Computation of domination sets in Von Koch curve using C program

Conference Paper *in* AIP Conference Proceedings · November 2022

DOI: 10.1063/5.0109214

2 authors:

P. Tharaniya
Vels University

Gopalakrishnan Jayalalitha
VELS Institute of Science Technology&Advanced studies

Computation of domination sets in Von Koch curve using C program

Cite as: AIP Conference Proceedings **2516**, 210021 (2022); https://doi.org/10.1063/5.0109214
Published Online: 30 November 2022

P. Tharaniya and and G. Jayalalitha

Computation of Domination Sets in Von Koch Curve Using C Program

P.Tharaniya[1*,a)] and G.Jayalalitha[2,b)]

[1]*Research Scholar, Department of Mathematics, VELS Institute of Science Technology and Advanced Studies, Chennai, India,*

[2]*Professor, Department of Mathematics, VELS Institute of Science Technology and Advanced Studies, Chennai, India,*

* *Department of Mathematics, Kings Engineering College, Sriprembudur, India.*

[a)] *P.Tharaniya : tharamanisharmi@gmail.com,*

[b)] *g.jayalalitha.sbs@velsuniv.ac.in*

Corresponding Author : tharamanisharmi@gmail.com

Keywords: Graph Theory, Fractal Graph, Domination, Classical Geometry
AMS Classification key: 05C30, 05C69

Abstract. The aim of this paper is to calculate the Dominating set in the Famous Fractal Graph Von Koch Curve which is created by Hedge Von Koch. Domination is the most development area in Graph Theory. Its application plays a vital role in our natural lives. Here this paper analyzes the characteristics, construction, properties, nature, implementation, Iteration and uses of the famous Fractal Graph like Von Koch Curve. Here to explain how the Fractal Graph can be constructed and implemented. It shows that each Iteration of Koch Curve has some constant ratio for implementation of Vertices and Edges in all Iteration. By using Proof of Iterative methods, it evaluates the Dominating Set Cardinality of Von Koch Curve. The calculation of dominating sets shows the common ratio for all the Iteration of Von Koch Curve. Application of this calculation also discussed in our life. Also the algorithm of the calculation of Dominating Set has written using C Programme.

1. INTRODUCTION

Graph Theory [1] is the branch of mathematics which is studying about Graphs in detail. A Graph is made by the set of vertices (called nodes or points) with the set of edges (called links or lines). Graph Theory applications and its models involve connection with real world on the hand and also expressed in lucid and animated graphical terms.[2] It also deals with the concept of Graphical Drawing and Presentation of Graphs. It gives the algorithm which is used to planarity testing and drawing.

2. DOMINATION IN GRAPHS

Domination in Graphs is advanced research tool of Graph Theory [3]. It has well growth and development area in Graph Theory such as discrete optimization, combinatorial problem and classical algebraic problem for the last 30 years. It was studied from 1950 onwards. Richard Karp proved the set cover problem be NP- complete [4]. This problem implemented a straight forward vertex to set and edge to non-distinct intersection bisection between two problems. Dominating set [5] has much practical interest such as wireless networking which is used to find efficient route with ad-hoc mobile networks, document summarization and designing secure system for electrical grids. Dark nodes of Figure 1 show the vertices of dominating set.

2nd International Conference on Mathematical Techniques and Applications
AIP Conf. Proc. 2516, 210021-1–210021-7; https://doi.org/10.1063/5.0109214
Published by AIP Publishing. 978-0-7354-4234-4/$30.00

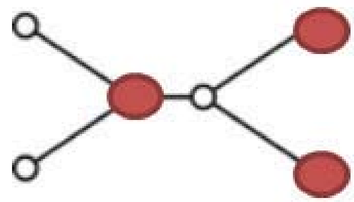

FIGURE 1: Examples of Dominating Set

2.1 Dominating Set [6]

A Graph G (V, E) consists the set of Vertices V and the set of edges has a subset S of dominating set if every vertex in S is non adjacent itself and every vertex in S which is adjacent with all the vertices not in S.

