

MAXIMUM MATCHING IN HILBERT CURVE

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ABSTRACT. The goal of this paper analyses the fractal Graphs in Fractal Antenna like Hilbert Fractal Graph which is 2- dimensional Fractal Graph. Fractal Antenna which is the most famous and worth Structure of Antenna when compared to other model of Antennas. In this paper discuss the History, development, Construction, Raw Materials and Advantages of Fractal Antenna. Here this paper shows the structures, properties and calculating the number of Vertices and Edges for such a fractal Graph. The concept of Matching is also major part of this paper. It finds the constant ratio of increasing number of vertices in the every level Iteration. There are two methods are used to find the Maximum Matching Cardinality in this paper. The methods are Iterative Function method and Mathematical Induction Method. These methods give the relationship between the cardinality of matching and the number of vertices increasing in the corresponding Iteration which is implemented by the theorem. Evaluation is shown as in the tabulated format.

1. INTRODUCTION

Fractals display everywhere in nature. It takes the core place of our lives. Most part of the world creation is done by God should projects the Structure of Fractals like leaves, clouds, mountains, the way of rivers, landscapes and so on. Fractal Graph is a major part of the Study of Graph Theory in Mathematics [4]. It has the surprising number of applications in Engineering, Medical Science, and

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Biological Science and so on. Fractal is derived for the word "Fractus" which is Latin word by Mathematician Beniot Mandelbrot. Fractals are compound pattern with self-similarity at every range [9]. Fractal Geometry has many precious equations for solving the complex device than regular Geometry. Fractal dimension has Hausdroff Dimension which is better than topological dimension. Its ratio gives a statistical index of complex pattern changes with the scale for measuring. This paper shows the basic knowledge of Fractals and their structures and properties with that in mind, it analyse with the main topic of the notes in matching. Here to analyse some of the famous Fractal Graph like Cantor Set and Hilbert Curve has a constant ratio of Maximum Matching Cardinality in all the Iterations.

2. MATCHING

It is a one of the major part of the Graph Theory. It plays a vital role in natural life. It has many application in real life, Science, Medical and Technology. It is nothing but a collection of non-adjacent edges from the given graph [1]. This set of edges does not share a common vertex between them. The set of vertices included in the Matching set is called M -saturated [2]. If $uv \in M$, u and v are called M -saturated. Otherwise the set of vertices whose are not included in the Matching set is unsaturated. Matching Cardinality is the number of edges included in the set of Matching [10]. It has three types. They are:

- (1) Maximum Matching
- (2) Maximal Matching
- (3) Perfect Matching

2.1. Maximum Matching. It has the maximum number of non-touching edges collected from the given graph. It is the highest collection of non-adjacent edges. It means that it is not possible to add another edge in the matching set. Maximum Matching Cardinality is the counting of edges in the Maximum Matching Set. It is denoted by $M(G)$.

2.2. Maximal Matching. It is the minimum number of collection of non-touching edges which is covered and adjacent with all other edges of the given graph. It is not possible to eliminate any edge in the collection of Maximal Matching set. Maximal Matching Cardinality is the counting of edges in the given Maximal Matching Set. It is denoted by $M'(G)$ [8].

2.3. Perfect Matching. This collection of non-adjacent edges in the given graph covers all the vertices in the given graph. In other words, in the given graph, all the vertices are included in the Matching set. All the vertices are M -Saturated. This Matching Set Cardinality is denoted by $P(G)$.

3. FRACTAL ANTENNA

3.1. History.

It is a Fractal Design consists of self-similar properties. Dr.Nathan Cohen discovered Fractal Antennas in 1988 which is accepted over the entire world. Self-similarity fractal antenna design gives highest interpretation, effective length and variations when compared to the other antenna element available in the world [5]. It can transmit electromagnetic radiation over the total surface area. It can be built by Metamaterial. Here to analyse about Hilbert Curve Antenna. This is the best example of Fractal Antenna Structure.

