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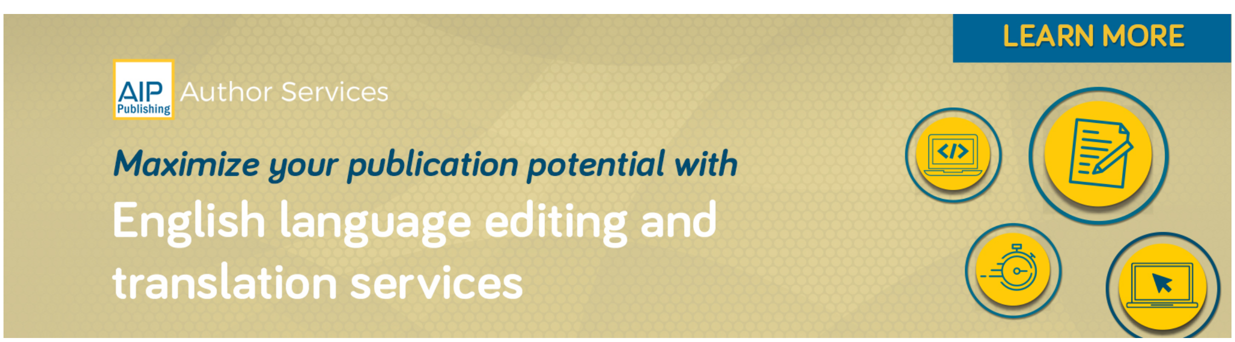

Interval Valued Picture Fuzzy Graphs

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Abstract. In this research paper, we propose a new speculationof fuzzy graph called interval valued picture fuzzy graph to models.describe uncertain information in the data analysis. We presented new operations like Cartesian product and Composition on two interval valued picture fuzzy graphs. Likewise, we demonstrated their properties. Further, we examined these operations with appropriate mathematical

Keywords. Picture fuzzy graph, interval valued picture fuzzy graph, product interval valued picture fuzzy graph.

INTRODUCTION

In 1975, Zadeh presented the thought of interval valued fuzzy sets as speculation of fuzzy sets. A fuzzy set can be defined mathematically assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represent by the fuzzy set. A fuzzy set is a crisp set, where components of the set have levels of membership. The single worth of the membership degree can't deal with the vulnerabilities because of the absence of information on the issue. In 1986, Atanassov proposed intuitionistic fuzzy sets to deal with this sort of questionable circumstance utilizing a non-membership degree. Recently, Ahmed [1] proposed the operation on interval valued picture fuzzy set and fuzzy soft set and their applications.Also, Krassimir [4] has presented the different types of intervals valued intuitionistic fuzzy graphs and interval valued Pythagorean fuzzy graphs concentrated by Yahya Mohamed [9]. Cen and Pal [2] introduced few meaningsof picture fuzzy graphs with their application in the social networks. Akram [5] discussed the interval valued Pythagorean fuzzy graphs and analysis to group decision approach. Ismayil [7] discussed the concept of stronginterval valued intuitionistic fuzzy graphs and also studied by definition of the interval valued fuzzy graph by Akram and Dudek [6]. In this paper, we define the interval valued picture fuzzy graph. We present the operations like cartesian product and composition on two interval valued picture fuzzy graphs. Additionally,their properties are demonstrated and it is explained with suitable examples.

PRELIMINARIES

Definition 1:[2]Let A be a picture fuzzy set. A in X is describedby $A = \{x, \mu_A(x), \eta_A(x), \gamma_A(x)/x \in X\}$ Where $\mu_A(x), \eta_A(x) and \gamma_A(x)$ follow the condition $0 \leq \mu_A(x) + \eta_A(x) + \gamma_A(x) \leq 1$. The $\mu_A(x), \eta_A(x), \gamma_A(x) \in [0,1]$, shows separatelythe positive membership degree, neutral membership degree and negative membership degree of the segment in the set A.

Definition 2:[2]Let G = (V, E) be a graph. A couple G = (C, D) is called a picture fuzzy graph on G*=(V, E), where C = $(\mu_C, \eta_C, \gamma_C)$ is a picture fuzzy set on V and D = $(\mu_D, \eta_D, \gamma_D)$ is a picture fuzzy set on $E \subseteq VXV$with that for each is$uv \in E$, $\mu_D(u,v) \leq \mu_C(u) \wedge \mu_C(v)$;$\eta_D(uv) \leq \eta_C(u) \wedge \eta_C(v)$and
$\gamma_D(uv) \geq \gamma_C(u) \vee \gamma_C(v)$.

Definition 3:[7] A fuzzy set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a couple of functions $G = (\theta, \delta)$ where σ is a fuzzy subset of a non-void set V and δ is symmetric fuzzy relation on θ, (i.e) $\delta(u,v) = \min(\theta(u), \theta(v))$. The underlying crisp graph $G = (\theta, \delta)$ is indicated by $G^* = (V, E)$ where $E \subseteq V \times V$.

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Let D [0,1] be the set of all closed subintervals of the interval [0, 1] and component of this set are signified by capitalized letters. Assuming $S \in D$ [0,1], it tends to be addressed as $S = [S_L, S_U]$ S_U *and* S_L are the upper and lower limit of S.

Definition 4:[7]An intuitionistic fuzzy graph with underlying set V is characterized to be a couple $G = (A, B)$ were

- the function $\tau_A: V \to D\,[0,1]$ and $\rho_A: V \to D\,[0,1]$indicate the membership degree and non-membership degree of the component $x \in V$separately, with that the condition $0 \leq \tau_A(x) + \rho_A(x) \leq 1$ for all $x \in V$.
- The function $\tau_A: V \times \mathrm{V} \to D\,[0,1]$ AND$\rho_A: V \times \mathrm{V} \to D\,[0,1]$ are characterized by $\tau_B(x, \mathrm{y}) \leq \tau_A(x) \wedge \tau_{\mathrm{A}}(y)$ and $\rho_B(x, \mathrm{y}) \leq \rho_A(x) \vee \rho_{\mathrm{A}}(y)$ such that $0 \leq \tau_B(x, y) + \rho_B(x, \mathrm{y}) \leq 1$ for all $(x, y) \in E$.

Definition5: [1]Let $I^{\Delta} = \{([w_1, x_1], [w_2, x_2], [w_3, x_3]) /\ w = (w_1, w_2, w_3) \in I, x = (x_1, x_2, x_3) \in I^2\}$ with an order $\leq$ defined by (w, x)$\leq$ (y, z)$\Leftrightarrow$ $[w_1, x_1] \supseteq [y_1, z_1]$, $[w_2, x_2] \supseteq [y_{21}, z_2], and\ [w_3, x_3] \supseteq [y_3, z_3](y = (y_1, y_2, y_3),\ z = (z_1, z_2, z_3))$. At that point a mapping $\Phi: X \to I^{\Delta}$ is called an interval valued picture fuzzy set (shorten form of IVPFS) on X. The arrangement of all IVPFSs on X is indicated by $I^{\Delta x}$.

TYPES OF INTERVALS VALUED PICTURE FUZZY GRAPHS

Definition 6: An interval valued picture fuzzy graph(IVPFG) with underlying set V is defined to be a pair$\mathcal{G} = (\mathcal{P}, \mathcal{Q})$where $\mathcal{P} =< (\mu_{\mathcal{P}L}, \eta_{\mathcal{P}L}, \gamma_{\mathcal{P}L};\ \mu_{\mathcal{P}U}, \eta_{\mathcal{P}U}, \gamma_{\mathcal{P}U}) >$ is an interval valued picture fuzzy set on V and $\mathcal{Q} =< (\mu_{\mathcal{Q}L}, \eta_{\mathcal{Q}L}, \gamma_{\mathcal{Q}L};\ \mu_{\mathcal{Q}U}, \eta_{\mathcal{Q}U}, \gamma_{\mathcal{Q}U}) >$ is an interval valued picture fuzzy set on E satisfies the following conditions:

(i) the function ,$\mu_{\mathcal{P}}: V \to K[0,1]$; $\eta_{\mathcal{P}}: V \to K[0,1]$ and $\gamma_{\mathcal{P}}: V \to K[0,1]$indicates positive membership degree, neutral membership degree and negative membership degree of the component$x \in V$, respectively, such that $0 \leq \mu_{\mathcal{P}}(x) + \eta_{\mathcal{P}}(x) + \gamma_{\mathcal{P}}(x) \leq 1$ for all $x \in V$ and