2.2 Maximum Dominating Set [7]

A dominating set is called Maximum dominating set if it has the collection of maximum possible non-adjacent vertices in the given graph. The count of vertices is called as cardinality. The Example is given below in the Figure 2. Here the dark brown nodes are selected for Maximum Dominating set.

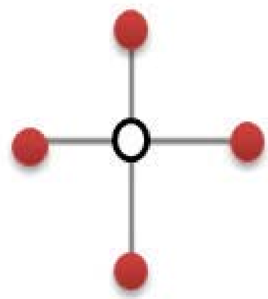

FIGURE 2: Example of Maximum Dominating Set

2.3 Maximum Domination Cardinality [8]

Maximum Domination Cardinality is the collection of non-touching vertices in the graph. It is denoted by D (G). In Figure 2 shows Maximum Domination Cardinality is 4 of the graph.

2.4 Maximal Dominating Set

A subset S of V consists non-touching vertices and not possible to add more than one vertex in the dominating set. The dark Brown nodes in the below example figure 3 shows the vertices of Maximal Dominating Set.

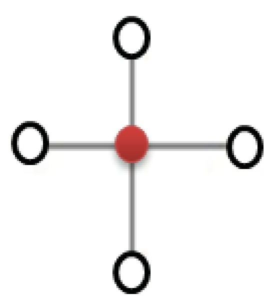

FIGURE 3: Example of Maximal Dominating Set

2.5 Maximal Dominating Cardinality

It has minimum number of possible collection of non-touching vertices in the given graph. It is denoted by D' (G). In Figure 3 shows Maximum Domination Cardinality is 1

2.6 Independent Set [9]

A set is called an independent set if no two vertices are adjacent in the set. The count of non-adjacent vertices is called an Independent Cardinality. It is denoted by i(G).

2.7 Independent Dominating Set

A set which has independent set as well as dominating set is called as Independent Dominating set. Obviously all dominating set are independent sets. [10]

2.8 Independent Domination Cardinality

Independent dominating set is obviously equal to Maximal Dominating set. Independent Domination Cardinality is the minimum number of possible collection of dominating sets.

3. FRACTAL GRAPHS

Fractal [11] is the self-similarity graphs which are fragmented geometric shape, it has subdivided into small parts, each of the part structure has seen in the whole of the shape. It has fine structured graph as well as Hausdroff dimension [12]. It was implemented by a Great Mathematician Beniot Mandelbrot [18]. The concept of Fractal Geometry gives unlimited way of analysing, measuring and protecting the natural appearance and circumstances. It is applied in many fields. Karl Weierstrass's explains non-Inherent property of being everywhere continuous and nowhere differentiable. In 1904, George Cantor dissatisfied his concept and explains the structures which have drawn by own hand geometric figures.

3.1 Von Koch Curve

Von Koch Curve was discovered by Hedge Von Koch in 1904[14]. It begins with topological dimension [15]. It has self-similarity for all iteration which is the earliest Fractal Graph. The unit interval straight line is the initial Iteration of Von Koch Curve. It has been divided into three parts with interval (0, 1/3), (1/3, 2/3) and (2/3, 1). Middle part has replaced by two sides of an equilateral triangle (without base) of the same length as the segment being removed. It is continued in all Iteration. Number of Vertices and Edges has increased in all Iteration at a constant ratio. It has an infinite length. Each Iteration of Koch Curve creates four times of line segments in the previous Iteration. Von Koch Curve [16] has 2^{2n} edges (n is an integer, n is increased by 1, started at 0) and $2^{2n}+1$ vertices in n^{th} Iteration. Dominating sets are calculated by the following formulae (1) and (2).

3.2 Maximum Dominating Set and Maximal Dominating Set in Von Koch Curve

In this paper analyses the calculation of Dominating set in famous Fractal Graph Von Koch Curve. It found a formula (1) & (2) for Maximal Dominating set and Minimal Dominating set for all iteration in common[13]. Also this calculation has implemented into C program.

$$\textbf{Maximum Dominating Cardinality} = \left|\frac{E}{2}\right| + 1 \quad (1)$$

$$\textbf{Maximal Dominating Cardinality} = \left\lfloor\frac{E}{3}\right\rfloor + 1 \quad (2)$$

First Three Iteration is discussed in the following steps. In n^{th} iteration, the general formulae have been derived by Iterative methods and also it can be implemented into C program. Dark nodes are dominating sets in the following figures 3,4,5,6,7,8,9.

