Triangular graphs are proper lucky and lucky

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Triangular Graphs are Proper Lucky and Lucky

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Abstract. The labelling is lucky when the sum over adjacent nodes have different labeling. The proper labeling says that adjacent nodes have different labeling. We proved a proper lucky and lucky for specific type of snake graphs like triangular, double triangular, alternate triangular and double alternate triangular snake graph, also derived a proper lucky and lucky number.

INTRODUCTION

Labeling has a connection between structure and number of the graph. Rosa introduced it in 1967 [8]. Gallian [4] conducted a study on graph labeling. Many academician have devised various graph labeling.. Karonsik initiated the proper labeling [5] which says adjacent nodes have different labeling. Ahadi initiated lucky labeling [1] which says that sum over adjacent nodes have different labelling. Comparison between the proper and lucky labeling done by Akbari [2]. Badr derived that double triangular graphs are odd graceful in 2013 [3]. Somasundram and Sandhya have found the triangular snake graph is root square mean graph in 2014 [9]. Meenakshi derived proper lucky and lucky of quadrilateral graphs in 2021[6]. In this paper $f(u)$ denotes labelling of nodes, $s(u)$ denotes sum over adjacent labelling and $\Delta(G)$ maximum degree of a graph. The proper lucky number denoted by $\eta_p(G)$ and lucky number denoted by $\eta(G)$ [7].

Preliminaries

Triangular snake

The triangular snake $\mathrm{TS_n}$ is constructed with the $2\mathrm{n}-1$ nodes where $\mathrm{n}>2, \mathrm{u_i}$ have n node, $\mathrm{v_i}$ have $\mathrm{n}-1$ nodes and the edge $\mathrm{E(TS_n)} = \{\mathrm{u_i u_{i+1}}: 1 \le \mathrm{i} \le \mathrm{n}\} \cup \{\mathrm{u_i v_i}: 1 \le \mathrm{i} \le \mathrm{n}\} \cup \{\mathrm{u_{i+1} v_i}: 1 \le \mathrm{i} \le \mathrm{n}\}$ refer figure 1 [10].

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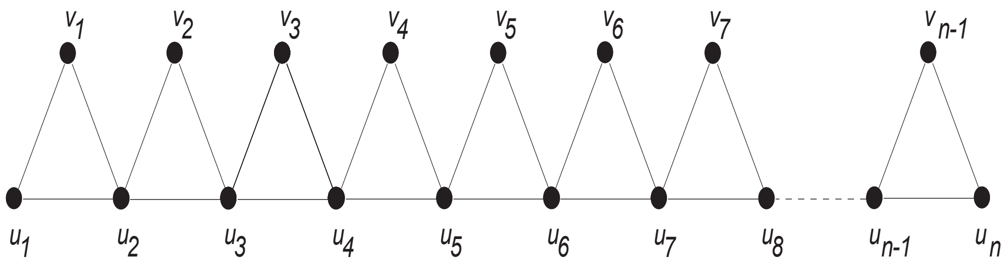

FIGURE 1. TS_n

Double triangular snake

A double triangular snake DTS_n is constructed with the $3n-2$ nodes where $n > 2, u_i$ have n nodes, v_i have $n-1$ nodes, w_i have $n-1$ nodes and the edge set $E(DTS_n) = E(TS_n) \cup \{u_i w_i : 1 \leq i \leq n\} \cup \{u_{i+1} w_i : 1 \leq i \leq n\}$ refer figure 2 [10].

