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Article · July 2022

DOI: 10.31838/ijpr/2021.13.01.054

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ISSN 0975-2366
DOI: https://doi.org/10.31838/ijpr/2021.13.01.054

Research Article

Domination in different graphs

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Received: 07.08.20, Revised: 07.09.20, Accepted: 07.10.20

ABSTRACT

In this paper analyses the concept of domination and its characteristics in graph theory. Here the concept of Domination, Maximum Dominating Set and Maximal Dominating Set as well as the concept of Independent set, Maximum Independent set and Maximal Independent Set are discussed in further. Dominating set is the collection of possible non-touching vertices through the single edge. Maximum Dominating set consist maximum number of possible non-touching vertices in the given graph. Maximal Domination Set consist minimum number of possible non-touching vertices which is adjacent with all the edges in the graph. Various types of Graphs, Structure and their properties also have to be discussed. Major part of this paper considers the cardinality calculation of Maximum and Maximal Dominating sets in the various graphs. After calculation, it derived a general formula for the calculation of Maximum Dominating cardinality and Maximal Dominating Cardinality. The calculation values should be representing in table format.

Keywords: Domination, Independent set, Graph, Vertex Degrees, Path, Cycle.

INTRODUCTION

A Graph is the collection of vertices and edges which is adjacent with vertices [2]. Graph Theory is the new branch development oriented about Graphs in Mathematics. It plays a vital role in modern world. It has many applications in various fields. Many types of graph is included in Graph Theory, It has various sectors like Matching [4], Coloring [1], and Domination [5] and so on. In this paper shows the calculation of domination number in various types of graphs.

DOMINATION

History: Domination in graphs is the recent research branch of Graph Theory [6]. It plays a vital role of application in various fields like Engineering, Physical Science and Social Science, etc. In 1960, the study of dominating sets began. In 1990, 86th issue of the Journal of Discrete Mathematics established the coverage "Topics on Domination in Graphs" [7]. After that, the theory of Domination became very popular for research area in Graph Theory. The study of Dominating Set had started around the world. It is used to find the efficient routes in Wireless Networking, designing Electrical Grids and Document Summarization [8]. In 1972, Richard Karp Proved NP Complete set cover problem. The concept of domination is applied in coding Theory, Problems on Facility Location which is minimized the distance that to travel for getting the closest facility [9]. Concept of domination involved monitoring in communication and land surveying. The problems of Dominating are used to minimize the number of places which is surveyor stand and take highest measurement for all entire regions [10].

Definition of domination graphs: The dominating set D (G) of the Graph G (V, E) consists of the collection of non-adjacent vertices through the edge. It means that the intersection of vertices those are incident with the edges is empty.

Condition for domination: The vertices of the graphs follow any one of the conditions: It may be included in dominating set (Or) It may be adjacent with the set of vertices in dominating set. If it is satisfied, these set of vertices dominates the given graph.

Domination cardinality [3]: The minimum possible number of vertices included in the dominating set is called as domination number. It is denoted by $\propto (\mathbf{G})$.

Independent set

Independent Set is the set of vertices that do not adjacent with each other. It means that, there are no edges connecting these set of vertices

Maximum independent set: An Independent Set is called as Maximum Independent set if it consist maximum possible number of vertices included in the set. It is denoted by I′ (G).

Maximal independent set: In this Independent set consist minimum possible number of collection of non-adjacent vertices and also it is not proper subset of any Independent sets [12]. It is denoted by I(G).

Maximum dominating set

The collection of maximum possible number of non-adjacent vertices in the Dominating Set is called Maximum Dominating Set [11]. It is denoted by D′ (G). It is also Maximum Independent Set.

Minimal or maximal dominating set

The collection of Minimum possible number of non-adjacent vertices in the Dominating Set is called Maximal Dominating Set. It is denoted by D (G). It is also Maximal Independent Set.

Whole dominating set

This set of Vertices D′(G) of Dominating set D(G) is the whole dominating set if all the vertices in the given graph is adjacent to a vertex of D′(G).

Connected dominating set

A Connected sub graph of dominating set is called as Connected Dominating Set [13].

DOMINATION IN VARIOUS GRAPHS

Here to find Maximal Dominating Cardinality and Maximum Dominating Cardinality for the various Graphs. The concept of Domination is analysed in various types of the Graph one by one.

Trivial graph

A single vertex without any edges is called as Trivial Graph. The calculation of Maximal Domination Cardinality and Maximum Domination Cardinality of Trivial Graph gives one [14]. It is shown in the given Figure 1.

Fig.1: Trivial Graph

Maximum Domination Cardinality = Maximal Domination Cardinality = 1.

Null graph

Null Graph has only more than one vertices without edges. Maximal Domination Cardinality and Maximum Domination Cardinality of Null Graph are equal to the number of vertices included in the Graph [15].

Maximum Domination Cardinality = Maximal Domination Cardinality = Number of vertices.

Complete graph: Each vertex of the graph is adjacent with other vertices in the graph through the edges, it is denoted C_n where n is the count of vertices in the graph. The connectivity of the vertex through edges is too strong as well as its structures also closed. If the graph has n vertices, it has degree n-1. Maximum Domination Cardinality and Maximal Domination Cardinality both have same value as one.