3.2.1 Iteration I

In the first Iteration of Von Koch Curve has $E = 2^2=4$ edges. In the following figure 4 & 5 shows Maximal Dominating Cardinality and Maximum Dominating Cardinality of Von Koch Curve. Maximum Dominating nodes are coloured by black nodes in figure 4 and Maximal Dominating nodes are coloured by red nodes in figure 5.

$$\textbf{Maximum Dominating Cardinality} = \left|\frac{E}{2}\right| + 1 = \left|\frac{4}{2}\right| + 1 = 3$$

Maximal Dominating Cardinality $= \left\lfloor\frac{E}{3}\right\rfloor + 1 = \left\lfloor\frac{4}{3}\right\rfloor + 1 = 2$

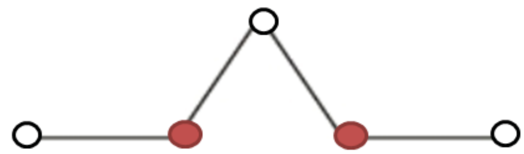

FIGURE 4: Maximal Dominating set of Von Koch Curve

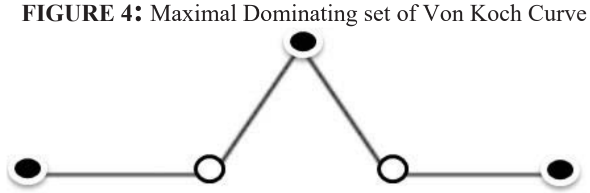

FIGURE 5: Maximum Dominating set of Von Koch Curve

3.2.2 Iteration II

In the second Iteration of Von Koch Curve has E (G) = 2^4=16 edges. In the following figure 6 & 7 shows Maximal Dominating Cardinality and Maximum Dominating Cardinality of Second Iteration of Von Koch Curve [17]. Maximum Dominating nodes are coloured by black nodes in figure 7 and Maximal Dominating nodes are denoted by red nodes in figure 6.

Maximum Dominating Cardinality $= \left|\frac{E}{2}\right| + 1 = \left|\frac{16}{2}\right| + 1 = 9$

Maximal Dominating Cardinality $= \left\lfloor\frac{E}{3}\right\rfloor + 1 = \left\lfloor\frac{16}{3}\right\rfloor + 1 = 6$

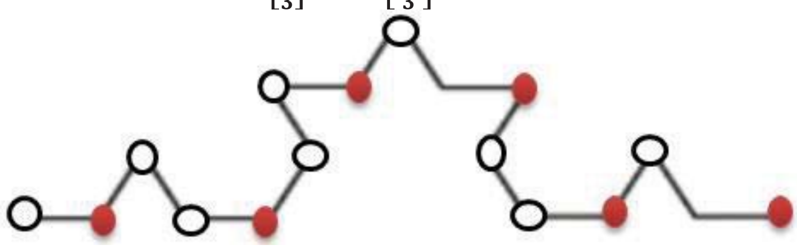

FIGURE 6: Maximal Dominating set of Second Iteration of Von Koch Curve

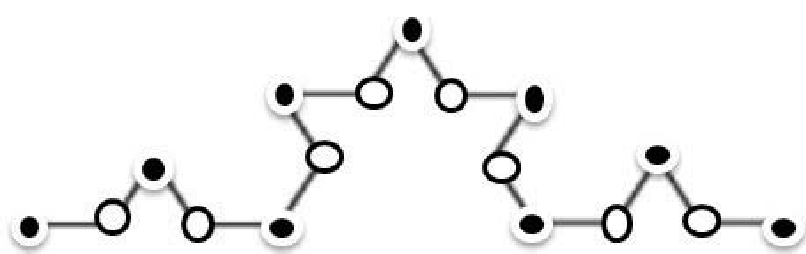

FIGURE 7: Maximum Dominating set of Second Iteration of Von Koch Curve

3.2.3 Iteration III

In the third Iteration of Von Koch Curve has E (G) = 2^6 =64 edges. In the following figure 8 & 9 shows Maximal Dominating Cardinality and Maximum Dominating Cardinality of Third Iteration of Von Koch Curve. Maximum Dominating nodes are colored by black nodes in figure 9 and Maximal Dominating nodes are colored by red nodes in figure 8.

