Lucky labeling of jelly fish graph J(m, n), cocktail party graph CPk and crown graph Cn*

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Lucky Labeling Of Jelly Fish Graph $J(m, n)$, Cocktail Party Graph CP_k And Crown Graph C_n^*

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Abstract. The Lucky labeling graph is the totals of labels over neighbouring nodes in the graph are not the identical, and if a node is an isolated then the sum is zero. The lucky number of G is indicated by $\eta(G)$. The graphs which admit the above condition are called lucky graph. Here we proved that Jelly fish graph $J(m, n)$, cocktail party graph CP_k and crown graph $C_n^*$are lucky graph with their lucky number $\eta(G)$.

INTRODUCTION

Graph labeling is a broad subject in graph theory. It has a solid relation between the numbers and the graph's structure. Rosa first introduced it in 1967 [8]. Labeling is a function that assigns a number to a graph vertex, edge, or both, depending on the condition. Gallian [5] provided a lively overview of graph labeling. Ahadi (2012) [1] investigated lucky labeling for three colorable graphs. Ahadi (2012) [1] investigated lucky labeling for three colorable graphs, which means that the total of labels over neighboring nodes in the graph are not identical, and if a node is isolated then the sum is zero. The graph which fulfills the lucky labeling conditions are called lucky graph. The Lucky number of G is indicated by η(G).

Sriram computed 1 -Near Mean Cordial Labeling of Jelly FishJ(m, n) graphs in 2015 [9]. Gregory computed Clique partitions of the cocktail party graph [6]. Amit H. Rokad derived Cordial Labeling of jelly fish Graphs in 2017 [3]. Daoud computed Cocktail Party Graph and Crown Graph's Complexity [4]. K.Thirusangu computed ZumkellerLabeling of Jelly Fish Graph in 2019 [10]. AleksandarJurisic computed 1-Homogeneous Graphs with Cocktail Party [7].

PRELIMINARIES

JELLY FISH

Jelly fish graph $J(m, n)$ is derive from $m + n + 4$ vertices i.e., $V\big(J(m, n)\big) = \{u, v, x, y\} \cup \{u_1, u_2, \dots, u_m\} \cup \{v_1, v_2, \dots, v_n\}$ and $m + n + 5$ edges i.e., $E\big(J(m, n)\big) = \{(u, x), (u, y), (v, x), (v, y), (x, y)\} \cup \{u, u_i; 1 \leq i \leq m\} \cup \{v, v_i; 1 \leq i \leq n\}$ refer figure 1 [11].

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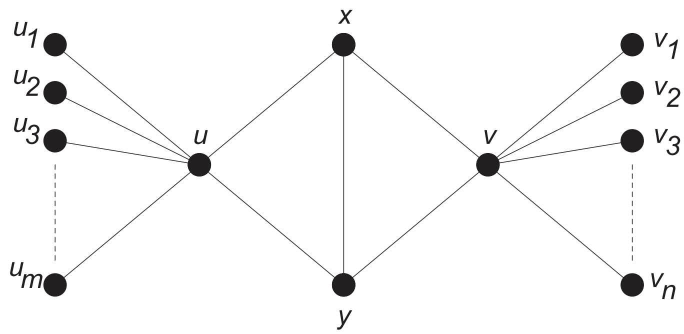

Figure 1: Jelly fish graph $J(m,n)$

COCKTAIL PARTY GROUP

A cocktail party graph CP_k, where k is the $2n$ vertices i.e., $V(CP_k) = \{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$ and $n + n^2$ edges i.e., $E(CP_k) = \{u_i, u_j;\ 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\} \cup \{(v_i, v_j;\ 1 \leq i \leq n, 1 \leq j \leq n, i \neq j)\} \cup \{u_i, v_i; i \neq j\}$. It is also known as hyper octahedral graph or Robert's graph refer figure 2(a) [4].

CROWN GRAPH

A crown graph C_n^* is obtained from $2n$ vertices i.e., $V(C_n^*) = \{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$ and $n(n-1)$ edges ie.,$EC_n^* = \{u_i, v_j; 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$ refer figure 2(b) [4].