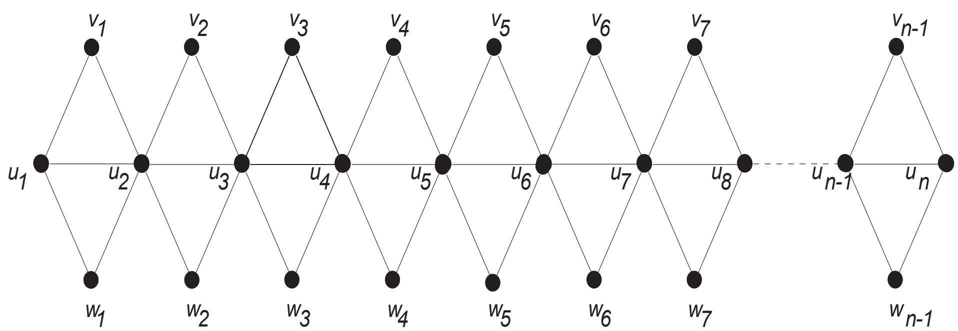

FIGURE 2. DTS_n

Alternate triangular snake

An alternate triangular snake ATS_n is constructed with the $\frac{3n}{2}$ nodes where $n > 4, u_i$ have n nodes, v_i have $\frac{n}{2}$ nodes and the edge set $E(ATS_n) = \{u_i u_{i+1} : 1 \leq i \leq n\} \cup \{u_{2i-1} v_i : 1 \leq i \leq n\} \cup \{u_{2i} v_i : 1 \leq i \leq n\}$ refer figure 3 [10].

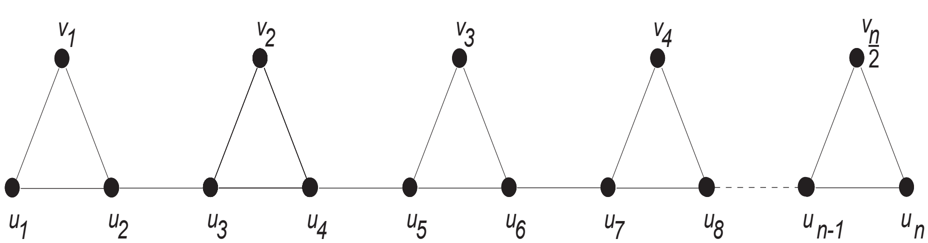

FIGURE 3. ATS_n

Double alternate triangular snake

A double alternate triangular snake $DATS_n$ is constructed with the 2n nodes where $n > 2, u_i$ have n nodes, v_i have $\frac{n}{2}$ nodes, w_i have $\frac{n}{2}$ nodes and the edge set $E(DATS_n) = E(ATS_n) \cup \{u_{2i-1} w_i : 1 \leq i \leq n\} \cup \{u_{2i} w_i : 1 \leq i \leq n\}$ refer figure 4 [10].

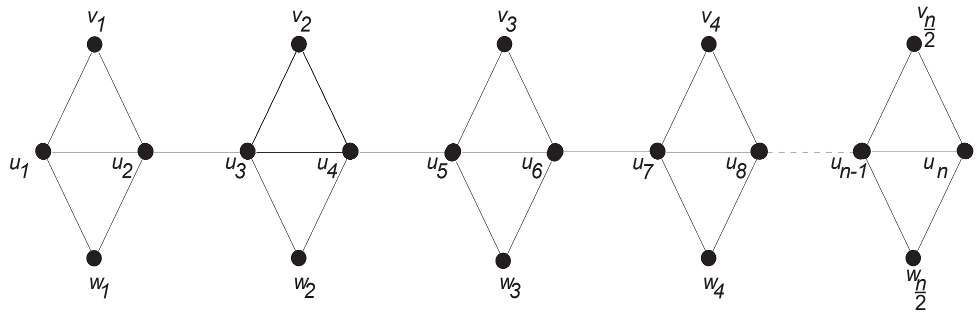

FIGURE 4. $DATS_n$

Main Results

Theorem 1

The triangular snake TS_n with $n > 2$ is lucky graph with $\eta(TS_n) = \frac{\Delta(TS_n)}{2}$.

