

## PERFORMANCE MEASUREMENTS OF AN UNSIGNALIZED TRAFFIC FLOW USING FUZZY QUEUING THEORY

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**ABSTRACT.** This paper's main objective is to analyze the traffic flow of an unsignalized intersection roadway by comparing the queuing model of a multi-channel system with the fuzzy queuing model of a multi-channel system. The numerical example illustrates the observation is based on the problem of waiting time in the traffic system, and the performance measures of the fuzzy queuing theory model lie in the range of the performance measures of the queuing theory model.

### 1. INTRODUCTION

Queuing theory has many applications in various fields such as telecommunication, bank ATM, computer networking, traffic system, railway system, etc. A queuing model is a study about the nature of queues. Queuing theory analyzes every aspect of a customer's waiting time together with the analysis of the service process. Decreasing the customer's waiting time and piling up the customer's satisfaction through remodeling their service quality. Fuzzy Queuing is a new approach to real-life situations. The performance of fuzzy numbers will guide the administrator to promote the service time with vagueness to raise the profit. This study concentrates on the queuing system of an unsignalized roadways. The unsignalized intersections are the most common in roadways. An

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unsignalized roadway means an unsignalized intersection, which is a junction of two or more public roads. This paper will note the similarity between the queuing theory model's performance measures and the fuzzy queuing theory.

Further references can be found in [1]-[10].

## 2. PRELIMINARIES

**2.1. Fuzzy Set.** A fuzzy set  $\tilde{A}$  is defined by  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$ . In the pair of an ordered set  $(x, \mu_{\tilde{A}}(x))$ , the first element  $x$  belongs to the Universe  $X$ , the second element  $\mu_{\tilde{A}}(x)$ , belongs to the interval  $[0, 1]$ , then the set  $\tilde{A}$  is called a fuzzy set.  $\mu_{\tilde{A}}(x)$  is called the Membership function.

**2.2. Pentagonal Fuzzy Number (PFN).** A PFN  $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$  should satisfy the following condition:

- (i)  $\mu_{\tilde{A}}(x)$  is a continuous function in the interval  $[0, 1]$
- (ii)  $\mu_{\tilde{A}}(x)$  is strictly non-decreasing continuous function on the intervals  $[a_1, a_2]$  and  $[a_2, a_3]$
- (iii)  $\mu_{\tilde{A}}(x)$  is strictly non-increasing continuous function on the intervals  $[a_3, a_4]$  and  $[a_4, a_5]$ .

**2.3. Linear PFN with Symmetry.** A linear PFN with symmetry is written as  $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$  whose corresponding membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} r \frac{x-a_1}{a_2-a_1}; & \text{for } a_1 \leq x \leq a_2 \\ 1 - (1-r) \frac{x-a_2}{a_3-a_2}; & \text{for } a_2 \leq x \leq a_3 \\ 1; & \text{for } x = a_3 \\ 1 - (1-r) \frac{x-a_3}{a_4-a_3}; & \text{for } a_3 \leq x \leq a_4 \\ r \frac{x-a_4}{a_4-a_5}; & \text{for } a_4 \leq x \leq a_5 \\ 0; & \text{for } x > a_5 \end{cases}$$

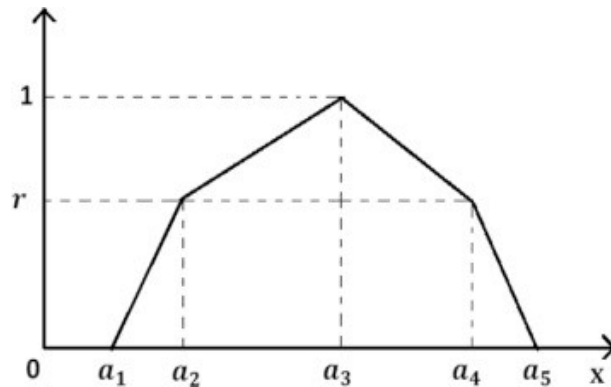


FIGURE 1. Linear PFN with Symmetry.

**2.4. Linear PFN with Asymmetry.** A linear PFN with asymmetry is given as  $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$  whose corresponding membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} r\left(\frac{x-a_1}{a_2-a_1}\right); & \text{for } a_1 \leq x \leq a_2 \\ 1 - (1-r)\left(\frac{x-a_2}{a_3-a_2}\right); & \text{for } a_2 \leq x \leq a_3 \\ 1; & \text{for } x = a_3 \\ 1 - (1-s)\frac{x-a_3}{a_4-a_3}; & \text{for } a_3 \leq x \leq a_4 \\ s\left(\frac{x-a_4}{a_4-a_5}\right); & \text{for } a_4 \leq x \leq a_5 \\ 0; & \text{for } x > a_5 \end{cases}$$

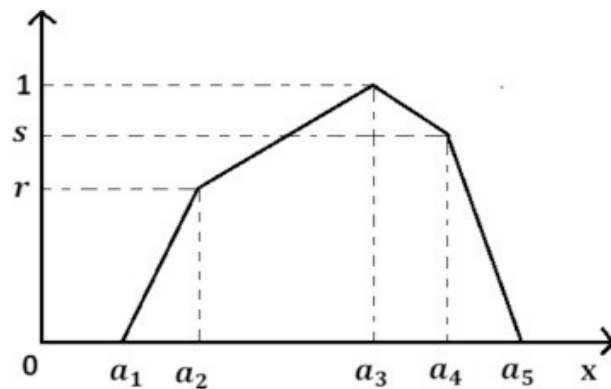


FIGURE 2. Linear PFN with Asymmetry.

