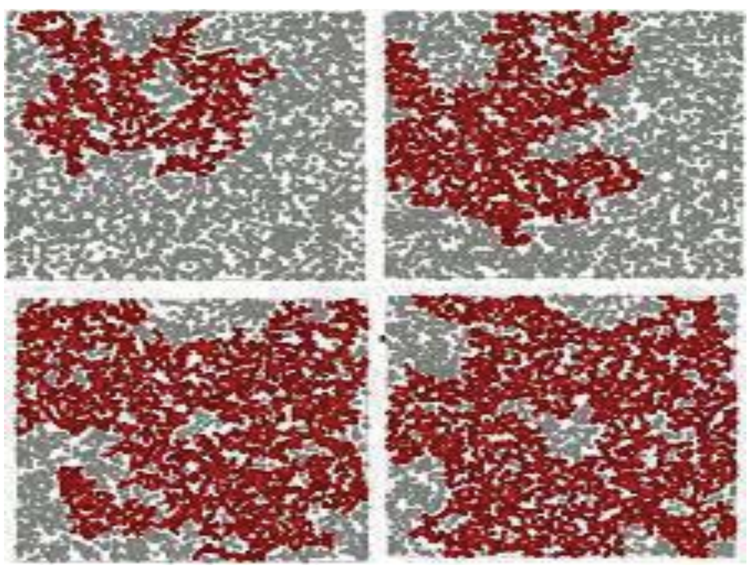

FIGURE 5. The percolation theory in the Road map

The upper left picture depicts the India map, with percolation theory (red dotted path), and the upper right picture depicts the Tamil Nadu map, India, also with lower right picture depicting the Chennai fort to the Vandalur Zoo map. Percolation is clearly present here. This percolation theory can be applied to a variety of streets and states.

Topological Features

By quantifying the spatial distribution of its paths, the topological function provides details on the road-side map. This method accomplishes this by encoding the routes' spatial interdependency prior to feature extraction [7].

The Delaunay triangulation of Voronoi diagrams can be used to estimate route independence. The voronoi diagram on the routes image creates convex polygons for each path. Any point of polygon in the road-side map is closer to itself than to another path for the specific routes. The Delaunay triangulation is the Voronoi diagram's dual graph. Fig. 1 shows a Voronoi diagram of a sample city routes picture and its Delaunay triangulation (a). The area and shape of a Voronoi diagram's polygon are two of the properties specified for them. The area and shape of a Voronoi diagram's polygon are two of the properties specified for them. The number of route connections and the average length of these connections are two properties described for Delaunay triangulation. It's also possible to find a Delaunay triangulation spanning tree and use properties like total tree length to classify topological features [12].