Proof:

Let $f: V(TS_n) \to \{1,2\}$ be defined by

Case (i): n = even

$$f(u_i) = \begin{cases} 1 & \text{odd i} \\ 2 & \text{even i} \end{cases}$$

$$f(v_i) = \begin{cases} 1 & i > 1 \\ 2 & i = 1 \end{cases}$$

$$s(u_1) = 4$$

$$s(u_2) = 5$$

$$s(u_n) = 2$$

$$s(u_i) = \begin{cases} 4 & \text{even i} > 2, i < n \\ 6 & \text{odd i} > 1 \end{cases}$$

$$s(v_i) = 3$$

Therefore TS_n with $n > 2$ is lucky graph with $\eta(TS_n) = 2$ for even n.

Case (ii): n = odd

$$f(u_i) = \begin{cases} 1 & \text{odd i}, i < n \\ 2 & \text{even i}, i = n \end{cases}$$

$$f(v_i) = \begin{cases} 1 & i > 1 \\ 2 & i = 1 \end{cases}$$

$$s(u_1) = 4$$

$$s(u_2) = 5$$

$$s(u_{n-1}) = 5$$

$$s(u_n) = 3$$

$$s(u_i) = \begin{cases} 4 & \text{even i} > 2, i < n-1 \\ 6 & \text{odd i} > 1, i < n \end{cases}$$

$$s(v_i) = \begin{cases} 3 & i < n \\ 4 & i = n \end{cases}$$

Therefore TS_n with $n > 2$ is lucky graph with $\eta(TS_n) = 2$ for odd n. Hence TS_n with $n > 2$ is lucky graph with $\eta(TS_n) = \frac{\Delta(TS_n)}{2}$ refer figure5.

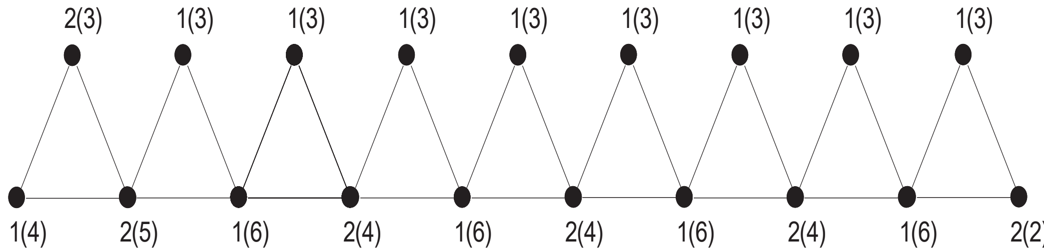

FIGURE 5. Lucky triangular snake graph TS_{10}

Theorem 2

The double triangular snake DTS_n with $n > 2$ is lucky graph with $\eta(DTS_n) = \frac{\Delta(DTS_n)}{3}$.

Proof:

Let $f: V(TS_n) \to \{1,2\}$ be defined by

Case (i): $n =$ even

$$f(u_i) = \begin{cases} 1 & \text{odd i, i} = 2, \text{i} = \text{n} \\ 2 & \text{even i, i} > 2, i < n \end{cases}$$

$$f(v_i) = 1$$

$$f(w_i) = 1$$

$$s(u_1) = 3$$

$$s(u_n) = 3$$

$$s(u_i) = \begin{cases} 6 & \text{even i} < n \\ 7 & \text{i} = 3, \text{i} = \text{n} - 1 \\ 8 & \text{odd i} > 3, i < n - 1 \end{cases}$$

$$s(v_i) = \begin{cases} 2 & \text{i} = 1, \text{i} = 2, \text{i} = \text{n} \\ 3 & \text{i} > 2, i < n \end{cases}$$

$$s(w_i) = \begin{cases} 2 & \text{i} = 1, \text{i} = 2, \text{i} = \text{n} \\ 3 & \text{i} > 2, i < n \end{cases}$$

Therefore DTS_n with $n > 2$ is lucky graph with $\eta(DTS_n) = 2$ for even n.

