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A NEW APPROACH FOR SOLVING FLOW SHOP SCHEDULING PROBLEM WITH TRIANGULAR INTUITIONISTIC FUZZY NUMBER

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ABSTRACT. Scheduling is one of the most critical decision-making problems widely studied in the operations research domain. This paper deals with finding an optimal solution to the scheduling problem under the Triangular Intuitionistic fuzzy Environment. Here processing times are being taken as Triangular I.F.N., which are further defuzzified into crisp values by the ranking procedure. We formulate a new algorithm to get an optimal sequence, and calculate the total elapsed time is minimum. We discussed the machines' minimum rental cost under the specified rental Policy and compared the results with Johnson's Algorithm.

1. INTRODUCTION

Initially, the idea of fuzzy set theory was formulated by Zadeh [9] in 1965. It deals with the information of membership function to handle Imprecise situation. Later, to overcome this uncertain situation, Atanassov started the intuitionistic fuzzy set and formulated the idea of belongings & non-belongings. The purpose of ranking the Intuitionistic fuzzy numbers plays a vital role in achieving an accurate solution for decision-makers. Atanassov (1994) defined various arithmetic operators in intuitionistic fuzzy sets, which are very helpful

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in applications of different areas of decision making. Further intuitionistic fuzzy optimization problem involves the comparison of fuzzy numbers.

Fuzzy sets are applied when the knowledge about the processing time is incomplete. Ishibuchi & Lee [5] formulated the fuzzy flow shop scheduling problem with fuzzy processing time Shakeela Sathish, K. Ganesan [7] studied 3 stage flow shop scheduling to minimize the rental cost of machines.

In this paper, We consider the four-machine flow-shop scheduling with a triangular intuitionistic fuzzy number as processing time. Here processing times are being taken as Triangular intuitionistic fuzzy number, which is further defuzzified into crisp values by the ranking procedure. We propose a new algorithm proposed to get an optimal sequence and calculate the total elapsed time. Minimizing the rental cost of machines under the specified rental Policy was discussed and compared the results with Johnson's Algorithm.

2. Preliminaries

Fuzzy Set

Let X be a nonempty set, and define a fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A\}$. In the pair $(x, \mu_{\tilde{A}}(x))$, the first element belongs to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1] is called the membership function.

Fuzzy number

$\tilde{A}$ is a fuzzy set on the real lineR, must satisfy the following conditions.

(1) $\mu_{\tilde{A}}(x_0)$ is piecewise continuous.
(2) There exist at least one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$.
(3) $\tilde{A}$ must be normal and convex.

Triangular Fuzzy Numbers

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{(a_3-a_2)} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } x > a_3 \end{cases},$$

where $a_1 \leq a_2 \leq a_3$ are real numbers.

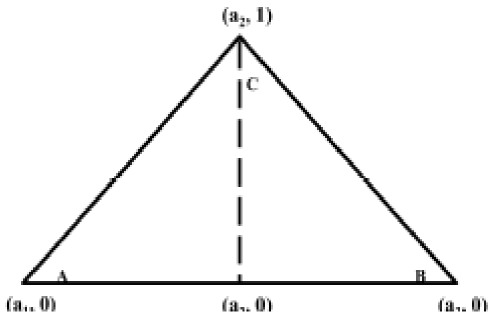


Fig. 1. Graphical representation of Triangular fuzzy number

Intuitionistic Fuzzy number

An Intuitionistic fuzzy subset $A^I = \{(x_i, \mu_{A^I}(x), \gamma_{A^I}(x)/x_i \in X)$ of the real line R is named as an intuitionistic fuzzy number if the following holds.

(1) There exist $\theta \in R$, $\mu_{A^I}(\theta) = 1$ and $\gamma_{A^I}(\theta) = 0$, where θ is the mean value of $^{A.I}$.