**Remark 2.1.** (i) *The asymmetry PFN becomes symmetry PFN when  $r=s$ .*  
 (ii) *For any asymmetry, PFN might be  $r < s$  or  $r > s$ .*

### 3. SYMBOLS AND NOTATIONS (FM/FM/C) :( FCFS/ $\infty$ / $\infty$ )

The following symbols and notations are used to compute the performance measures of the model (FM/FM/C) :( FCFS/ $\infty$ / $\infty$ )

$\lambda$  - Average number of customers being serviced per unit of time,

$\mu$  - Average number of customers arriving per unit of time,

$s$  - Number of parallel service channels,

The utilization factor,

$$\rho = \frac{\lambda}{s\mu}.$$

Probability of zero customers in the system,  $P_0$

$$P_0 = \frac{1}{\left[ \left( \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n \right) + \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \left( \frac{1}{1-\rho} \right) \right]}.$$

The average number of customers in the waiting line,

$$L_q = \frac{\rho \left( \frac{\lambda}{\mu} \right)^s}{s! (1-\rho)^2}.$$

The average number of customers in the system,

$$L_s = L_q + \frac{\lambda}{\mu}.$$

The average waiting time of a customer waiting for service,

$$W_q = \frac{L_q}{\lambda}.$$

The average waiting time of a customer waiting in the queue,

$$W_s = \frac{L_s}{\lambda}.$$

**3.1. Yager's ranking method.** Yager's ranking index for the fuzzy quantities is,

$$Y_1 = \int_0^{hgt(\tilde{A})} M(\tilde{A}_\alpha) d\alpha.$$

Here  $hgt(\tilde{A})$  denotes  $\sup_{x \in \text{sup}\tilde{A}} \tilde{A}(x)$ , which is the height of  $\tilde{A}$ , and  $M$  denotes the mean value operator. In this case,  $hgt(\tilde{A}) = 1$  and  $M(\tilde{A}) = \frac{a_\alpha^- + a_\alpha^+}{2}$ .

#### 4. NUMERICAL EXAMPLE

An unsignalized roadway is considered and analyzed to compare the performance measurements of queuing theory models with the performance measurements of fuzzy queuing theory models. In this paper, the unsignalized intersections with existing left-turn lanes to be considered. The data on the frequency of domestic four-wheelers were collected from Monday to Friday from 8 a.m. to 10 a.m. in a particular intersection.



FIGURE 3. Unsignalized intersections with a left turn lane.



FIGURE 4. Unsignalized intersections with left turn lane signboard.

**Table: the 1-Data collection**

Days	Average arrival rate ( $\lambda$ )	Average service rate ( $\mu$ )
Monday	124	150
Tuesday	102	125
Wednesday	97	112
Thursday	107	140
Friday	131	145

Three steps follow the result of this work,

- (i) To compute the performance measures of this problem by using queuing theory model.
- (ii) To calculate the performance measures by applying the fuzzy queuing theory model.
- (iii) To compare the performance measures between the queuing theory model and the fuzzy queuing theory model.

**Table:2 Performance measures obtained by queuing theory model**

Days	Average arrival rate ( $\lambda$ )	Average service rate ( $\mu$ )	$L_q$	$L_s$	$W_q$	$W_s$
Monday	124	150	0.4104	1.2371	0.0033	0.0100
Tuesday	102	125	0.3876	1.2036	0.0038	0.0118
Wednesday	97	112	0.5051	1.3712	0.0052	0.0141
Thursday	107	140	0.2923	1.0566	0.0027	0.0099
Friday	131	145	0.6128	1.5162	0.0047	0.0116

**Table:3 Performance measures obtained by fuzzy queuing theory model using Yager's Ranking Index**

Performance Measures	Results
Average No. of vehicles in the queue, $L_q$	0.2142
Average No. of vehicles in the system, $L_s$	1.0844
Average time spends in the queue, $W_q$	0.0019
Average time spends in the system, $W_s$	0.0095

**Table:4 Comparison performance measures between the Queuing Theory Model and Fuzzy Queuing Model**

Performance Measures	Queuing Theory	Fuzzy Queuing Theory
Average No. of vehicles in the queue, $L_q$	0.4416	0.2142
Average No. of vehicles in the system, $L_s$	1.27694	1.0844
Average time spends in the queue, $W_q$	0.0003	0.0019
Average time spends in the system, $W_s$	0.0115	0.0095

**4.1. Result.** From the above table, we can quickly summarize that the range of the performance measures derived from fuzzy queuing models, is entirely within the scope of the performance measures, which is calculated using queuing theory. Based on the result, the Fuzzy Queuing Model is much more effective and efficient in measuring the performance of multi-server in a queuing system since the Fuzzy set theory is more easily adaptable than other approaches.

## 5. CONCLUSIONS

In this paper, the result shows that the performance measures  $L_q$ ,  $L_s$ ,  $W_q$ , and  $W_s$  for both Queuing Theory Model and Fuzzy Queuing Model were computed and compared. The results of the quantities in both models are precisely similar. In a particular unsignalized intersection, the frequency of an average number

of vehicles was evaluated, so the average time spends in that system was also computed. Suppose the frequency of the average number of vehicles and the average time spends in the system will increase, then it should be understood that the system is in traffic gridlock. To avoid traffic congestion, it is advisable to install an optical signal in the future to improve safety measures. Thus, this study concludes that fuzzy queuing is an alternative way to compute the performance measures since the information obtained from the application is much easier to understand and interpret. Therefore, the Fuzzy Queuing Model is an alternative way to measure multi-server performance in a queuing system.

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