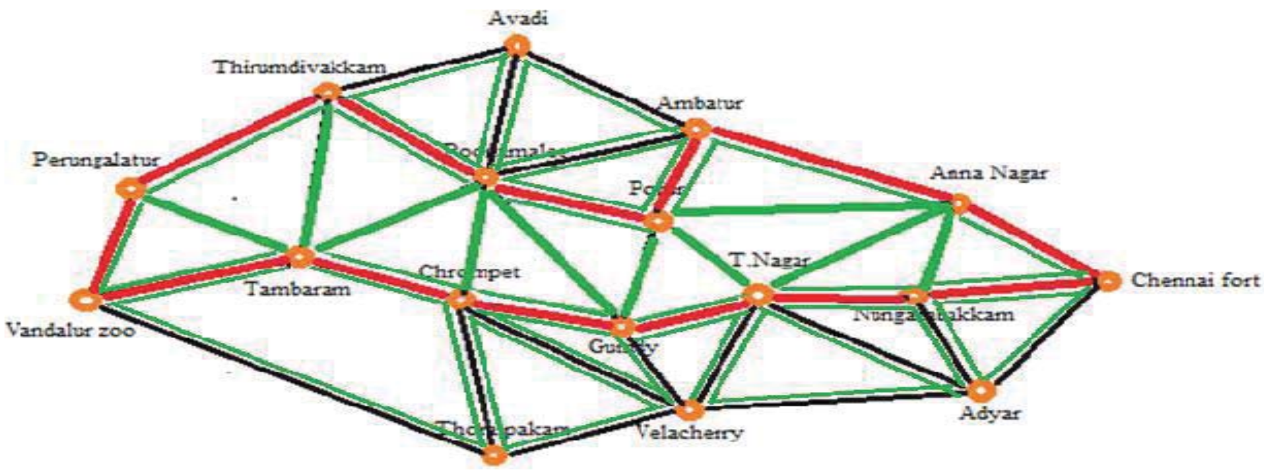

FIGURE 6(A). Topological Features

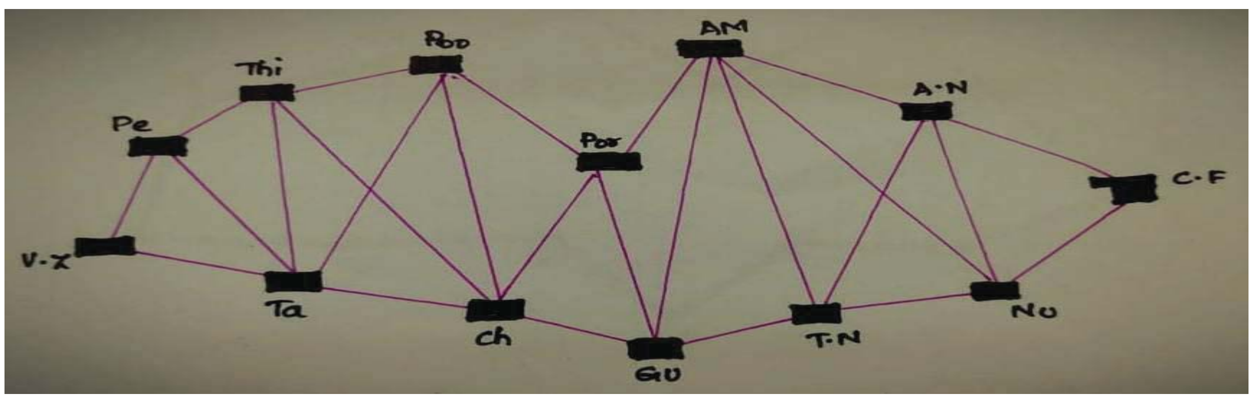

FIGURE 6(B). Topological Features

fig.(a) the voronoi diagram of the image (red lines) and its Delaunay triangulation (green solid lines), Fig.(6b) the route-graph of the image.

To create a graph for an image, encode the dependence between any pair of routes. To increasing Distance function between the nodes u and v in this graph, the probability of being an edge between them decays; the probability of being an edge between them is defined as follows:

$$P(u,v) = \alpha \cdot \exp(-d(u,v)/ \beta L)$$

Where d(u,v) denotes the Distance function between vertices u and v, L denotes the largest possible Distance function between any two vertices, and that govern the number of connections and connectivity of these routes-graph. Figure (b) depicts a route-graph of the sample image in figure 1. In this case, the route cluster can be thought of as sub-images made up of a set of routes. As a result, the local metrics correspond to the sub-properties. Furthermore, the local feature distributions are used to describe global features like the average degree [6]. Figure 7 depicts Voronoi diagrams and Delaunay triangulation.

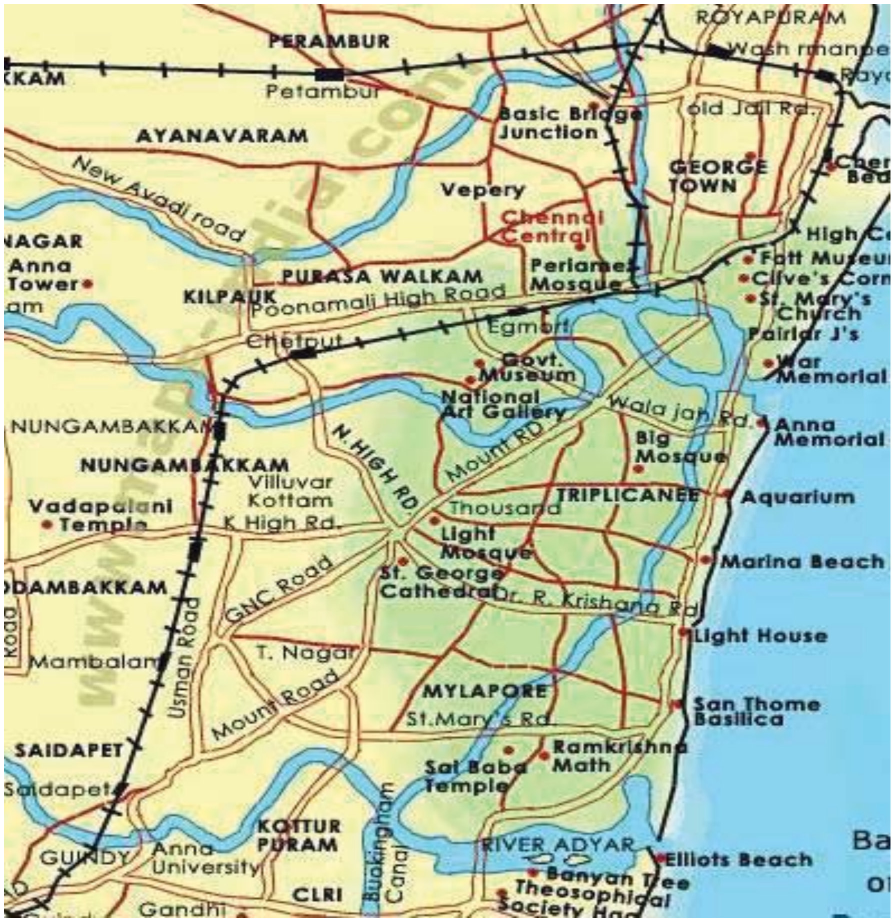

FIGURE 7. The Chennai City

RESULT

The Delaunay triangulation is shown in red lines in Fig.6. The geodesic route is analyzed using the random walk and complexity methods. Each path forms a triangulation, and the random walk's shape is irregular, illustrating that it is fractal. Delaunay triangulation is used to display the number of connections of the routes and the average length of the connection. Voronoi diagrams are used to measure the area and shapes of their polygons.

CONCLUSION

The routes of the road map are percolation theory in network analysis, and the Delaunay triangulation is formed by the diagrams in the chosen road map, and each route is irregular in shape, resulting in fractals. It is used to quantify the maximum and minimum flow rates. In comparison to other roads, the road Chennai fort – Nungambakkam – T.Nagar – Guindy – Chrompet – Tambaram – Vandalur zoo is the shortest. The road map's defect can be observed in the study Poisson distribution. This is also true for Google Maps. All rotes in Tamil Nadu, India, need it.

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