Case (ii): $n =$ odd

$$f(u_i) = \begin{cases} 1 & \text{odd i, i} = 2 \\ 2 & \text{even i, i} > 2 \end{cases}$$

$$f(v_i) = 1$$

$$f(w_i) = 1$$

$$s(u_1) = 3$$

$$s(u_n) = 4$$

$$s(u_i) = \begin{cases} 6 & \text{even i} \\ 7 & \text{i} = 3 \\ 8 & \text{odd i} > 3, i < n \end{cases}$$

$$s(v_i) = \begin{cases} 2 & \text{i} = 1, \text{i} = 2 \\ 3 & \text{i} > 2 \end{cases}$$

$$s(w_i) = \begin{cases} 2 & \text{i} = 1, \text{i} = 2 \\ 3 & \text{i} > 2, \end{cases}$$

Therefore DTS_n with $n > 2$ is lucky graph with $\eta(DTS_n) = 2$ for odd n. Hence DTS_n with $n > 2$ is lucky graph with $\eta(DTS_n) = \frac{\Delta(DTS_n)}{3}$ refer figure 6.

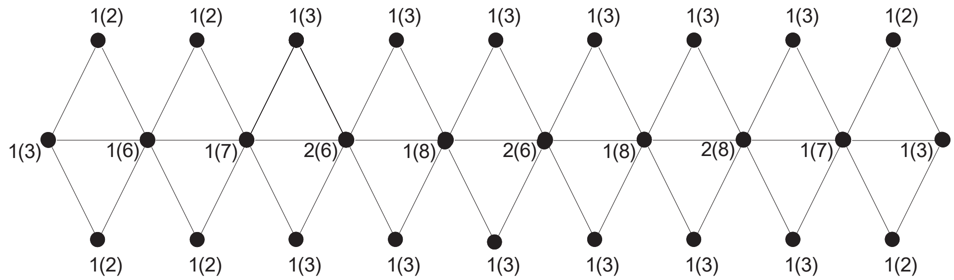

FIGURE 6. Lucky double triangular snake graph DTS_{10}

Theorem 3

The alternate triangular snake $\mathrm{ATS_n}$ with $\mathrm{n} > 4$ is lucky graph with $\eta(\mathrm{ATS_n}) = \Delta(\mathrm{ATS_n}) - 1$

Proof:

Let $\mathrm{f}: \mathrm{V}(\mathrm{ATS_n}) \rightarrow \{1,2\}$ be defined by

$$\mathrm{f}(\mathrm{u_i}) = \begin{cases} 1 & \text{odd i}, \mathrm{i} \neq 3, \mathrm{i} = \mathrm{n} \\ 2 & \text{even i}, \mathrm{i} = 3, \mathrm{i} \neq \mathrm{n} \end{cases}$$

$$\mathrm{f}(\mathrm{v_i}) = 2$$

$$\mathrm{s}(\mathrm{u_1}) = 4$$

$$\mathrm{s}(\mathrm{u_n}) = 3$$

$$\mathrm{s}(\mathrm{u_i}) = \begin{cases} 4 & \text{even i} > 4, i \neq n \\ 5 & \text{even i} \leq 4, \mathrm{i} = \mathrm{n} - 1 \\ 6 & \text{odd i} > 1, i < n - 1 \end{cases}$$

$$\mathrm{s}(\mathrm{v_i}) = 2 \text{ for } \mathrm{i} = \frac{\mathrm{n}}{2}$$

$$\mathrm{s}(\mathrm{v_2}) = 4$$

$$\mathrm{s}(\mathrm{v_i}) = 3 \text{ for } \mathrm{i} \neq \frac{\mathrm{n}}{2}, \mathrm{i} \neq 2$$

Therefore $\mathrm{ATS_n}$ with $\mathrm{n} > 4$ is lucky graph with $\eta(\mathrm{ATS_n}) = \Delta(\mathrm{ATS_n}) - 1$ refer figure 7.