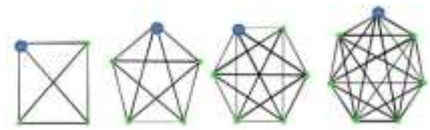


Fig.2: Examples of Complete Graph

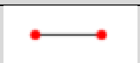
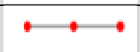
The dark vertex is Maximal Dominating Vertex. Here all the vertices have an equivalent chance for selection Maximal Dominating set

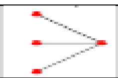
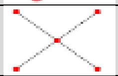
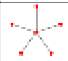
Star graph: All the vertices of the graph are joined by one vertex (centre) through the edges. It is Complete Bipartite Graph as well as path or tree. Star Graph is denoted by S (1, n), n is the count of edges and it is greater than 2 in the given Graph. It looks like open but the Connectivity with the centre vertex in the Graph. Here the number of vertices is n then the number of edges is one higher than n. Here single edge of the graph is considered as Maximal Dominating set. Maximum Dominating set has all the vertices except centre vertex which is adjacent with all the vertices in the graph. Maximum Dominating Cardinality is equal to the number of edges in the given graph. All-star graph has one for Maximal Domination Cardinality. The given table shows the calculation of Maximum Domination Cardinality and Maximal Domination Cardinality. It is used to find the general formula for the calculation of same.

Maximal Dominating Cardinality = 1

Maximum Dominating Cardinality = Number of Edges

Table 1: Calculation of maximum dominating cardinality and maximal dominating cardinality for star graph

SNO	n- STAR GRAPH	NO OF VERTICES	NO. OF EDGES	MAXIMUM DOMINATING CARDINALITY	MAXIMAL DOMINATING CARDINALITY
1		2	1	1	1
2		3	2	2	1

3		4	3	3	1
4		5	4	4	1
5		6	5	5	1

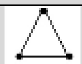
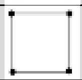
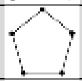
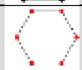
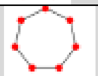
Cycle graph
It is closed and connected path. It is denoted by C_n where $n \geq 3$. In this type of graphs have same number of vertices and number of edges. The general formulae for Maximum Domination Cardinality are given for all cycle graphs. Here the calculation used only the number of vertices in the given graph.

Maximum Domination Cardinality= $\left|\frac{\textbf{Number of vertices}}{\textbf{2}}\right|$

Maximal Domination Cardinality= $\begin{cases} \left|\frac{n}{3}\right| & \textbf{if n is multiple of 3} \\ \left|\frac{n}{3}\right| + 1 & \textbf{Otherwise} \end{cases}$

Here modulus attains the whole number only.

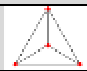
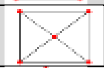
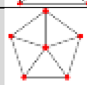
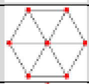
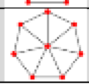
Table 2: calculation of maximum dominating cardinality and maximal dominating cardinality for cycle graph

SNO	CYCLE GRAPH	NO OF VERTICES n	MAXIMUM DOMINATION CARDINALITY	MAXIMAL DOMINATION CARDINALITY
1		3	1	1
2		4	2	2
3		5	2	2
4		6	3	2
5		7	3	3

WHEEL GRAPH
Its Structure is closed as well as loop. Here the number of edges is calculated by multiple of 2 X number of vertices. All the vertices can be connected to a single vertex through an edge as well as they can be joined themselves by an edge in order wise forming a boundary and closed loop. Maximal Domination Cardinality gives the same value for all wheel graphs. It is one for all Wheel Graph. Maximum Matching Cardinality is derived by the following formula and it can be tabulated in the given Table 3.

Maximum Domination Cardinality= $\left|\frac{\textbf{Number of vertices}}{\textbf{2}}\right|$

Table 3: Calculation of maximum dominating cardinality and maximal dominating cardinality for wheel graph

SNO	NUMBER OF VERTICES	WHEEL GRAPH	MAXIMUM DOMINATION CARDINALITY	MAXIMAL DOMINATION CARDINALITY
1	4		2	2
2	5		2	2
3	6		3	3
4	7		3	3
5	8		3	3

CONCLUSION

This paper gives the explanation of structure, number of vertices, number of edges and properties of some types of the graph like Trivial graph, Null graph, Complete graph, Cycle Graph and Wheel Graph. This paper is used to calculate the Maximal Dominating Cardinality and Maximum Dominating Cardinality of the above graphs. It finds that there is relation between number of edges, number of vertices with the Concept of Cardinality for Dominating Set. Then it can be implemented by general formulae also. In future work, it has to implement into all the various type of complicated graph like Fractal Graph, Peterson Graph and so on. Concept of Domination has a main rule in real life world. Some of the examples has explained such as Fixing the camera into the right place where it gives enlarge focus area, Finding Neural system which is most connected to all organs into the human body, Choosing place for fixing the stone into ornaments which makes more beauty and so on. The application of Domination has to be analyzed into the future work.

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