(ii) the function $\mu_{\mathcal{Q}}: V \times \mathrm{V} \to K[0,1]$; $\eta_{\mathcal{Q}}: V \times \mathrm{V} \to K[0,1]$ and $\gamma_{\mathcal{Q}}: V \times \mathrm{V} \to K[0,1]$are defined by $\mu_{\mathcal{Q}L}(uv) \leq \min(\mu_{\mathcal{P}L}(u), \mu_{\mathcal{P}L}(v))$; $\eta_{\mathcal{Q}L}(uv) \leq \min(\eta_{\mathcal{P}L}(u), \eta_{\mathcal{P}L}(v))$ and $\gamma_{\mathcal{Q}L}(uv) \geq \max\,(\gamma_{\mathcal{P}L}(u), \gamma_{\mathcal{P}L}(v))$ and $\mu_{\mathcal{Q}U}(uv) \leq \min(\mu_{\mathcal{P}U}(u), \mu_{\mathcal{P}U}(v))$; $\eta_{\mathcal{Q}U}(uv) \leq \min(\eta_{\mathcal{P}U}(u), \eta_{\mathcal{P}U}(v))$ and $\gamma_{\mathcal{Q}U}(uv) \geq \max\,(\gamma_{\mathcal{P}U}(u), \gamma_{\mathcal{P}U}(v))$ such that $0 \leq \mu_{\mathcal{Q}U}(uv) + \eta_{\mathcal{Q}U}(uv) + \gamma_{\mathcal{Q}U}(uv) \leq 1$ for all $uv \in E$, where K[0, 1] be the set of closed subintervals of [0, 1].

Example 1: Consider a graph$\mathcal{G}^* = (V, E)$ where the vertex set {a, b, c} and the edge set {ab, bc, ca}. Let $\mathcal{P}$ be an interval valued picture fuzzy set of V and let $\mathcal{Q}$ be interval valued picture fuzzy set of $E \subseteq V \times V$ defined by

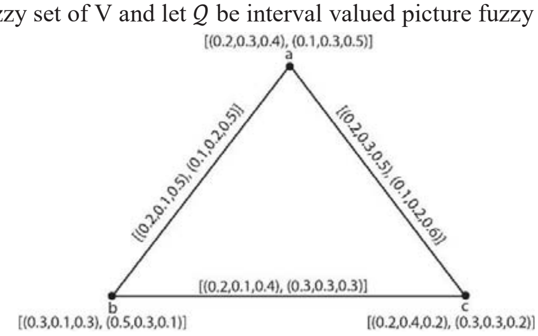

FIGURE 1. Interval valued picture fuzzy graph.

[ab, (0.2, 0.1, 0.5), (0.1, 0.2, 0.5)]; [bc, (0.2,0.1, 0.4), 0.3, 0.3, 0.3)]; [ac, (0.2, 0.3, 0.5), (0.1, 0.2, 0.6)]

Definition7: The cartesian product of two interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{P}_1, \mathcal{Q}_1)$ and$\mathcal{G}_2 = (\mathcal{P}_2, \mathcal{Q}_2)$ with underlying crisp graphs $\mathcal{G}_1{}^* = (V, E)$ and $\mathcal{G}_2{}^* = (V, E)$ is signified by$\mathcal{G}_1 \times \mathcal{G}_2 = (\mathcal{P}_1 \times \mathcal{P}_2, \mathcal{Q}_1 \times \mathcal{Q}_2)$ and is defined as follows

(i) $(\mu_{\mathcal{P}_1L} \times \mu_{\mathcal{P}_2L})(x_1, x_2) = \min\{\mu_{\mathcal{P}_1L}(x_1), \mu_{\mathcal{P}_2L}(x_2)\}$

$$(\mu_{\mathcal{P}_1U} \times \mu_{\mathcal{P}_2U})(x_1, \mathrm{x}_2) = \min\{\mu_{\mathcal{P}_1U}(x_1), \mu_{\mathcal{P}_2U}(x_2)\}$$

$$(\eta_{\mathcal{P}_1L} \times \eta_{\mathcal{P}_2L})(\mathrm{x}_1, x_2) = \min\{\eta_{\mathcal{P}_1L}(x_1), \eta_{\mathcal{P}_2L}(x_2)\}$$
$$(\eta_{\mathcal{P}_1U} \times \eta_{\mathcal{P}_2U})(x_1, \mathrm{x}_2) = \min\{\eta_{\mathcal{P}_1U}(x_1), \eta_{\mathcal{P}_2U}(x_2)\}$$

$$(\gamma_{\mathcal{P}_1L} \times \gamma_{\mathcal{P}_2L})(x_1, x_2) = \max\{\gamma_{\mathcal{P}_1L}(x_1), \gamma_{\mathcal{P}_2L}(x_2)\}$$
$(\gamma_{\mathcal{P}_1U} \times \gamma_{\mathcal{P}_2U})(x_1, x_2) = \max\{\gamma_{\mathcal{P}_1U}(x_1), \gamma_{\mathcal{P}_2U}(x_2)\}$, for every $x_1 \in V_1 \& x_2 \in \mathrm{V}_2$

(ii) $(\mu_{\mathcal{Q}_1L} \times \mu_{\mathcal{Q}_2L})(x, \mathrm{x}_2)(x, y_2) = \min\{\mu_{\mathcal{P}_1L}(\mathrm{x}), \mu_{\mathcal{Q}_2L}(\mathrm{x}_2, y_2)\}$
$(\mu_{\mathcal{Q}_1U} \times \mu_{\mathcal{Q}_2U})(x, x_2)(x, y_2) = \min\{\mu_{\mathcal{P}_1U}(x), \mu_{\mathcal{Q}_2U}(x_2, y_2)\}$

$(\eta_{\mathcal{Q}_1L} \times \eta_{\mathcal{Q}_2L})(x, x_2)(x, y_2) = \min\{\eta_{\mathcal{P}_1L}(x), \eta_{\mathcal{Q}_2L}(x_2, y_2)\}$
$(\eta_{\mathcal{Q}_1U} \times \eta_{\mathcal{Q}_2U})(x, x_2)(x, y_2) = \min\{\eta_{\mathcal{P}_1U}(x), \eta_{\mathcal{Q}_2U}(x_2, y_2)\}$

$(\gamma_{\mathcal{Q}_1L} \times \gamma_{\mathcal{Q}_2L})(x, x_2)(x, y_2) = \max\{\gamma_{\mathcal{P}_1L}(\mathrm{x}), \gamma_{\mathcal{Q}_2L}(x_2, y_2)\}$
$(\gamma_{\mathcal{Q}_1U} \times \gamma_{\mathcal{Q}_2U})(x, x_2)(x, y_2) = \max\{\gamma_{\mathcal{P}_1U}(x), \gamma_{\mathcal{Q}_2U}(x_2, y_2)\}$, for every x$\in V_1 \& (\mathrm{x}_2, y_2) \in E_2$

(iii) $(\mu_{\mathcal{Q}_1L} \times \mu_{\mathcal{Q}_2L})(\mathrm{x}_{1,}z)(y_1, z) = \min\{\mu_{\mathcal{Q}_1L}(x_1, y_1), \mu_{\mathcal{P}_2L}(z)\}$
$(\mu_{\mathcal{Q}_1U} \times \mu_{\mathcal{Q}_2U})(x_{1,}z)(y_1, z) = \min\{\mu_{\mathcal{Q}_1U}(x_1, y_1), \mu_{\mathcal{P}_2U}(z)\}$

$(\eta_{\mathcal{Q}_1L} \times \eta_{\mathcal{Q}_2L})(x_{1,}z)(y_1, z) = \min\{\eta_{\mathcal{Q}_1L}(x_1, y_1), \eta_{\mathcal{P}_2L}(z)\}$
$(\eta_{\mathcal{Q}_1U} \times \eta_{\mathcal{Q}_2U})(\mathrm{u}_{1,}w)(v_1, w) = \min\{\eta_{\mathcal{Q}_1U}(\mathrm{u}_1, v_1), \eta_{\mathcal{P}_2U}(z)\}$

$(\gamma_{\mathcal{Q}_1L} \times \gamma_{\mathcal{Q}_2L})(x_{1,}z)(y_1, z) = \max\{\gamma_{\mathcal{Q}_1L}(x_1, y_1), \gamma_{\mathcal{P}_2L}(z)\}$
$(\gamma_{\mathcal{Q}_1U} \times \gamma_{\mathcal{Q}_2U})(x_{1,}z)(y_1, z) = \max\{\gamma_{\mathcal{Q}_1U}(x_1, y_1), \gamma_{\mathcal{P}_2U}(z)\}$, for every $(\mathrm{x}_1, y_1) \in E_1 \& z \in V_2$

Theorem 1: The cartesian product $\mathcal{G}_1 \times \mathcal{G}_2 = (\mathcal{P}_1 \times \mathcal{P}_2, \mathcal{Q}_1 \times \mathcal{Q}_2)$ of two interval valued picture fuzzy graphs is an interval valued picture fuzzy graph.