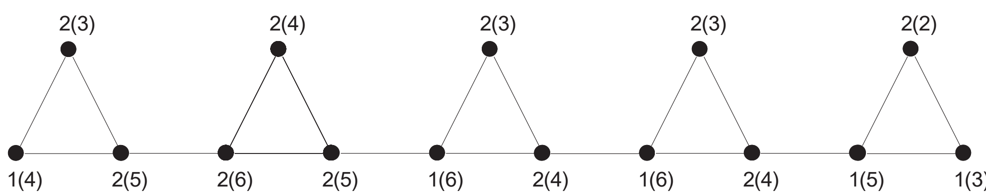

FIGURE 7. Lucky alternate triangular snake graph ATS_{10}

Theorem 4

The double alternate triangular snake $\mathrm{DATS_n}$ with $\mathrm{n} > 2$ is lucky graph with $\eta(\mathrm{DATS_n}) = \frac{\Delta(\mathrm{DATS_n})}{2}$.

Proof:

Let $\mathrm{f}: \mathrm{V}(\mathrm{DATS_n}) \rightarrow \{1,2\}$ be defined by

$$\mathrm{f}(\mathrm{u_i}) = \begin{cases} 1 & \text{odd i}, \mathrm{i} = 2 \\ 2 & \text{even i} > 2 \end{cases}$$

$$\mathrm{f}(\mathrm{v_i}) = 1$$

$$\mathrm{f}(\mathrm{w_i}) = 1$$

$$\mathrm{s}(\mathrm{u_1}) = 3$$

$$\mathrm{s}(\mathrm{u_n}) = 3$$

$$s(u_i) = \begin{cases} 4 & \text{even } i < n \\ 5 & i = 3 \\ 6 & \text{odd } i > 3 \end{cases}$$

$$s(v_i) = \begin{cases} 2 & i = 1 \\ 3 & i > 1 \end{cases}$$

$$s(w_i) = \begin{cases} 2 & i = 1 \\ 3 & i > 1 \end{cases}$$

Therefore $DATS_n$ with $n > 2$ is lucky graph with $\eta(DATS_n) = \frac{\Delta(DATS_n)}{2}$ refer figure 8.

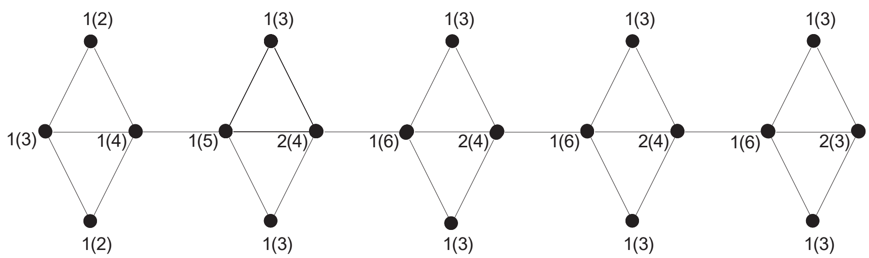

FIGURE 8. Lucky double alternate triangular snake graph DTS_{10}

Theorem 5

The triangular snake TS_n with $n > 2$ is proper lucky graph with $\eta_p(TS_n) = \Delta(TS_n) - 1$.

Proof:

Let $f: V(TS_n) \rightarrow \{1,2,3\}$ be defined by

$$f(u_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

$$f(v_i) = 3$$

$$s(u_1) = 5$$

$$s(u_n) = \begin{cases} 4 & \text{even } n \\ 5 & \text{odd } n \end{cases}$$

$$s(u_i) = \begin{cases} 8 & \text{even } i < n \\ 10 & \text{odd } i > 1, i < n \end{cases}$$

$$s(v_i) = 3$$

Therefore TS_n with $n > 2$ is proper lucky graph with $\eta_p(TS_n) = \Delta(TS_n) - 1$ refer figure 9.