3.2. Metamaterial.

Fractal Antenna is built by Metamaterial [6]. It is not created in Natural rising materials. It can be made from multiple elements form the amalgamated stuff such as metals and plastics. The substances are arranged in iterating patterns at small scales than the wavelength of cautions authority. Metamaterial is not derived from the properties of base materials. It is derived their properties which are used to give newly generated designed structures. The properties of electromagnetic waves equal the precise properties of Metamaterial such as shape, geometry, orientation and arrangement which is used to the activities of impeding, consuming, enhancing or bending waves to achieve benefits.

3.3. Advantages.

- (1) Fractal Antenna accepted the most wideband and multiband antenna in the world. It is reduced the count of antenna needed and the cost of antenna. This single antenna only enables to meet many wireless needs.
- (2) It is used to minimize their size and weight. These smaller and less presuming factors provided maximum design flexibility, best attractiveness and small feasibility interactions with the other antenna and electrical.
- (3) Fractal Antenna provides small, weightless solution of Antenna which is easier to install and consuming mechanical support.

- (4) Fractal Antenna consumes the components whatever is needed to create than the traditional design to achieve the same work. It is used to reduce the failure components of antenna and cost of materials consuming.
- (5) Fractal Antenna gives better coverage area, reduce number of antenna needed.

3.4. Hilbert Curve.

Hilbert Curve Antenna Structure is the most famous and worth of Fractal Antenna. It was discovered by German Mathematician David Hilbert. It is a continuous fractal space filling curve. Space filling Curve is a curve whose scale contains two dimensional units square. Obviously Hilbert curve is 2 - dimensional Hausdroff unit square. It is a graph is a compact set which is homeomorphic to the closed unit interval. Geometrically, it is an open path length with two degree regular graph. H_n is the n dimensional Hilbert space [8]. The length of H_n is

$$(3.1) \quad 2^n - \frac{1}{2^n},$$

where n is the number of Iteration of the limiting curve. It is bounded by a square with finite surface area. Here analyse the construction of Hilbert curve, properties and finding Matching Cardinality in all Iteration.

3.5. Maximum Matching In Hilbert Curve.

Here to find the Maximum Matching in all the Iteration of Hilbert Curve. Hilbert Curve is a path of odd length with even number of vertices. Maximum Matching Set started at an initial edge of the graph and then it consists of alternative edges from the graph. This paper concludes that Maximum Matching Cardinality is equal to the half of the number of vertices in the corresponding Iteration. It will be proved in the following Iteration of Hilbert curve. It derived the General Formulae for Maximum Matching Cardinality by using vertices. It can be tabulated in table 1.

Iteration 1

In this Iteration of Hilbert Curve consists of 4 numbers of Vertices. There are two non-adjacent edges taken from the graph for Maximum Matching Set [8]

$$M(G) = \frac{V(G)}{2} = 4/2 = 2.$$

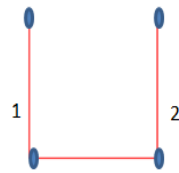


FIGURE 1. Maximum Matching in First Iteration of Hilbert Curve

Iteration 2

In the Second Iteration of Hilbert Curve consists of 16 numbers of vertices in the Graph. There are eight non adjacent edges selected for Maximum Matching Set

$$M(G) = \frac{V(G)}{2} = 16 / 2 = 8.$$

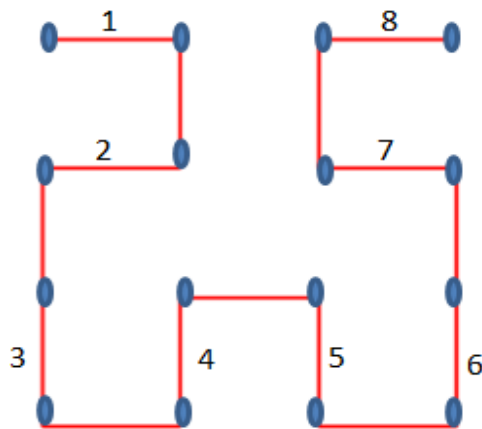


FIGURE 2. Maximum Matching in Second Iteration of Hilbert Curve

Iteration 3

In the third Iteration of Hilbert Curve consists 64 numbers of vertices. Half of the count of vertices i.e. 32 numbers of edges are selected for the Maximum Matching Cardinality

$$M(G) = \frac{V(G)}{2} = 64 / 2 = 32.$$

Result

In the same process will continue in all Iteration, it will conclude the General Formulae for Maximum Matching Cardinality in all Iteration