(2) μ_{A^I} is a continuous mapping from R to [0,1] for all $x \in R$, the relation $0 \leq \mu_{A^I}(x) + \gamma_{A^I}(x) \leq 1$ holds. The membership and non-membership function of $^{A.I}$ is of the following form,

$$\mu_{A^I}(x) = \begin{cases} 0, & if -\alpha < x < \theta - \alpha \\ f_1(x), & if x \in [\theta - \alpha, \theta] \\ 1, & if x = \theta \\ g_1(x), & if x \in [\theta, \theta + \beta] \\ 0, & if \theta + \beta \leq x < \alpha \end{cases},$$

$$\gamma_{A^I}(x) = \begin{cases} 1, & if -\alpha < x < \theta - \alpha' \\ f_2(x), & if x \in [\theta - \alpha', \theta]; 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0, & if x = \theta \\ g_2(x), & if x \in [\theta, \theta + \beta']; 0 \leq g_1(x) + g_2(x) \leq 1 \\ 1, & if \theta + \beta' \leq x \leq \alpha \end{cases},$$

where $f_i(x)$ and $g_i(x)$; i =1,2 which are strictly increasing and decreasing functions in $[\theta - \alpha, \theta]$, $[\theta, \theta + \beta]$, $[\theta - \alpha', \theta]$ and $[\theta, \theta + \beta']$ respectively. α, β, α' and β' are left and right spreads of $\mu_{A^I}(x)$ and $\gamma_{A^I}(x)$.

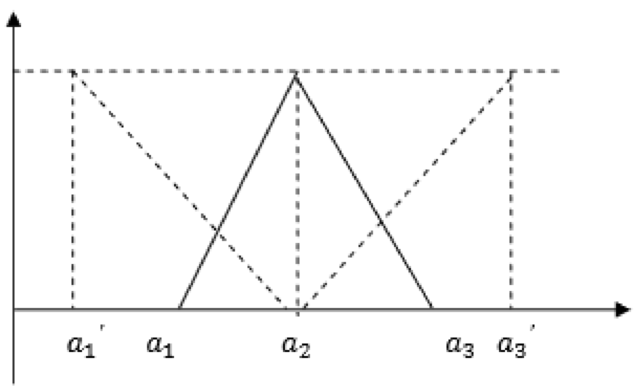
3. TRIANGULAR INTUITIONISTIC FUZZY NUMBER (TIFN)

An intuitionistic fuzzy number $A^I = \{(a_1, a_2, a_3)(a_1', a_2, a_3')\}$ is said to be triangular intuitionistic fuzzy number (TIFN) if its membership and non-membership functions are respectively given by

$$\mu_{A^I}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & if a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & if a_2 \leq x \leq a_3, \\ 0, & Otherwise \end{cases}$$

$$\gamma_{A^I}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & if a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2}, & if a_2 \leq x \leq a_3' \ , \\ 1, & Otherwise \end{cases}$$

Here $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and $\mu_{A^I}(x), \gamma_{A^I}(x) \leq 0.5$ for $\mu_{A^I}(x) = \gamma_{A^I}(x)$ for every $x \in R$.



Membership and non membership functions of TrIFN

Arithmetic Operators

Let $A = (a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$ and $B = (b_1, b_2, b_3)(b_1^{'}, b_2, b_3^{'})$ be the two TrIFN then

1. Addition Let $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)(a_1^{'} + b_1^{'}, a_2 + b_2, a_3^{'} + b_3^{'})$
2. Subtraction Let $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)(a_1^{'} - b_3^{'}, a_2 - b_2, a_3^{'} - b_1^{'})$

4. Ranking of TIFN

The Ranking of Triangular intuitionistic fuzzy number is defined as

$$R(A) = \frac{1}{3}[\frac{(a_3^{'} - a_1^{'})(a_2 - 2a_3^{'} - 2a_1^{'}) + (a_3 - a_1)(a_1 + a_2 + a_3) + 3(a_3^{'2} - a_1^{'2})}{(a_3^{'} - a_1^{'} + a_3 - a_1)}].$$

If $R.A. \leq R.B.$, then $A \leq B$.

5. Flow Shop Scheduling Problem With Intuitionistic Fuzzy Processing Time

Rental Policy

Policy I All the machines are taken and rent at one time and are returned at one time

Policy II All the tools are taken on rent at once and come back as and when they are no longer required.