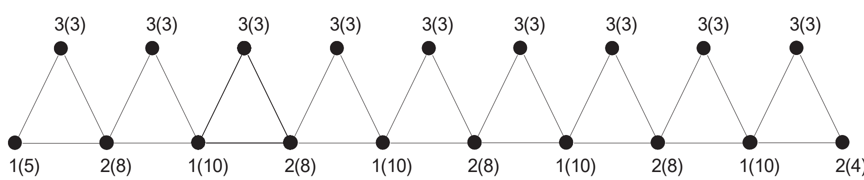

FIGURE 9. Proper lucky triangular snake graph TS_{10}

Theorem 6

The double triangular snake DTS_n with $n > 2$ is proper lucky graph with $\eta_p(DTS_n) = \frac{\Delta(DTS_n)}{2}$.

Proof:

Let $f: V(DTS_n) \rightarrow \{1,2,3\}$ be defined by

$$f(u_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

$$f(v_i) = 3$$

$$f(w_i) = 3$$

$$s(u_1) = 8$$

$$s(u_n) = \begin{cases} 7 & \text{even n} \\ 8 & \text{odd n} \end{cases}$$
$$s(u_i) = \begin{cases} 14 & \text{even i} < n \\ 16 & \text{odd i} > 1, i < n \end{cases}$$
$$s(v_i) = 3$$
$$s(w_i) = 3$$

Therefore DTS_n with $n > 2$ is proper lucky graph with $\eta_p(DTS_n) = \frac{\Delta(DTS_n)}{2}$ refer figure 10.

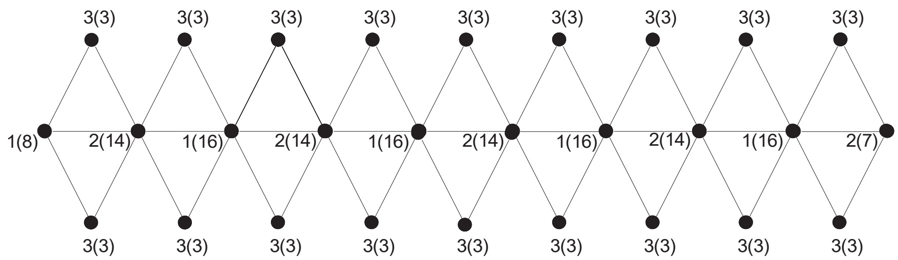

FIGURE 10. Proper lucky double triangular snake graph DTS_{10}

Theorem 7

The alternate triangular snake ATS_n with $n > 4$ is proper lucky graph with $\eta_p(ATS_n) = \Delta(ATS_n) + 1$.

Proof:

Let $f: V(ATS_n) \to \{1,2,3,4\}$ be defined by

$$f(u_i) = \begin{cases} 1 & \text{odd i} \\ 2 & \text{even i}, i \neq n \\ 4 & i = n \end{cases}$$
$$f(v_i) = 3$$
$$s(u_1) = 4$$
$$s(u_{n-1}) = 8$$
$$s(u_n) = 5$$
$$s(u_i) = \begin{cases} 5 & \text{odd i} > 1, i < n-1 \\ 7 & \text{even i} < n \end{cases}$$
$$s(v_i) = \begin{cases} 3 & i \neq n \\ 6 & i = n \end{cases}$$

Therefore ATS_n with $n > 4$ is proper lucky graph with $\eta_p(ATS_n) = \Delta(ATS_n) + 1$ refer figure 11.

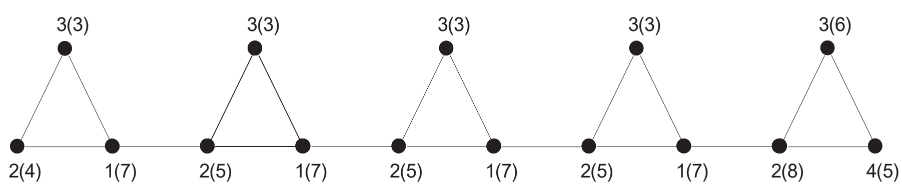

FIGURE 11. Proper lucky alternate triangular snake graph ATS_{10}

Theorem 8

The double alternate triangular snake $DATS_n$ with $n > 2$ is proper lucky labeling with $\eta_p(DATS_n) = \Delta(DATS_n) - 1$.