Proof: Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two interval valued picture fuzzy graphs.
Let $E = \{(x, x_2)(x, y_2)/x \in V_1 \& (\mathrm{x}_2, y_2) \in E_2\} \cup \{(\mathrm{x}_1, z)(y_1, z)/u \in V_2 \& (\mathrm{u}_1, v_1) \in E_1\}$
Consider$(x, x_2)(x, y_2) \in E$, by Definition 6 and7 we have

$$\begin{aligned}(\mu_{\mathcal{Q}_1L} \times \mu_{\mathcal{Q}_2L})(x, x_2)(x, y_2) &= \min\{\mu_{\mathcal{P}_1L}(x), \mu_{\mathcal{Q}_2L}(\mathrm{x}_2, y_2)\} \\ &\leq \min\{\mu_{\mathcal{P}_1L}(x), [\mu_{\mathcal{P}_2L}(\mathrm{x}_2), \mu_{\mathcal{P}_2L}(y_2 \\ &= \min(\min(\mu_{\mathcal{P}_1L}(x), \mu_{\mathcal{P}_2L}(\mathrm{x}_2)), \min((\mu_{\mathcal{P}_1L}(x), \mu_{\mathcal{P}_2L}(y_2))) \\ &= \min\{(\mu_{\mathcal{P}_1L} \times \mu_{\mathcal{P}_2L})(x, x_2), (\mu_{\mathcal{P}_1L} \times \mu_{\mathcal{P}_2L})(x, y_2)\}\end{aligned}$$

$$\begin{aligned}(\mu_{\mathcal{Q}_1U} \times \mu_{\mathcal{Q}_2U})(x, x_2)(x, y_2) &= \min\{\mu_{\mathcal{P}_1U}(x), \mu_{\mathcal{Q}_2U}(\mathrm{x}_2, y_2)\} \\ &\leq \min\{\mu_{\mathcal{P}_1U}(x), [\mu_{\mathcal{P}_2U}(\mathrm{x}_2), \mu_{\mathcal{P}_2U}(y_2)]\} \\ &= \min(\min(\mu_{\mathcal{P}_1U}(x), \mu_{\mathcal{P}_2U}(\mathrm{x}_2)), \min((\mu_{\mathcal{P}_1U}(x), \mu_{\mathcal{P}_2U}(y_2))) \\ &= \min\{(\mu_{\mathcal{P}_1U} \times \mu_{\mathcal{P}_2U})(x, x_2), (\mu_{\mathcal{P}_1U} \times \mu_{\mathcal{P}_2U})(x, y_2)\}\end{aligned}$$

$$\begin{aligned}(\eta_{\mathcal{Q}_1L} \times \eta_{\mathcal{Q}_2L})(x, x_2)(x, y_2) &= \min\{\eta_{\mathcal{P}_1L}(x), \eta_{\mathcal{Q}_2L}(\mathrm{x}_2, y_2)\} \\ &\leq \min\{\eta_{\mathcal{P}_1L}(x), [\eta_{\mathcal{P}_2L}(\mathrm{x}_2), \eta_{\mathcal{P}_2L}(y_2)]\} \\ &= \min(\min(\eta_{\mathcal{P}_1L}(x), \eta_{\mathcal{P}_2L}(\mathrm{x}_2)), \min((\eta_{\mathcal{P}_1L}(x), \eta_{\mathcal{P}_2L}(y_2))) \\ &= \min\{(\eta_{\mathcal{P}_1L} \times \eta_{\mathcal{P}_2L})(x, x_2), (\eta_{\mathcal{P}_1L} \times \eta_{\mathcal{P}_2L})(x, y_2)\}\end{aligned}$$

$$(\eta_{Q_1U} \times \eta_{Q_2U})(x,x_2)(x,y_2) = \min\{\eta_{\mathcal{P}_1U}(x), \eta_{Q_2U}(\mathrm{x}_2, y_2)\}$$
$$\leq \min\{\eta_{\mathcal{P}_1U}(x), [\eta_{\mathcal{P}_2U}(\mathrm{x}_2), \eta_{\mathcal{P}_2U}(y_2)]\}$$
$$= \min(\min(\eta_{\mathcal{P}_1U}(x), \eta_{\mathcal{P}_2U}(\mathrm{x}_2)), \min((\eta_{\mathcal{P}_1U}(x), \eta_{\mathcal{P}_2U}(y_2)))$$
$$= \min\{(\eta_{\mathcal{P}_1U} \times \eta_{\mathcal{P}_2U})(x,x_2), (\eta_{\mathcal{P}_1U} \times \eta_{\mathcal{P}_2U})(x,y_2)\}$$

$$(\gamma_{Q_1L} \times \gamma_{Q_2L})(x,x_2)(x,y_2) = \max\{\gamma_{\mathcal{P}_1L}(x), \gamma_{Q_2L}(\mathrm{x}_2, y_2)\}$$
$$\geq \max\{\gamma_{\mathcal{P}_1L}(x), [\gamma_{\mathcal{P}_2L}(\mathrm{x}_2), \gamma_{\mathcal{P}_2L}(y_2)]\}$$
$$= \max(\max(\gamma_{\mathcal{P}_1L}(x), \gamma_{\mathcal{P}_2L}(\mathrm{x}_2)), \max((\gamma_{\mathcal{P}_1L}(x), \gamma_{\mathcal{P}_2L}(y_2)))$$
$$= \max\{(\gamma_{\mathcal{P}_1L} \times \gamma_{\mathcal{P}_2L})(x,x_2), (\gamma_{\mathcal{P}_1L} \times \gamma_{\mathcal{P}_2L})(x,y_2)\}$$

$$(\gamma_{Q_1U} \times \gamma_{Q_2U})(x,x_2)(x,y_2) = \max\{\gamma_{\mathcal{P}_1U}(x), \gamma_{Q_2U}(\mathrm{x}_2, y_2)\}$$
$$\geq \max\{\gamma_{\mathcal{P}_1U}(x), [\gamma_{\mathcal{P}_2U}(\mathrm{x}_2), \gamma_{\mathcal{P}_2U}(y_2)]\}$$
$$= \max(\max(\gamma_{\mathcal{P}_1U}(x), \gamma_{\mathcal{P}_2U}(\mathrm{x}_2)), \max((\gamma_{\mathcal{P}_1U}(x), \gamma_{\mathcal{P}_2U}(y_2)))$$
$$= \max\{(\gamma_{\mathcal{P}_1U} \times \gamma_{\mathcal{P}_2U})(x,x_2), (\gamma_{\mathcal{P}_1U} \times \gamma_{\mathcal{P}_2U})(x,y_2)\}$$