Policy III We required all the machines are taking on rent and returned and no longer needed for processing.

Rental Situation

The problem formulated here is under Policy III; whenever the job under machine I completed, it is returned and deal with the job(appointment) to the next device.

Assumption

(1) The tasks to be processed are independent of each other
(2) Pre-emption of employment are not allowed
(3) An appointment is not available to the next machine until and unless we complete the current processing device.
(4) Machines never breakdown and are available throughout the scheduling process.
(5) We conclude each job must be once when it starts.

Notations

f_{ij}- Processing time of i^{th} job on a j^{th} machine
R(S)- Total rental cost for the sequence (S)
$U_k(S_K)$-Utilisation time of each machine
Cm -Cost for each rent (m =1...4)

Problem Formulation
Assume that some jobs i(i=1,2,...n) are to be processed on machines j(j=1,2,...m) under the specified rental policy.

Let f_{ij} be the processing time of i^{th} job on the j^{th} machine described by the triangular intuitionistic fuzzy number. We aim to find the minimal rental cost $R(S) = \sum_{i=1}^{n} f_{ij} * C1 + U_2(S_K) * C2 + U_3(S_K) * C3 + U_4(S_K) * C4$

Division Algorithm (Proposed method)
Step1: Defuzzify the triangular intuitionistic fuzzy number into a crisp number
Step2: Choose the highest processing time in each row, and all the entries are divided by it, enter the results in the top right of the table. **Step3:** Choose the highest processing time in each column, and all the entries are divided by it, enter the results in the bottom left of the table.
Step4: Add both the entries and note the minimum entry in the table.
Step5: Delete the corresponding row of the minimum entry and mark the sequence as S_k (k=1...5)
Step6: Repeat the procedure until we arrange all the jobs in proper order
Step7: Calculate the minimum total elapsed time and idle time of each Machine.
Step8: Calculate the total rental cost.

6. Numerical Example

Prestigious management school in a city is giving an order to stitch the uniform to all the students studying from sixth to the tenth standard to the X, Y, Z. Shop, the works are to stitch: 1) Boys Pants, 2) Boys Shirts, 3) Girls Pants, 4) Girls Tops, 5) Girls Overcoat. For each job, the owner has to arrange some persons for cutting, stitching, fixing school emblem and ironing.

Therefore he has to pay for each person who is doing the work. He is planning to pay Rs.10, Rs.20, Rs.15, and Rs.5 for each task. Triangular Intuitionistic Fuzzy Number is using for the time taken in each job. Calculate the total amount of time he takes to complete the work and the total amount he spends for each job.

Jobs	Cutting(C)	Stitching (S)	Fixing School Emblem(F)	Ironing(I)
1	((3,5,6); (2,5.1,7))	((1,2,3); (0.5,2,4))	((0.5,0.6,0.7); (0.4,0.6,0.8))	((4,5,6); (3,5,7))
2	((6,7,8); (4,7,9))	((2,4,5); (1,4.1,6))	((1,2,3); (0.2,2,5))	((6,7,8); (5,7.1,9))
3	((8,9,11); (7,9.2,12))	((1,2,3); (0.5,2,4))	((1,2,3); (0.5,2,4))	((5,6,7); (4,6.2,8))
4	((10,11,12); (8,11,13))	((0.6,0.7,0.8); (0.2,0.7,0.8))	((2,3,4); (1,3,5))	((6,8,9); (5,7,10))
5	((13,15,16); (12,15.1,17))	((1,2,3); (0.5,2,4))	((2,4,5); (1,3.5,6))	((3,5,6); (2,5.1,7))

Solution

The given TIFN is defuzzified into crisp value by using the proposed ranking procedure as follows

Jobs	C	S	F	I
1	4.688	2.106	0.600	5
2	6.762	3.688	2.282	7.022
3	9.375	2.106	2.106	6.044
4	10.762	0.600	3	7.458
5	14.688	2.106	3.563	4.688

Choose the highest processing time in each row, and all the entries are divided by it, enter the results in the top right of the table.