Proof:

Let $f: V(DATS_n) \to \{1,2,3\}$ be defined by

$$f(u_i) = \begin{cases} 1 & \text{even i} < n \\ 2 & \text{odd i} \\ 3 & i = n \end{cases}$$

$$f(v_i) = \begin{cases} 1 & i = \frac{n}{2} \\ 3 & i < \frac{n}{2} \end{cases}$$

$$f(w_i) = \begin{cases} 1 & i = \frac{n}{2} \\ 3 & i < \frac{n}{2} \end{cases}$$

$$s(u_1) = 7$$

$$s(u_{n-1}) = 6$$

$$s(u_n) = 4$$

$$s(u_i) = \begin{cases} 8 & \text{odd } i > 1, i < n-1 \\ 10 & \text{even } i < n \end{cases}$$

$$s(v_i) = \begin{cases} 3 & i < \frac{n}{2} \\ 5 & i = \frac{n}{2} \end{cases}$$

$$s(w_i) = \begin{cases} 3 & i < \frac{n}{2} \\ 5 & i = \frac{n}{2} \end{cases}$$

Therefore $DATS_n$ with $n > 2$ is proper lucky graph with $\eta_p(DATS_n) = \Delta(DATS_n) - 1$ refer figure 12.

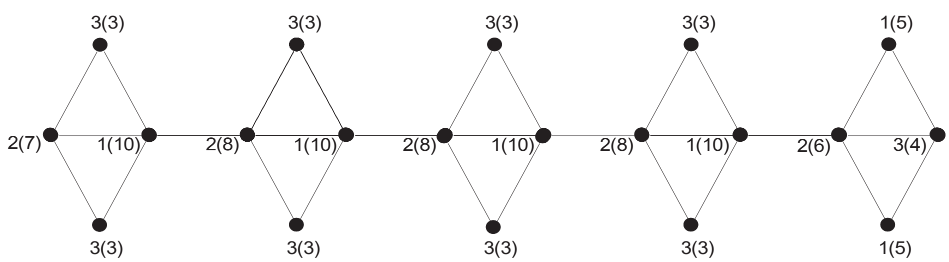

FIGURE 12. Proper lucky double alternate triangular snake graph $DATS_{10}$

CONCLUSION

We showed triangular, alternate triangular, double triangular and double alternate triangular snake graph are proper lucky and lucky. Also we investigated the proper lucky number and lucky number.

REFERENCES

1. Ahadi A, Dehghan A, Kazemi M and Mollaahmadi E. Information processing letters, **112,** 109-112 (2012).
2. Akbari S, Ghanbari M, Manaviyat R and Zare S. Graphs and combinatorics, **29**, 157-163 (2013).
3. Badr E M and Abdel-aal M E. Odd International Journal of Soft Computing, Mathematics and Control (IJSCMC) **2,** (2013).
4. Gallian Joseph. Electron J Combin **DS6** (2019).
5. Karoński M, Łuczak T and Thomason A. Journal of Combinatorial Theory B, **91**, 151-157 (2004).
6. Kumar T V Sateesh and S. Meenakshi. Lucky and Proper Lucky Labeling of Quadrilateral Snake Graphs. *IOP Conference Series: Materials Science and Engineering*. **1085**. (2021). doi: 10.1088/1757-899X/1085/1/012039.
7. Kumar TV Sateesh and S. Meenakshi. Proper Lucky Labeling of Graph. *Proceedings of First International Conference on Mathematical Modeling and Computational Science: ICMMCS* (2020), Springer Nature.
8. Rosa. In Theory of Graphs (Rome: Int. Symp.), 349-355 (1996).
9. Sandhya S S, Somasundaram S and Anusa S. International Journal of Contemporary Mathematical Sciences, **9**, 667-676 (2014).
10. Senthurpriya S and Meenakshi S. Journal of Recent Technology and Engineering (IJRTE,) **8**, 20-23 (2019).