Consider $(\mathrm{x}_1, z)(y_1, z) \in E$, we have

$$(\mu_{Q_1L} \times \mu_{Q_2L})(x_1,z)(y_1,z) = \min\{\mu_{Q_1L}(\mathrm{x}_1, \mathrm{y}_1), \mu_{\mathcal{P}_2L}(z)\}$$
$$\leq \min\{[\mu_{\mathcal{P}_1L}(\mathrm{x}_1), \mu_{\mathcal{P}_1L}(y_1)], \mu_{\mathcal{P}_2L}(z)\}$$
$$= \min(\min\left(\mu_{\mathcal{P}_1L}(\mathrm{x}_1), \mu_{\mathcal{P}_2L}(z)\right), \min\left(\mu_{\mathcal{P}_1L}(y_1), \mu_{\mathcal{P}_2L}(z)\right))$$
$$= \min\{(\mu_{\mathcal{P}_1L} \times \mu_{\mathcal{P}_2L})(x_1,z), (\mu_{\mathcal{P}_1L} \times \mu_{\mathcal{P}_2L})(y_1,z)\}$$

$$(\mu_{Q_1U} \times \mu_{Q_2U})(x_1,z)(y_1,z) = \min\{\mu_{Q_1U}(\mathrm{x}_1, \mathrm{y}_1), \mu_{\mathcal{P}_2U}(z)\}$$
$$\leq \min\{[\mu_{\mathcal{P}_1U}(\mathrm{x}_1), \mu_{\mathcal{P}_1U}(y_1)], \mu_{\mathcal{P}_2U}(z)\}$$
$$= \min(\min\left(\mu_{\mathcal{P}_1U}(\mathrm{x}_1), \mu_{\mathcal{P}_2U}(z)\right), \min\left(\mu_{\mathcal{P}_1U}(y_1), \mu_{\mathcal{P}_2U}(z)\right))$$
$$= \min\{(\mu_{\mathcal{P}_1U} \times \mu_{\mathcal{P}_2U})(x_1,z), (\mu_{\mathcal{P}_1U} \times \mu_{\mathcal{P}_2U})(y_1,z)\}$$

$$(\eta_{Q_1L} \times \eta_{Q_2L})(x_1,z)(y_1,z) = \min\{\eta_{Q_1L}(\mathrm{x}_1, \mathrm{y}_1), \eta_{\mathcal{P}_2L}(z)\}$$
$$\leq \min\{[\eta_{\mathcal{P}_1L}(\mathrm{x}_1), \eta_{\mathcal{P}_1L}(y_1)], \eta_{\mathcal{P}_2L}(z)\}$$
$$= \min(\min\left(\eta_{\mathcal{P}_1L}(\mathrm{x}_1), \eta_{\mathcal{P}_2L}(z)\right), \min\left(\eta_{\mathcal{P}_1L}(y_1), \eta_{\mathcal{P}_2L}(z)\right))$$
$$= \min\{(\eta_{\mathcal{P}_1L} \times \eta_{\mathcal{P}_2L})(x_1,z), (\eta_{\mathcal{P}_1L} \times \eta_{\mathcal{P}_2L})(y_1,z)\}$$

$$(\eta_{Q_1U} \times \eta_{Q_2U})(x_1,z)(y_1,z) = \min\{\eta_{Q_1U}(\mathrm{x}_1, \mathrm{y}_1), \eta_{\mathcal{P}_2U}(z)\}$$
$$\leq \min\{[\eta_{\mathcal{P}_1U}(\mathrm{x}_1), \eta_{\mathcal{P}_1U}(y_1)], \eta_{\mathcal{P}_2U}(z)\}$$
$$= \min(\min\left(\eta_{\mathcal{P}_1U}(\mathrm{x}_1), \eta_{\mathcal{P}_2U}(z)\right), \min\left(\eta_{\mathcal{P}_1U}(y_1), \eta_{\mathcal{P}_2U}(z)\right))$$
$$= \min\{(\eta_{\mathcal{P}_1U} \times \eta_{\mathcal{P}_2U})(x_1,z), (\eta_{\mathcal{P}_1U} \times \eta_{\mathcal{P}_2U})(y_1,z)\}$$

$$(\gamma_{Q_1L} \times \gamma_{Q_2L})(x_1,z)(y_1,z) = \max\{\gamma_{Q_1L}(\mathrm{x}_1, \mathrm{y}_1), \gamma_{\mathcal{P}_2L}(z)\}$$
$$\geq \max\{[\gamma_{\mathcal{P}_1L}(\mathrm{x}_1), \gamma_{\mathcal{P}_1L}(y_1)], \gamma_{\mathcal{P}_2L}(z)\}$$
$$= \max(\max\left(\gamma_{\mathcal{P}_1L}(\mathrm{x}_1), \gamma_{\mathcal{P}_2L}(z)\right), \max\left(\gamma_{\mathcal{P}_1L}(y_1), \gamma_{\mathcal{P}_2L}(z)\right))$$
$$= \max\{(\gamma_{\mathcal{P}_1L} \times \gamma_{\mathcal{P}_2L})(x_1,z), (\gamma_{\mathcal{P}_1L} \times \gamma_{\mathcal{P}_2L})(y_1,z)\}$$

$$(\gamma_{Q_1U} \times \gamma_{Q_2U})(x_1,z)(y_1,z) = \max\{\gamma_{Q_1U}(\mathrm{x}_1, \mathrm{y}_1), \gamma_{\mathcal{P}_2U}(z)\}$$
$$\geq \max\{[\gamma_{\mathcal{P}_1U}(\mathrm{x}_1), \gamma_{\mathcal{P}_1U}(y_1)], \gamma_{\mathcal{P}_2U}(z)\}$$
$$= \max(\max\left(\gamma_{\mathcal{P}_1U}(\mathrm{x}_1), \gamma_{\mathcal{P}_2U}(z)\right), \max\left(\gamma_{\mathcal{P}_1U}(y_1), \gamma_{\mathcal{P}_2U}(z)\right))$$
$$= \max\{(\gamma_{\mathcal{P}_1U} \times \gamma_{\mathcal{P}_2U})(x_1,z), (\gamma_{\mathcal{P}_1U} \times \gamma_{\mathcal{P}_2U})(y_1,z)\}$$

Hence $\mathcal{G}_1 \times \mathcal{G}_2$ is an interval valued picture fuzzy graph.

Example 2: Consider two interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{P}_1, \mathcal{Q}_1)$ and $\mathcal{G}_2 = (\mathcal{P}_2, \mathcal{Q}_2)$ with underlying crisp graphs$\mathcal{G}_1{}^* = (V_1, E_1)$ and $\mathcal{G}_2{}^* = (V_2, E_2)$ such that the vertices $V_1 = \{x_1, y_1, z_1\}$ and$V_2 = \{x_2, y_2\}$ and the edge sets $E_1 = \{x_1y_1, y_1z_1\}$ and $E_2 = \{x_2y_2\}$ respectively, where

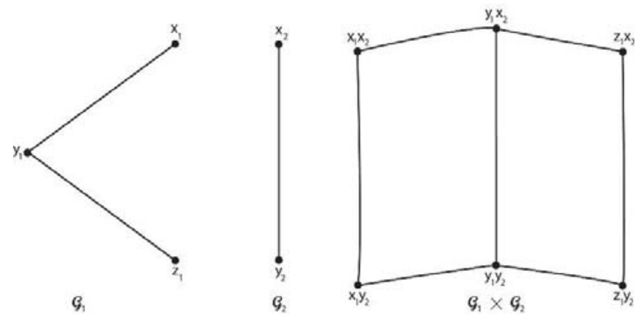

FIGURE 2. Cartesian product of twointerval valued picture fuzzy graphs.

The vertex sets of $\mathcal{G}_1$ *and* $\mathcal{G}_2$

$$\mathcal{P}_1 = \begin{matrix} \langle x_1; (0.2, 0.3, 0.4), (0.4, 0.1, 0.4) \\ \langle y_1; (0.3, 0.3, 0.3), (0.3, 0.4, 0.1) \\ \langle z_1; (0.4, 0.3, 0.2), (0.3, 0.1, 0.4) \end{matrix}$$

$$\mathcal{P}_2 = \begin{matrix} \langle x_2; (0.1, 0.2, 0.5), (0.2, 0.4, 0.3) \\ \langle y_2; (0.3, 0.6, 0.1), (0.4, 0.3, 0.2) \end{matrix}$$

The edge sets of $\mathcal{G}_1$ *and* $\mathcal{G}_2$

$$\mathcal{Q}_1 = \begin{matrix} \langle x_1y_1; (0.2, 0.2, 0.5), (0.2, 0.1, 0.6) \\ \langle y_1z_1; (0.1, 0.2, 0.4), (0.3, 0.1, 0.5) \end{matrix}$$

$$\mathcal{Q}_2 = \{\langle x_2y_2; (0.1, 0.1, 0.6), (0.2, 0.2, 0.4)$$

The Cartesian Product of$\mathcal{G}_1 \times \mathcal{G}_2$

$$\mathcal{P}_1 \times \mathcal{P}_2 = \begin{matrix} \langle x_1x_2; (0.1, 0.2, 0.5), (0.2, 0.1, 0.4) \\ \langle x_1y_2; (0.2, 0.3, 0.4), (0.4, 0.2, 0.4) \\ \langle y_1x_2; (0.1, 0.2, 0.5), (0.2, 0.4, 0.3) \\ \langle y_1y_2; (0.3, 0.3, 0.2), (0.3, 0.3, 0.2) \\ \langle z_1x_2; (0.1, 0.2, 0.5), (0.2, 0.1, 0.4) \\ \langle z_1y_2; (0.3, 0.3, 0.2), (0.3, 0.1, 0.4) \end{matrix}$$

$$\mathcal{Q}_1 \times \mathcal{Q}_2 = \begin{matrix} \langle (x_1x_2, x_1y_2); (0.1, 0.1, 0.6), (0.2, 0.1, 0.4) \\ \langle (y_1x_2, y_1y_2); (0.1, 0.1, 0.6), (0.2, 0.2, 0.4) \\ \langle (z_1x_2, z_1y_2); (0.1, 0.1, 0.6), (0.2, 0.1, 0.4) \\ \langle (x_1x_2, y_1x_2); (0.2, 0.1, 0.5), (0.2, 0.1, 0.6) \\ \langle (y_1x_2, z_1x_2); (0.1, 0.2, 0.5), (0.2, 0.1, 0.5) \\ \langle (x_1y_2, y_1y_2); (0.2, 0.2, 0.5), (0.2, 0.1, 0.6) \\ \langle (y_1y_2, z_1y_2); (0.1, 0.2, 0.4), (0.3, 0.1, 0.5) \end{matrix}$$