Similarly, choose the highest processing time in each column, and all the entries are divided by it, enter the results in the bottom left of the table. Adding the effects of each cell, choose the minimum entry, and delete the corresponding row.

Repeat the procedure until we arrange all the jobs

Jobs	**C**	**S**	**F**	**I**
1	0.938 **4.688** 0.319	0.421 **2.106** 0.571	0.120 **0.600** 0.168	1 **5** 0.670
2	0.963 **6.762** 0.460	0.525 **3.688** 1	0.325 **2.282** 0.640	1 **7.022** 0.942
3	1 **9.375** 0.638	0.225 **2.106** 0.571	0.225 **2.106** 0.591	0.645 **6.044** 0.810
4	1 **10.762** 0.733	0.056 **0.600** 0.163	0.279 **3** 0.842	0.695 **7.458** 1
5	1 **14.688** 1	0.143 **2.106** 0.571	0.243 **3.563** 1	0.319 **4.688** 0.629

The resultant value of the simplification is

Jobs	**C**	**S**	**F**	**I**
1	1.257	0.992	0.288	1.670
2	1.423	1.525	0.965	1.942
3	1.638	0.796	0.816	1.455
4	1.733	**0.219**	1.121	1.695
5	2	0.714	1.243	0.948

Among all the entries, the value in the fourth row is minimum. Delete the corresponding row and form the sequence as S_4. Proceed to the above algorithm until all the jobs arranged.

Here we get the sequence as $S_4 - S_1 - S_5 - S_2 - S_3$

The in-out table for the above series is under the following

Machines	**A cutting (C)**		**Stitching (S)**		**Fixing School Emblem(F)**		**Ironing(I)**	
JOBS	In	Out	In	In	In	Out	In	Out
4	--	10.762	10.762	11.362	11.362	14.362	14.362	21.820
1	10.762	15.450	15.450	7.556	17.556	18.156	21.820	26.820
5	15.450	30.138	30.138	32.244	32.244	35.807	35.807	40.495
2	30.138	36.900	36.900	40.588	40.588	42.870	42.870	49.892
3	36.900	46.275	46.275	48.381	48.381	50.487	50.487	56.531

- Minimum total elapsed time = 56.531 hrs
- Idle time of Cutting (C), C = 56.531 - 46.275= 10.256 hrs
- Idle time of Stitching (S),
  S =10.762+4.088+12.582+4.656+5.687+8.150= 45.925hrs
- Idle time of Fixing School Emblem (F),
  F = 11.362+3.194+14.088+4.781+5.511+6.044 = 44.980hrs
- Idle time of Ironing(I), I = 14.362+8.987+2.375+0.595= 26.319 hrs
- Rental Cost for Cutting (C), C = 46.275 *10 = Rs 462.75
- Rental Cost for Stitching (S), S =48.381 - 45.925 = 2.456* 20 = Rs.49.12
- Rental Cost for Fixing School Emblem (F), F = 50.487-44.980= 5.507 *15 = Rs.82.605
- Rental Cost for Ironing (I), I = 56.531 - 26.319 = 30.212*5 = Rs. 151.06
- Total Cost given for all the work = 462.75+49.12+82.605+151.06 = Rs.745.53

Comparison with Existing Methods

We tabulate the comparison of the proposed method with the existing process, which clearly shows that the proposed method provides the same results.

	Proposed Method			Johnson's Algorithm		
Machines	**Idle time**	**Total time**	**Rental Cost**	**Idle time**	**Total time**	**Rental Cost**
Cutting(C)	10.256	46.275	462.75	10.256	46.275	462.75
Stitching (S)	45.925	48.381	49.12	45.925	48.381	49.12
Fixing School Emblem(F)	44.980	50.487	82.605	44.980	50.487	82.605
Ironing(I)	26.319	56.531	151.06	26.319	56.531	151.06
		Total Cost	**Rs.745.53**			**Rs.745.53**

7. Conclusion

Here flow shop scheduling problem is formulated under an intuitionistic fuzzy number and discussed a new approach is to find the optimal sequence and rental cost calculated and compared. The application of the above procedure is useful in many uncertain conditions.

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