(3.2) Maximum Matching Cardinality = $M(G) = \frac{\text{Number of Vertices}}{2} = \frac{V(G)}{2}.$

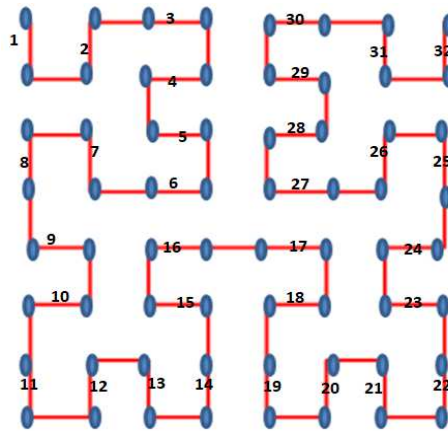


FIGURE 3. Maximum Matching in Third Iteration of Hilbert Curve

Theorem also derived for Maximum Matching Cardinality in all the Iteration of Hilbert Curve. It gives the relationship between number of vertices and Maximum Matching Cardinality for all Iteration of Hilbert Curve.

Theorem 3.1. *In all the Iteration of Hilbert Curve consists of 2^n number of vertices has 2^{n-1} number of edges in Matching Cardinality set. Apply Mathematical Induction Method to prove this theorem. Here n is started at 2 and it is increased by two (i.e., an even number) in the next upcoming Iteration.*

Proof. Let to prove the theorem by Induction method [3].

Step 1: This theorem can be proved for an Initial Iteration. Here $n = 2$.

Number of Vertices = $2^n = 2^2 = 4$.

Matching Cardinality = $2^{n-1} = 2^1 = 2$.

Theorem is proved.

Step 2: Let us assume that the theorem is true for all $n = k$.

That means, Hilbert Curve consists of 2^k number of vertices has 2^{k-1} number of edges in Matching Cardinality set in the corresponding Iteration.


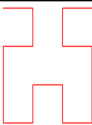
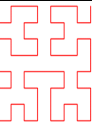
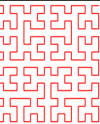
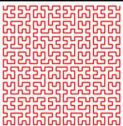
Step 3: Let us prove that the theorem is true for $n = k + 1$.

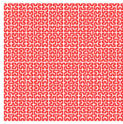
Here no of vertices is 2^{k+1} . Let us prove that in this Iteration has 2^k number of edges. It can be splitted into two parts (i.e.,) 2^k and 2. In the previous step assumes that 2^k number of vertices has 2^{k-1} number of edges in Matching Cardinality set. Another two edges are combined with Permutation method which is the

best method in Combinatorics [7]. It provides $2^{k-1} * 2! = 2^k$ number of vertices in the corresponding Iteration. The theorem is proved for all n where n is an even number.

It can be tabulated in table.1.

Table 1: Calculation of Maximal Matching Cardinality

Iteration	n	Figure	No. of Vertices 2^n	Maximum Matching Cardinality 2^{n-1}
I	2		4	2
II	4		16	8
III	6		64	32
IV	8		256	128
V	10		1024	512

VI	12		4096	2048
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4. CONCLUSION

In this paper apply the concept of Maximum Matching in the Fractal Graph like Hilbert Curve which is the famous structure of Fractal Antenna. Here it analyse the Structure, Properties, Calculation of number of Vertices as well as edges and at last give two General Formula for the calculation of Maximum Matching Cardinality in those Fractal Graph by the method of Iterative Function and Mathematical Induction Method. In the future work, let us derive the General Formula for Maximal Matching Cardinality in Various Fractal Antenna Curve like Peano Curve and Hilbert Curve. This is new study to combine the concepts of Matching and Fractal Graphs.

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