Definition 8: The composition of interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{P}_1, \mathcal{Q}_1)$ and $\mathcal{G}_2 = (\mathcal{P}_2, \mathcal{Q}_2)$ with underlying crisp graphs $\mathcal{G}_1{}^* = (V, E)$ and $\mathcal{G}_2{}^* = (V, E)$ is denoted by$\mathcal{G}_1 \circ \mathcal{G}_2 = (\mathcal{P}_1 \circ \mathcal{P}_2, \mathcal{Q}_1 \circ \mathcal{Q}_2)$
and is defined as follows

(i) $(\mu_{\mathcal{P}_1L} \circ \mu_{\mathcal{P}_2L})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\mu_{\mathcal{P}_1L}(\mathrm{u}_1), \mu_{\mathcal{P}_2L}(\mathrm{u}_2)\}$
$(\mu_{\mathcal{P}_1U} \circ \mu_{\mathcal{P}_2U})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\mu_{\mathcal{P}_1U}(\mathrm{u}_1), \mu_{\mathcal{P}_2U}(\mathrm{u}_2)\}$

$(\eta_{\mathcal{P}_1L} \circ \eta_{\mathcal{P}_2L})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\eta_{\mathcal{P}_1L}(\mathrm{u}_1), \eta_{\mathcal{P}_2L}(\mathrm{u}_2)\}$
$(\eta_{\mathcal{P}_1U} \circ \eta_{\mathcal{P}_2U})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\eta_{\mathcal{P}_1U}(\mathrm{u}_1), \eta_{\mathcal{P}_2U}(\mathrm{u}_2)\}$

$(\gamma_{\mathcal{P}_1L} \circ \gamma_{\mathcal{P}_2L})(\mathrm{u}_1, \mathrm{u}_2) = \max\{\gamma_{\mathcal{P}_1L}(\mathrm{u}_1), \gamma_{\mathcal{P}_2L}(\mathrm{u}_2)\}$
$(\gamma_{\mathcal{P}_1U} \circ \gamma_{\mathcal{P}_2U})(\mathrm{u}_1, \mathrm{u}_2) = \max\{\gamma_{\mathcal{P}_1U}(\mathrm{u}_1), \gamma_{\mathcal{P}_2U}(\mathrm{u}_2)\}$, for every $\mathrm{u}_1 \in V_1$&$\mathrm{u}_2 \in \mathrm{V}_2$

(ii) $(\mu_{\mathcal{Q}_1L} \circ \mu_{\mathcal{Q}_2L})(u, \mathrm{u}_2)(u, v_2) = \min\{\mu_{\mathcal{P}_1L}(\mathrm{u}), \mu_{\mathcal{Q}_2L}(\mathrm{u}_2, v_2)\}$
$(\mu_{\mathcal{Q}_1U} \circ \mu_{\mathcal{Q}_2U})(u, \mathrm{u}_2)(u, v_2) = \min\{\mu_{\mathcal{P}_1U}(\mathrm{u}), \mu_{\mathcal{Q}_2U}(\mathrm{u}_2, v_2)\}$

$(\eta_{\mathcal{Q}_1L} \circ \eta_{\mathcal{Q}_2L})(u, \mathrm{u}_2)(u, v_2) = \min\{\eta_{\mathcal{P}_1L}(\mathrm{u}), \eta_{\mathcal{Q}_2L}(\mathrm{u}_2, v_2)\}$
$(\eta_{\mathcal{Q}_1U} \circ \eta_{\mathcal{Q}_2U})(u, \mathrm{u}_2)(u, v_2) = \min\{\eta_{\mathcal{P}_1U}(\mathrm{u}), \eta_{\mathcal{Q}_2U}(\mathrm{u}_2, v_2)\}$

$(\gamma_{\mathcal{Q}_1L} \circ \gamma_{\mathcal{Q}_2L})(u, \mathrm{u}_2)(u, v_2) = \max\{\gamma_{\mathcal{P}_1L}(\mathrm{u}), \gamma_{\mathcal{Q}_2L}(\mathrm{u}_2, v_2)\}$
$(\gamma_{\mathcal{Q}_1U} \circ \gamma_{\mathcal{Q}_2U})(u, \mathrm{u}_2)(u, v_2) = \max\{\gamma_{\mathcal{P}_1U}(\mathrm{u}), \gamma_{\mathcal{Q}_2U}(\mathrm{u}_2, v_2)\}$,for every $u \in V_1$&$(\mathrm{u}_2, v_2) \in E_2$

(iii) $(\mu_{\mathcal{Q}_1L} \circ \mu_{\mathcal{Q}_2L})(\mathrm{u}_1, w)(v_1, w) = \min\{\mu_{\mathcal{Q}_1L}(\mathrm{u}_1, v_1), \mu_{\mathcal{P}_2L}(w)\}$
$(\mu_{\mathcal{Q}_1U} \circ \mu_{\mathcal{Q}_2U})(\mathrm{u}_1, w)(v_1, w) = \min\{\mu_{\mathcal{Q}_1U}(\mathrm{u}_1, v_1), \mu_{\mathcal{P}_2U}(w)\}$

$(\eta_{\mathcal{Q}_1L} \circ \eta_{\mathcal{Q}_2L})(\mathrm{u}_1, w)(v_1, w) = \min\{\eta_{\mathcal{Q}_1L}(\mathrm{u}_1, v_1), \eta_{\mathcal{P}_2L}(w)\}$
$(\eta_{\mathcal{Q}_1U} \circ \eta_{\mathcal{Q}_2U})(\mathrm{u}_1, w)(v_1, w) = \min\{\eta_{\mathcal{Q}_1U}(\mathrm{u}_1, v_1), \eta_{\mathcal{P}_2U}(w)\}$

$(\gamma_{\mathcal{Q}_1L} \circ \gamma_{\mathcal{Q}_2L})(\mathrm{u}_1, w)(v_1, w) = \max\{\gamma_{\mathcal{Q}_1L}(\mathrm{u}_1, v_1), \gamma_{\mathcal{P}_2L}(\mathrm{w})\}$
$(\gamma_{\mathcal{Q}_1U} \circ \gamma_{\mathcal{Q}_2U})(\mathrm{u}_1, w)(v_1, w) = \max\{\gamma_{\mathcal{Q}_1U}(\mathrm{u}_1, v_1), \gamma_{\mathcal{P}_2U}(w)\}$, for every $w \in V_2$& $(\mathrm{u}_1, v_1) \in E_1$

(iv) $(\mu_{\mathcal{Q}_1L} \circ \mu_{\mathcal{Q}_2L})(\mathrm{u}_1, \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\mu_{\mathcal{P}_2L}(\mathrm{u}_2), \mu_{\mathcal{P}_2L}(v_2), \mu_{\mathcal{Q}_1L}(\mathrm{u}_1, v_1)\}$
$(\mu_{\mathcal{Q}_1U} \circ \mu_{\mathcal{Q}_2U})(\mathrm{u}_1, \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\mu_{\mathcal{P}_2U}(\mathrm{u}_2), \mu_{\mathcal{P}_2U}(v_2), \mu_{\mathcal{Q}_1U}(\mathrm{u}_1, v_1)\}$