Maximum Dominating Cardinality $= \left|\frac{E}{2}\right| + 1 = \left|\frac{64}{2}\right| + 1 = 32$

Maximal Dominating Cardinality $= \left\lfloor\frac{E}{3}\right\rfloor + 1 = \left\lfloor\frac{64}{3}\right\rfloor + 1 = 22$

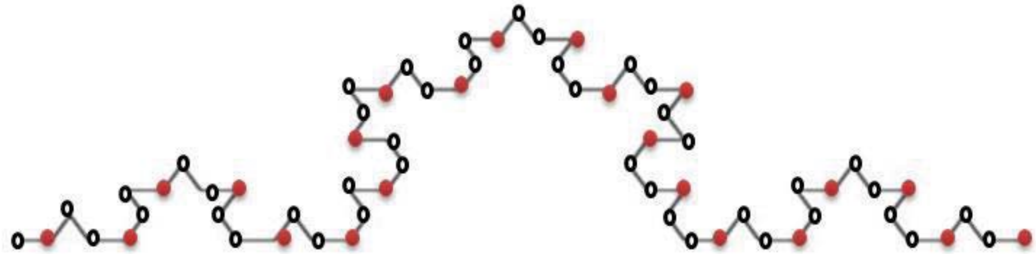

FIGURE 8: Maximal Matching of Third Iteration of Von Koch Curve

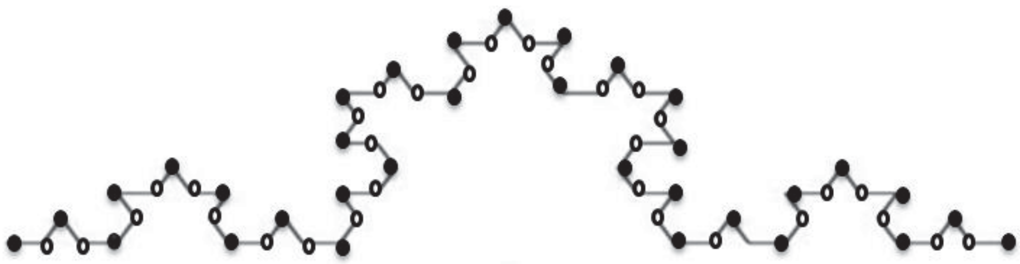

FIGURE 9: Maximum Matching of Third Iteration of Von Koch Curve

3.2.4 Iteration n

In the n^{th} Iteration of Von Koch Curve has 2^{2n} number of edges and 2^{2n} +1 number of vertices. (n is whole even number and increased by 2). Maximum Dominating Cardinality and Maximal Dominating Cardinality are evaluated by the following formulae in the n^{th} iteration.

$$\textbf{Maximum Dominating Cardinality} = \left\lfloor \frac{2^{2n}}{2} \right\rfloor + 1$$

$$\textbf{Maximal Dominating Cardinality} = \left\lfloor \frac{2^{2n}}{3} \right\rfloor + 1$$

4. APPLICATION

The vertices of Dominating Sets in Von Koch Curve has selected in constant ratio. The selection of non-touching Vertices in dominating sets is the main ethics of this paper. Domination sets has too much application in our real lives. This summary gives some application of Dominating Sets. The first example of Dominating set is Area Selection of Towers which is connecting many areas under network surveillance, the second example is Minimizing Manual Junction in Sanitary Pipeline for the construction of new model Apartment, the third example is Consuming travelling time for the postman who is distributing the letters never repeat the same area, the fourth example is Selecting the persons who has many friends are passing messages through Facebook, WhatsApp, Twitter, etc. to many people at a time, and the last example is the process of sending messages through a single person WhatsApp group contact peoples.

