

INDEPENDENT DOMINATION NUMBER (IDN) FOR SOME SPECIAL NETWORKS IN ADAPTIVE MESH REFINEMENT (AMR)-WENO SCHEME

N. SENTHURPRIYA, S. MEENAKSHI, AND P. PUNITHAVATHI¹

ABSTRACT. Let $G(V,E)$ be a graph, V has a subset C , this set is a non-empty subset of V and the vertices in C is adjacent to the minimum of one vertex of the set V , then G has the dominating set C . If there is no adjacency between the vertices of C , then G has an independent dominating set C and so the number of vertices present in the set C represents the IDN, the minimum cardinality of the sets C . Here in our research, we find the same for some special networks, namely the polygons with nine, ten and eleven sides by above mentioned Scheme.

1. INTRODUCTION

In the past the ideas of domination, is started with the game of chess. Later the work was extended by various peoples as, Ahrens in 1901 [9] Berge in 1958 [2] and Ore in 1962 [5], by 1972 Cockayne and Hedetniemi [2,4] gone through domination and commenced to review it, thereby a survey was published in 1975 and there came into existence for the topic independent domination number. Thereon, many researchers started to work in that. Thus, Kostichka [7] Goddard et al. [1,6] and Lam et al. [13] researched for regular graph and cubic graph. Cockayne et al. [5] found its boundary and its complement, while Shiu et al. [8] gave for triangle-free graphs and thereby characterizing its upper bounds.

¹*corresponding author*

2020 *Mathematics Subject Classification.* 05C69.

Key words and phrases. ID, IDN, Domination, Adaptive Mesh Refinement (AMR), WENO Scheme.

Favaron [4] gave the sharp upper bounds for general graphs and Haviland [3] extended it.

When we come to AMR these are the techniques followed by steady or unsteady stimulation. And by this method we can get solutions for complex features easily, without refining the whole mesh.

Definition 1.1. [6] *Adaptive Mesh Refinement: The AMR will work on conformal method of hybrid meshes. The refinement of nodes is taken at the base level itself i.e., the base cell information is preserved and is thereby carried over the whole refining techniques i.e., the information is carries and that's the base for the whole process.*

Definition 1.2. [6] *WENO Scheme A process of approximation that takes place throughout the procedure without any oscillation in between.*

Definition 1.3. [11] *Let $G(V,E)$ be a graph, V has a subset C , this set is a non-empty subset of V and the vertices in C is adjacent to the minimum of one vertex of the set V , then G has the dominating set C .*

Definition 1.4. [12] *If there is no adjacency between the vertices of C , then G has an independent dominating set C and so the number of vertices present in the set C represents the IDN, the minimum cardinality of the sets C .*

As in [10] it gives the IDN for the mesh networks from triangle till octagon, so we extend our work for the next three networks such as nonagon graph (NG), decagon graph(DG) and hendecagon graph(HG).

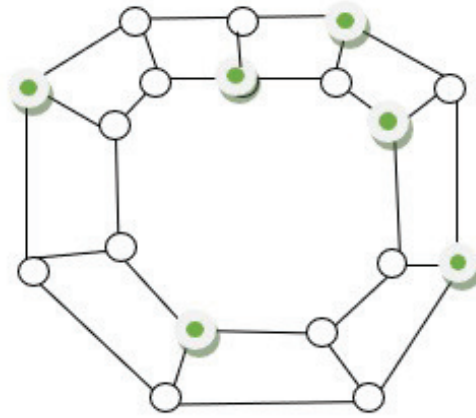
2. IDN IN AMR-WENO SCHEME NETWORKS

Theorem 2.1. *Let G be the NG. Then the IDN for our above mentioned scheme for at n^{th} level will be*

$$i(NG) = \begin{cases} \left\lceil \frac{n}{4} \right\rceil + 1 & \text{for the mesh level } 2, 3, \dots, (N-1)/4 \\ \left\lceil \frac{n}{4} \right\rceil + 2 & \text{for the mesh level which is perfectly divisible by 4} \end{cases} .$$

Proof. To prove

$$i(NG) = \begin{cases} \left\lceil \frac{n}{4} \right\rceil + 1 & \text{for the mesh level } 2, 3, \dots, (N-1)/4 \\ \left\lceil \frac{n}{4} \right\rceil + 2 & \text{for the mesh level which is perfectly divisible by 4} \end{cases} .$$



Naming NG as follows:

Naming the NG at n^{th} level as $(u_1, v_1, w_1, x_1, y_1, z_1, a_1, b_1, c_1), (u_2, v_2, w_2, x_2, y_2, z_2, a_2, b_2, c_2), \dots, (u_i, v_i, w_i, x_i, y_i, z_i, a_i, b_i, c_i)$.

Thus, we get n times the vertices in the base level (here in the base level we have nine vertices). Therefore, at each level of NG has $9n$ vertices. Hence,

(i) For the level $2, 3, \dots, (N - 1)/4$ we get the IDN as $\lceil \frac{n}{4} \rceil + 1$.

(ii) For levels which is perfectly divisible by 4 we get the IDN as $\lceil \frac{n}{4} \rceil + 2$.