$(\eta_{\mathcal{Q}_1L} \circ \eta_{\mathcal{Q}_2L})(\mathrm{u}_1, \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\eta_{\mathcal{P}_2L}(\mathrm{u}_2), \eta_{\mathcal{P}_2L}(v_2), \eta_{\mathcal{Q}_1L}(\mathrm{u}_1, v_1)\}$
$(\eta_{\mathcal{Q}_1U} \circ \eta_{\mathcal{Q}_2U})(\mathrm{u}_1, \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\eta_{\mathcal{P}_2U}(\mathrm{u}_2), \eta_{\mathcal{P}_2U}(v_2), \eta_{\mathcal{Q}_1U}(\mathrm{u}_1, v_1)\}$

$(\gamma_{\mathcal{Q}_1L} \circ \gamma_{\mathcal{Q}_2L})(\mathrm{u}_1, \mathrm{u}_2)(v_1, \mathrm{v}_2) = \max\{\gamma_{\mathcal{P}_2L}(\mathrm{u}_2), \gamma_{\mathcal{P}_2L}(v_2), \gamma_{\mathcal{Q}_1L}(\mathrm{u}_1, v_1)\}$
$(\gamma_{\mathcal{Q}_1U} \circ \gamma_{\mathcal{Q}_2U})(\mathrm{u}_1, \mathrm{u}_2)(v_1, \mathrm{v}_2) = \max\{\gamma_{\mathcal{P}_2U}(\mathrm{u}_2), \gamma_{\mathcal{P}_2U}(v_2), \gamma_{\mathcal{Q}_1U}(\mathrm{u}_1, v_1)\}$
For all $E^0 = E \cup \{(\mathrm{u}_1, \mathrm{u}_2)(v_1, \mathrm{v}_2) / (\mathrm{u}_1, v_1) \in E_1\ \mathrm{u}_2 \neq v_2\}$

Theorem 2: The composition$\mathcal{G}_1 \circ \mathcal{G}_2$of twointerval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{P}_1, \mathcal{Q}_1)$ and $\mathcal{G}_2 = (\mathcal{P}_2, \mathcal{Q}_2)of\mathcal{G}_1{}^* = (V_1, E_1)$ and $\mathcal{G}_2{}^* = (V_2, E_2)$ is an interval valued picture fuzzy graph.

Proof: Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two interval valued picture fuzzy graphs.
Let $E = \{(x, x_2)(x, y_2) / x \in V_1 \& (\mathrm{x}_2, y_2) \in E_2\} \cup \{(\mathrm{x}_1, z)(y_1, z) / u \in V_2 \& (\mathrm{u}_1, v_1) \in E_1\}$
Consider $(x, x_2)(x, y_2) \in E$, by Definition6 and 8 we have
$(\mu_{\mathcal{Q}_1L} \circ \mu_{\mathcal{Q}_2L})(x, x_2)(x, y_2) = \min\{\mu_{\mathcal{P}_1L}(x), \mu_{\mathcal{Q}_2L}(\mathrm{x}_2, y_2)\}$
$\leq \min\{\mu_{\mathcal{P}_1L}(x), [\mu_{\mathcal{P}_2L}(\mathrm{x}_2), \mu_{\mathcal{P}_2L}(y_2$
$= \min(\min(\mu_{\mathcal{P}_1L}(x), \mu_{\mathcal{P}_2L}(\mathrm{x}_2)), \min((\mu_{\mathcal{P}_1L}(x), \mu_{\mathcal{P}_2L}(y_2)))$
$= \min\{(\mu_{\mathcal{P}_1L} \circ \mu_{\mathcal{P}_2L})(x, x_2), (\mu_{\mathcal{P}_1L} \circ \mu_{\mathcal{P}_2L})(x, y_2)\}$

$$(\mu_{Q_1U}\circ\mu_{QU})(x,x_2)(x,y_2)=\min\{\mu_{\mathcal{P}_1U}(x),\mu_{Q_2U}(\mathrm{x}_2,y_2)\}$$
$$\leq \min\{\mu_{\mathcal{P}_1U}(x),[\mu_{\mathcal{P}_2U}(\mathrm{x}_2),\mu_{\mathcal{P}_2U}(y_2)]\}$$
$$=\min(\min(\mu_{\mathcal{P}_1U}(x),\mu_{\mathcal{P}_2U}(\mathrm{x}_2)),\min((\mu_{\mathcal{P}_1U}(x),\mu_{\mathcal{P}_2U}(y_2)))$$
$$=\min\{(\mu_{\mathcal{P}_1U}\circ\mu_{\mathcal{P}_2U})(x,x_2),\left(\mu_{\mathcal{P}_1U}\circ\mu_{\mathcal{P}_2U}\right)(x,y_2)\}$$

$$(\eta_{Q_1L}\circ\eta_{Q_2L})(x,x_2)(x,y_2)=\min\{\eta_{\mathcal{P}_1L}(x),\eta_{Q_2L}(\mathrm{x}_2,y_2)\}$$
$$\leq \min\{\eta_{\mathcal{P}_1L}(x),[\eta_{\mathcal{P}_2L}(\mathrm{x}_2),\eta_{\mathcal{P}_2L}(y_2)]\}$$
$$=\min(\min(\eta_{\mathcal{P}_1L}(x),\eta_{\mathcal{P}_2L}(\mathrm{x}_2)),\min((\eta_{\mathcal{P}_1L}(x),\eta_{\mathcal{P}_2L}(y_2)))$$
$$=\min\{(\eta_{\mathcal{P}_1L}\circ\eta_{\mathcal{P}_2L})(x,x_2),\left(\eta_{\mathcal{P}_1L}\circ\eta_{\mathcal{P}_2L}\right)(x,y_2)\}$$

$$(\eta_{Q_1U}\circ\eta_{Q_2U})(x,x_2)(x,y_2)=\min\{\eta_{\mathcal{P}_1U}(x),\eta_{Q_2U}(\mathrm{x}_2,y_2)\}$$
$$\leq \min\{\eta_{\mathcal{P}_1U}(x),[\eta_{\mathcal{P}_2U}(\mathrm{x}_2),\eta_{\mathcal{P}_2U}(y_2)]\}$$
$$=\min(\min(\eta_{\mathcal{P}_1U}(x),\eta_{\mathcal{P}_2U}(\mathrm{x}_2)),\min((\eta_{\mathcal{P}_1U}(x),\eta_{\mathcal{P}_2U}(y_2)))$$
$$=\min\{(\eta_{\mathcal{P}_1U}\circ\eta_{\mathcal{P}_2U})(x,x_2),\left(\eta_{\mathcal{P}_1U}\circ\eta_{\mathcal{P}_2U}\right)(x,y_2)\}$$

$$(\gamma_{Q_1L}\circ\gamma_{Q_2L})(x,x_2)(x,y_2)=\max\{\gamma_{\mathcal{P}_1L}(x),\gamma_{Q_2L}(\mathrm{x}_2,y_2)\}$$
$$\geq \max\{\gamma_{\mathcal{P}_1L}(x),[\gamma_{\mathcal{P}_2L}(\mathrm{x}_2),\gamma_{\mathcal{P}_2L}(y_2)]\}$$
$$=\max(\max(\gamma_{\mathcal{P}_1L}(x),\gamma_{\mathcal{P}_2L}(\mathrm{x}_2)),\max((\gamma_{\mathcal{P}_1L}(x),\gamma_{\mathcal{P}_2L}(y_2)))$$
$$=\max\{(\gamma_{\mathcal{P}_1L}\circ\gamma_{\mathcal{P}_2L})(x,x_2),\left(\gamma_{\mathcal{P}_1L}\circ\gamma_{\mathcal{P}_2L}\right)(x,y_2)\}$$

$$(\gamma_{Q_1U}\circ\gamma_{Q_2U})(x,x_2)(x,y_2)=\max\{\gamma_{\mathcal{P}_1U}(x),\gamma_{Q_2U}(\mathrm{x}_2,y_2)\}$$
$$\geq \max\{\gamma_{\mathcal{P}_1U}(x),[\gamma_{\mathcal{P}_2U}(\mathrm{x}_2),\gamma_{\mathcal{P}_2U}(y_2)]\}$$
$$=\max(\max(\gamma_{\mathcal{P}_1U}(x),\gamma_{\mathcal{P}_2U}(\mathrm{x}_2)),\max((\gamma_{\mathcal{P}_1U}(x),\gamma_{\mathcal{P}_2U}(y_2)))$$
$$=\max\{(\gamma_{\mathcal{P}_1U}\circ\gamma_{\mathcal{P}_2U})(x,x_2),\left(\gamma_{\mathcal{P}_1U}\circ\gamma_{\mathcal{P}_2U}\right)(x,y_2)\}$$