5. CONCLUSION

This paper explains about the common formulae for Maximal Dominating Set and Maximum Dominating Set in the Famous Fractal Graph like Von Koch Curve. It is examined by an Iteration Method. In the future work, the formula of Maximum Dominating Cardinality and Maximal Dominating Cardinality can be calculated in further fractal graph like Koch Snowflake, Sierpenski Arrow Head Curve etc. Highest Iteration of Fractal Graph are too complicated and unmeasured. This derived formula is used to give cardinality value in a simple way in the highest value of iteration.

REFERENCES

1. Nithya, R., Kamali, R., and Jayalalitha, G. Ramsey numbers in Sierpinski triangle. *International Journal of Pure and Applied Mathematics*, *116(*4), 967-975, (2017).

2. Nithya R., Kamali R. and Jayalalitha G., Ramsey numbers in Sierpinski Triangle *International Journal of Pure and Applied Mathematics*, 116(4), (2017).
3. Provata A., Almiirantis Y., Fractal Cantor Patterns in the sequence structure of DNA, *Fractals*, 8, 15-27, (2011)
4. Mahapatra, T., Ghorai, G. and Pal, M. Fuzzy fractional coloring of fuzzy graph with its application. *Journal of Ambient Intelligence and Humanized Computing*, 11(11), 5771-5784, (2020)
5. Hao F, Park D.S, Li S and Lee H.M. (2016), Mining λ-maximal cliques from a fuzzy graph. *Sustainability*, 8(6), 553.
6. Vivek V., Raich., Archana Gawande. and Rakesh Kumar Triapath., Fuzzy Matrix Theory and its Application for Recognizing the Qualities of Effective Teacher, *International Journal of Fuzzy Mathematics and Systems*, 1(1), 111-120, (2011).
7. Bandt, C., Graf.S and Zähle, M. Fractal geometry and stochastics ,1. Birkhäuser, (1995).
8. Barnsely M.F. Fractals Everywhere, 2nd Edition, Academic Press, Boston,1993.
9. Falconer K.J , The geometry of Fractal Sets, Cambridge University Press, Cambridge.,1985.
10. Ore,O, Theory of Graphs. American Mathematical Society Colloquium Publications. *Amer. Math. Soc., Providence*, *38,* (1962)
11. Deo, Narsingh. Graph theory with applications to engineering and computer science. Courier Dover Publications, 2017.
12. Gyárfás, A. and Lehel, J. On-line and first fit coloring of graphs. *Journal of Graph theory*, 12(2), 217-227, (1988).
13. Tharaniya and Jayalalitha G, Maximum Matching in Fractal Graphs, The International Journal of Analytical and Experimental Modal Analysis, XI(X), 138-143(2019).
14. Kenneth Falconer, Fractal Geometry Mathematical Foundation and Applications, Third Edition, WILEY, 2014.
15. Mandelbrot, Benoit B., and Benoit B. Mandelbrot. *The fractal geometry of nature*, 1,New York: WH freeman, 1982.
16. Blackledge, Jonathan M., A. K. Evans, and Martin J. Turner. Fractal Geometry: Mathematical Methods, Algorithms, Applications. Elsevier, 2002.
17. West, Douglas Brent. Introduction to graph theory, 2, Upper Saddle River: Prentice hall, 2001.
18. Tharaniya P and Jayalalitha G, Maximal Matching in Fractal Graphs and Its Application, *International Journal of Pharmaceutical Research,* 13(1),(2021).

Appendix

Calculation of Cardinality of Dominating set is calculated in C program.

```
#include<stdio.h>
#include<math.h>
#include<conio.h>
Void main()
{
int i,E;
clrscr();
printf(" Enter the Iteration of Von Koch Curve:\n");
scanf("%d",&i);
if(i>=0)
{
E=pow(2,2*i);
printf("\n No. of Vertices:%d",E+1);
printf("\n No. of edges:%d",E);
printf("\n Maximum Dominating Cardinality:%d",(E/2)+1);
printf("\n Maximal Dominating Cardinality:%d",(E/3)+1);
```

```
}
else
{
Printf("\n Invalid Iteration number. Enter only positive integers");
}
getch();
}
```