Thus

$$i(NG) = \begin{cases} \lceil \frac{n}{4} \rceil + 1 & \text{for the mesh level } 2,3,\dots,(N-1)/4 \\ \lceil \frac{n}{4} \rceil + 2 & \text{for the mesh level which is perfectly divisible by 4} \end{cases}$$

For example, The coloured vertices represent the ID set. □

Theorem 2.2. Let G be the DG. Then the IDN for our above mentioned scheme for at n^{th} level will be $i(G) = \frac{n}{5} + N$, where N corresponds to the mesh level.

Proof. To prove $i(G) = \frac{n}{5} + N$, where N corresponds to the mesh level.

Naming DG as follows:

Naming the DG at n^{th} level as

$(u_1, v_1, w_1, x_1, y_1, z_1, a_1, b_1, c_1, d_1), (u_2, v_2, w_2, x_2, y_2, z_2, a_2, b_2, c_2, d_2), \dots, (u_i, v_i, w_i, x_i, y_i, z_i, a_i, b_i, c_i, d_i)$. Thus, we get n times the vertices in the base level (here in the base level we have ten vertices). At each level of DG we have $10n$ vertices.

Hence we get the IDN as $\frac{n}{5} + N$. Thus $i(DG) = \frac{n}{5} + N$, where N corresponds to the mesh level. □

Theorem 2.3. *Let G be the HG. Then the IDN for our above mentioned scheme for at n^{th} level will be $i(HG) = \frac{n}{6} + N$, where N corresponds to the mesh level.*

Proof. To prove $i(HG) = \frac{n}{6} + N$, where N corresponds to the mesh level.

Naming HG as follows:

Naming the HG at n^{th} level as: $(u_1, v_1, w_1, x_1, y_1, z_1, a_1, b_1, c_1, d_1, e_1), (u_2, v_2, w_2, x_2, y_2, z_2, a_2, b_2, c_2, d_2, e_2), \dots, (u_i, v_i, w_i, x_i, y_i, z_i, a_i, b_i, c_i, d_i, e_i)$. And thus we get n times the vertices in the base level (here in the base level we have eleven vertices). Therefore, at each level of HG has $11n$ vertices. Hence we get the IDN as $\frac{n}{6} + N$. Thus $i(HG) = \frac{n}{6} + N$, where N corresponds to the mesh level. \square

3. CONCLUSION

In our past research we have found IDN by AMR for polygons of vertices from 3-8 i.e., from triangular to octagons. By that concept we extended this paper by finding IDN for nonagon, decagon and restricted till hendecagon graphs. we limited our findings till hendecagon and by the same concept we can find for any number of vertices of polygons and thereby leaving the work to the readers as an open question.

REFERENCES

- [1] A.V. KOSTOCHKA: *The independent domination number of a cubic 3-connected graph are often much larger than its domination number*, Graphs Combin., **9** (1993), 235-237.
- [2] C. BERGE: *Theory of Graphs and its Applications*, Methuen, London., 1962.
- [3] E.J. COCKAYNE, S.T. HEDETNIEMI: *Independent graphs*, Congr. Numer., **10** (1974) 471-491.
- [4] E.J. COCKAYNE, S.T. HEDETNIEMI: *Towards a theory of domination in graphs*, Networks **7**, (1977), 247-261.
- [5] E.J. COCKAYNE, O. FAVARON, H. LI, G. MACGILLIVRAY: *The product of the $i(G)$ of a graph and its complement*, Discrete Mathematics, **90** (1991), 313-317.
- [6] HARSHAVARDHANA SRINIVASAN AND PANAGIOTIS TSOUTANIS: *Adaptive mesh refinement techniques for high-order finite- volume WENO scheme*, conference paper, January 2016.
- [7] J. HAVILAND: *Boundary for independent domination in regular graphs*, Discrete Math., **307** (2007) 2643-2646.
- [8] O. FAVARON: *Relations between the two parameters of independence and irredundance*, Discrete Math., **70** (1988) 17-20.
- [9] O. ORE: *Theory of graphs*, Amer. Math. Soc. Transl., **38** (1962) pp. 206-212.

- [10] P. LAM, W. SHIU AND L. SUN: *On $i(G)$ of regular graphs*, Discrete Mathematics Combin, **202** (1999), 135-144.
- [11] S. AO, E. G. COCKAYNE MACGILLIVRAY AND C. M. MYNHARDT: *Domination critical graphs with higher independent domination numbers*, J. Graph Theory, **22** (1996), 9-14.
- [12] T.W. HAYNES, S.M. HEDETNIEMI, S.T. HEDETNIEMI AND M.A. HENNING: *Power domination in graphs applied to electrical power networks*, SIAM Journal on Discrete Mathematics, **15**(4) (2002), 519-529.
- [13] W. GODDARD, M. HENNING, J. LYLE, J. SOUTHEY: *On the independent domination number of regular graphs*, Ann. Comb., **16** (2012), 719-732.

DEPARTMENT OF MATHEMATICS
VELS INSTITUTE OF SCIENCE, TECHNOLOGY AND ADVANCED STUDIES
CHENNAI 600 117,
INDIA.
Email address: pspriyasaha@gmail.com

DEPARTMENT OF MATHEMATICS
VELS INSTITUTE OF SCIENCE, TECHNOLOGY AND ADVANCED STUDIES
CHENNAI 600 117,
INDIA.
Email address: meenakshikarthikeyan@yahoo.co.in

DEPARTMENT OF MATHEMATICS
VELS INSTITUTE OF SCIENCE, TECHNOLOGY AND ADVANCED STUDIES
CHENNAI 600 117,
INDIA.
Email address: punilashok.ak@gmail.com