Consider $(\mathrm{x}_1,z)(y_1,z)\in E$, we have

$$(\mu_{Q_1L}\circ\mu_{Q_2L})(x_1,z)(y_1,z)=\min\{\mu_{Q_1L}(\mathrm{x}_1,\mathrm{y}_1),\mu_{\mathcal{P}_2L}(z)\}$$
$$\leq \min\{[\mu_{\mathcal{P}_1L}(\mathrm{x}_1),\mu_{\mathcal{P}_1L}(y_1)],\mu_{\mathcal{P}_2L}(z)\}$$
$$=\min(\min\left(\mu_{\mathcal{P}_1L}(\mathrm{x}_1),\mu_{\mathcal{P}_2L}(z)\right),\min\left(\mu_{\mathcal{P}_1L}(y_1),\mu_{\mathcal{P}_2L}(z)\right))$$
$$=\min\{(\mu_{\mathcal{P}_1L}\circ\mu_{\mathcal{P}_2L})(x_1,z),\left(\mu_{\mathcal{P}_1L}\circ\mu_{\mathcal{P}_2L}\right)(y_1,z)\}$$

$$(\mu_{Q_1U}\circ\mu_{Q_2U})(x_1,z)(y_1,z)=\min\{\mu_{Q_1U}(\mathrm{x}_1,\mathrm{y}_1),\mu_{\mathcal{P}_2U}(z)\}$$
$$\leq \min\{[\mu_{\mathcal{P}_1U}(\mathrm{x}_1),\mu_{\mathcal{P}_1U}(y_1)],\mu_{\mathcal{P}_2U}(z)\}$$
$$=\min(\min\left(\mu_{\mathcal{P}_1U}(\mathrm{x}_1),\mu_{\mathcal{P}_2U}(z)\right),\min\left(\mu_{\mathcal{P}_1U}(y_1),\mu_{\mathcal{P}_2U}(z)\right))$$
$$=\min\{(\mu_{\mathcal{P}_1U}\circ\mu_{\mathcal{P}_2U})(x_1,z),\left(\mu_{\mathcal{P}_1U}\circ\mu_{\mathcal{P}_2U}\right)(y_1,z)\}$$

$$(\eta_{Q_1L}\circ\eta_{Q_2L})(x_1,z)(y_1,z)=\min\{\eta_{Q_1L}(\mathrm{x}_1,\mathrm{y}_1),\eta_{\mathcal{P}_2L}(z)\}$$
$$\leq \min\{[\eta_{\mathcal{P}_1L}(\mathrm{x}_1),\eta_{\mathcal{P}_1L}(y_1)],\eta_{\mathcal{P}_2L}(z)\}$$
$$=\min(\min\left(\eta_{\mathcal{P}_1L}(\mathrm{x}_1),\eta_{\mathcal{P}_2L}(z)\right),\min\left(\eta_{\mathcal{P}_1L}(y_1),\eta_{\mathcal{P}_2L}(z)\right))$$
$$=\min\{(\eta_{\mathcal{P}_1L}\circ\eta_{\mathcal{P}_2L})(x_1,z),\left(\eta_{\mathcal{P}_1L}\circ\eta_{\mathcal{P}_2L}\right)(y_1,z)\}$$

$$(\eta_{Q_1U}\circ\eta_{Q_2U})(x_1,z)(y_1,z)=\min\{\eta_{Q_1U}(\mathrm{x}_1,\mathrm{y}_1),\eta_{\mathcal{P}_2U}(z)\}$$
$$\leq \min\{[\eta_{\mathcal{P}_1U}(\mathrm{x}_1),\eta_{\mathcal{P}_1U}(y_1)],\eta_{\mathcal{P}_2U}(z)\}$$
$$=\min(\min\left(\eta_{\mathcal{P}_1U}(\mathrm{x}_1),\eta_{\mathcal{P}_2U}(z)\right),\min\left(\eta_{\mathcal{P}_1U}(y_1),\eta_{\mathcal{P}_2U}(z)\right))$$

$$= \min\{(\eta_{\mathcal{P}_1U} \circ \eta_{\mathcal{P}_2U})(x_1, z), (\eta_{\mathcal{P}_1U} \circ \eta_{\mathcal{P}_2U})(y_1, z)\}$$

$$(\gamma_{Q_1L} \circ \gamma_{Q_2L})(x_1, z)(y_1, z) = \max\{\gamma_{QL}(x_1, y_1), \gamma_{\mathcal{P}_2L}(z)\}$$
$$\geq \max\{[\gamma_{\mathcal{P}_1L}(x_1), \gamma_{\mathcal{P}_1L}(y_1)], \gamma_{\mathcal{P}_2L}(z)\}$$
$$= \max(\max\left(\gamma_{\mathcal{P}_1L}(x_1), \gamma_{\mathcal{P}_2L}(z)\right), \max\left(\gamma_{\mathcal{P}_1L}(y_1), \gamma_{\mathcal{P}_2L}(z)\right))$$
$$= \max\{(\gamma_{\mathcal{P}_1L} \times \gamma_{\mathcal{P}_2L})(x_1, z), (\gamma_{\mathcal{P}_1L} \times \gamma_{\mathcal{P}_2L})(y_1, z)\}$$

$$(\gamma_{Q_1U} \circ \gamma_{Q_2U})(x_1, z)(y_1, z) = \max\{\gamma_{Q_1U}(x_1, y_1), \gamma_{\mathcal{P}_2U}(z)\}$$
$$\geq \max\{[\gamma_{\mathcal{P}_1U}(x_1), \gamma_{\mathcal{P}_1U}(y_1)], \gamma_{\mathcal{P}_2U}(z)\}$$
$$= \max(\max\left(\gamma_{\mathcal{P}_1U}(x_1), \gamma_{\mathcal{P}_2U}(z)\right), \max\left(\gamma_{\mathcal{P}_1U}(y_1), \gamma_{\mathcal{P}_2U}(z)\right))$$
$$= \max\{(\gamma_{\mathcal{P}_1U} \circ \gamma_{\mathcal{P}_2U})(x_1, z), (\gamma_{\mathcal{P}_1U} \circ \gamma_{\mathcal{P}_2U})(y_1, z)\}$$

Consider $(x_1, x_2)(y_1, y_2) \in E$, we have

$$(\mu_{Q_1L} \circ \mu_{Q_2L})(x_1, x_2)(y_1, y_2) = \min\{\mu_{Q_1L}(x_1, y_1), \mu_{\mathcal{P}_2L}(x_2), \mu_{\mathcal{P}_2L}(y_2)\}$$
$$\leq \min\{[\mu_{\mathcal{P}_1L}(x_1), \mu_{\mathcal{P}_1L}(y_1)], \mu_{\mathcal{P}_2L}(x_2), \mu_{\mathcal{P}_2L}(y_2)\}$$
$$= \min(\min\left(\mu_{\mathcal{P}_1L}(x_1), \mu_{\mathcal{P}_2L}(z)\right), \min\left(\mu_{\mathcal{P}_1L}(y_1), \mu_{\mathcal{P}_2L}(z)\right))$$
$$= \min\{(\mu_{\mathcal{P}_1L} \circ \mu_{\mathcal{P}_2L})(x_1, x_2), (\mu_{\mathcal{P}_1L} \circ \mu_{\mathcal{P}_2L})(y_1, y_2)\}$$

$$(\mu_{Q_1U} \circ \mu_{Q_2U})(x_1, x_2)(y_1, y_2) = \min\{\mu_{Q_1U}(x_1, y_1), \mu_{\mathcal{P}_2U}(x_2), \mu_{\mathcal{P}_2U}(y_2)\}$$
$$\leq \min\{[\mu_{\mathcal{P}_1U}(x_1), \mu_{\mathcal{P}_1U}(y_1)], \mu_{\mathcal{P}_2U}(x_2), \mu_{\mathcal{P}_2U}(y_2)\}$$
$$= \min(\min\left(\mu_{\mathcal{P}_1U}(x_1), \mu_{\mathcal{P}_2U}(z)\right), \min\left(\mu_{\mathcal{P}_1U}(y_1), \mu_{\mathcal{P}_2U}(z)\right))$$
$$= \min\{(\mu_{\mathcal{P}_1U} \circ \mu_{\mathcal{P}_2U})(x_1, x_2), (\mu_{\mathcal{P}_1U} \circ \mu_{\mathcal{P}_2U})(y_1, y_2)\}$$

$$(\eta_{Q_1L} \circ \eta_{Q_2L})(x_1, x_2)(y_1, y_2) = \min\{\eta_{Q_1L}(x_1, y_1), \eta_{\mathcal{P}_2L}(x_2), \eta_{\mathcal{P}_2L}(y_2)\}$$
$$\leq \min\{[\eta_{\mathcal{P}_1L}(x_1), \eta_{\mathcal{P}_1L}(y_1)], \eta_{\mathcal{P}_2L}(x_2), \eta_{\mathcal{P}_2L}(y_2)\}$$
$$= \min(\min\left(\eta_{\mathcal{P}_1L}(x_1), \eta_{\mathcal{P}_2L}(z)\right), \min\left(\eta_{\mathcal{P}_1L}(y_1), \eta_{\mathcal{P}_2L}(z)\right))$$
$$= \min\{(\eta_{\mathcal{P}_1L} \circ \eta_{\mathcal{P}_2L})(x_1, x_2), (\eta_{\mathcal{P}_1L} \circ \eta_{\mathcal{P}_2L})(y_1, y_2)\}$$

$$(\eta_{Q_1U} \circ \eta_{Q_2U})(x_1, x_2)(y_1, y_2) = \min\{\eta_{Q_1U}(x_1, y_1), \eta_{\mathcal{P}_2U}(x_2), \eta_{\mathcal{P}_2U}(y_2)\}$$
$$\leq \min\{[\eta_{\mathcal{P}_1U}(x_1), \eta_{\mathcal{P}_1U}(y_1)], \eta_{\mathcal{P}_2U}(x_2), \eta_{\mathcal{P}_2U}(y_2)\}$$
$$= \min(\min\left(\eta_{\mathcal{P}_1U}(x_1), \eta_{\mathcal{P}_2U}(z)\right), \min\left(\eta_{\mathcal{P}_1U}(y_1), \eta_{\mathcal{P}_2U}(z)\right))$$
$$= \min\{(\eta_{\mathcal{P}_1U} \circ \eta_{\mathcal{P}_2U})(x_1, x_2), (\eta_{\mathcal{P}_1U} \circ \eta_{\mathcal{P}_2U})(y_1, y_2)\}$$

$$(\gamma_{Q_1L} \circ \gamma_{Q_2L})(x_1, x_2)(y_1, y_2) = \max\{\gamma_{Q_1L}(x_1, y_1), \gamma_{\mathcal{P}_2L}(x_2), \gamma_{\mathcal{P}_2L}(y_2)\}$$
$$\geq \max\{[\gamma_{\mathcal{P}_1L}(x_1), \gamma_{\mathcal{P}_1L}(y_1)], \gamma_{\mathcal{P}_2L}(x_2), \gamma_{\mathcal{P}_2L}(y_2)\}$$
$$= \max(\max\left(\gamma_{\mathcal{P}_1L}(x_1), \gamma_{\mathcal{P}_2L}(z)\right), \max\left(\gamma_{\mathcal{P}_1L}(y_1), \gamma_{\mathcal{P}_2L}(z)\right))$$
$$= \max\{(\gamma_{\mathcal{P}_1L} \circ \gamma_{\mathcal{P}_2L})(x_1, x_2), (\gamma_{\mathcal{P}_1L} \circ \gamma_{\mathcal{P}_2L})(y_1, y_2)\}$$

$$(\gamma_{Q_1U} \circ \gamma_{Q_2U})(x_1, x_2)(y_1, y_2) = \max\{\gamma_{Q_1U}(x_1, y_1), \gamma_{\mathcal{P}_2U}(x_2), \gamma_{\mathcal{P}_2U}(y_2)\}$$
$$\geq \max\{[\gamma_{\mathcal{P}_1U}(x_1), \gamma_{\mathcal{P}_1U}(y_1)], \gamma_{\mathcal{P}_2U}(x_2), \gamma_{\mathcal{P}_2U}(y_2)\}$$
$$= \max(\max\left(\gamma_{\mathcal{P}_1U}(x_1), \gamma_{\mathcal{P}_2U}(z)\right), \max\left(\gamma_{\mathcal{P}_1U}(y_1), \gamma_{\mathcal{P}_2U}(z)\right))$$
$$= \max\{(\gamma_{\mathcal{P}_1U} \circ \gamma_{\mathcal{P}_2U})(x_1, x_2), (\gamma_{\mathcal{P}_1U} \circ \gamma_{\mathcal{P}_2U})(y_1, y_2)\}$$

Hence $\mathcal{G}_1 \circ \mathcal{G}_2$ is an interval valued picture fuzzy graph.

Example3: Consider two interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{P}_1, \mathcal{Q}_1)$ and $\mathcal{G}_2 = (\mathcal{P}_2, \mathcal{Q}_2)$ with underlying crisp graphs $\mathcal{G}_1{}^* = (V_1, E_1)$ and $\mathcal{G}_2{}^* = (V_2, E_2)$such that the vertices $V_1 = \{a, b\}$ and $V_2 = \{c, d\}$ and the edge sets $E_1 = \{ab\}$ and $E_2 = \{cd\}$ respectively, where

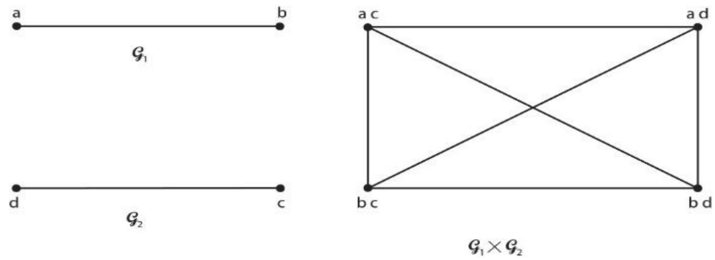

FIGURE 3. Composition of twointerval valued picture fuzzy graphs.

The vertex sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{P}_1 = \begin{matrix} \langle a; (0.2, 0.3, 0.4), (0.4, 0.1, 0.4) \\ \langle b; (0.3, 0.3, 0.3), (0.3, 0.4, 0.1) \end{matrix}$$

$$\mathcal{P}_2 = \begin{matrix} \langle c; (0.1, 0.2, 0.5), (0.2, 0.4, 0.3) \\ \langle d; (0.3, 0.6, 0.1), (0.4, 0.3, 0.2) \end{matrix}$$

The edge sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{Q}_1 = \{\langle ab; (0.2, 0.2, 0.5), (0.2, 0.1, 0.6)$$

$$\mathcal{Q}_2 = \{\langle cd; (0.1, 0.1, 0.6), (0.2, 0.2, 0.4)$$

The Composition of$\mathcal{G}_1[\mathcal{G}_2]$

$$\mathcal{P}_1 \circ \mathcal{P}_2 = \begin{matrix} \langle ac; (0.1, 0.2, 0.5), (0.2, 0.1, 0.4) \\ \langle ad; (0.2, 0.3, 0.4), (0.4, 0.1, 0.4) \\ \langle bc; (0.1, 0.2, 0.5), (0.2, 0.4, 0.3) \\ \langle bd; (0.3, 0.3, 0.3), (0.3, 0.3, 0.2) \end{matrix}$$

$$\mathcal{Q}_1 \circ \mathcal{Q}_2 = \begin{matrix} \langle ac, ad \rangle; (0.1, 0.2, 0.5), (0.2, 0.1, 0.4) \rangle \\ \langle bc, bd \rangle; (0.1, 0.2, 0.6), (0.2, 0.3, 0.3) \rangle \\ \langle ac, bc \rangle; (0.1, 0.2, 0.5), (0.2, 0.1, 0.4) \rangle \\ \langle ad, bd \rangle; (0.2, 0.3, 0.4), (0.3, 0.1, 0.4) \rangle \\ \langle ac, bd \rangle;\ (0.1, 0.2, 0.5), (0.2, 0.1, 0.4) \rangle \\ \langle ad, bc \rangle;\ (0.1, 0.2, 0.5), (0.2, 0.1, 0.4) \rangle \end{matrix}$$

CONCLUSION

In this paper, we described the new operations of interval valued picture fuzzy graphs. The fundamental properties of interval valued picture fuzzy graph of Cartesian product and Composition are obtained and furthermore examined for specific models.

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