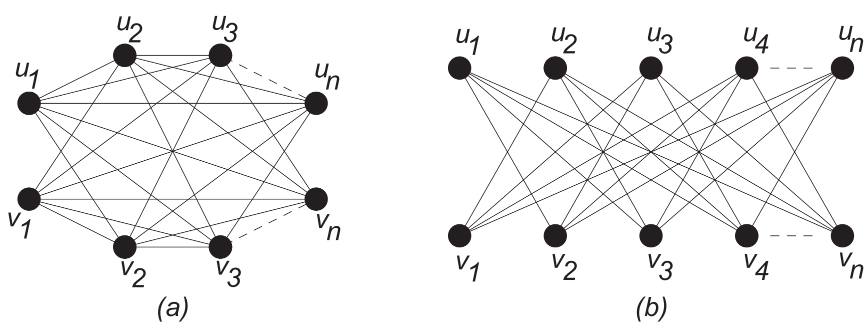

Figure 2: (a) cocktail party graph CP_k; (b) crown graph C_n^*

MAIN RESULTS

THEOREM 1

The jelly fish graph $J(m,n)$ is a lucky graph with lucky number $\eta(J(m,n)) = 2$

Proof

Let $f(J(m,n)) \to \{1,2\}$ for Jelly fish graph $J(m,n)$ with $m+n+4$ vertices i.e., $V(J(m,n)) = \{u, v, x, y\} \cup \{u_1, u_2, \dots, u_m\} \cup \{v_1, v_2, \dots, v_n\}$ and $m+n+5$ edges i.e., $E(J(m,n)) = \{(u,x), (u,y), (v,x), (v,y), (x,y)\} \cup \{uu_i; 1 \leq i \leq m\} \cup \{vv_i; 1 \leq i \leq n\}$ be defined by

Case(i) $m = n = 1$

$$f(x) = 1$$
$$f(u) = f(v) = f(y) = 2$$
$$f(u_m) = f(v_n) = 1$$

We obtain the sum of neighbor vertices as

$$s(x) = 6$$
$$s(y) = 5$$
$$s(u) = s(v) = 4$$
$$s(u_m) = s(v_n) = 2$$

Here a total of adjoining vertices isn't equal.. So jelly fish graph $J(m,n)$ is a lucky graph with lucky number $\eta\big(J(m,n)\big) = 2$when $n = m = 1$.

Case(ii) $n > 1, m > 1$

$$f(x) = f(u) = f(v) = 1$$
$$f(y) = 2$$
$$f(u_m) = f(v_n) = 1$$

We obtain the sum of neighbor vertices as

$$s(x) = 4$$
$$s(y) = 3$$
$$s(u) = 3 + m$$
$$s(v) = 3 + n$$
$$s(u_m) = s(v_n) = 1$$

Here a total of adjoining vertices isn't equal.. So jelly fish graph $J(m,n)$ is a lucky graph with lucky number $\eta\big(J(m,n)\big) = 2$when $n > 1, m > 1$.

Hence from case(i) and (ii), it is obvious the jelly fish graph $J(m,n)$ is a lucky graph with lucky number $\eta\big(J(m,n)\big) = 2$.

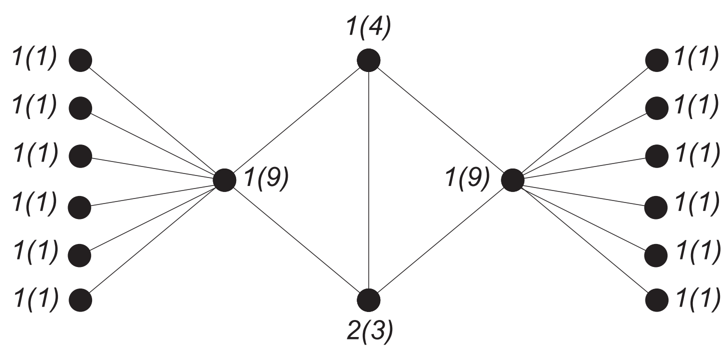

Figure 3: Lucky jelly fish graph $J(6,6)$

Illustration 1

The lucky jelly fish graph $J(6,6)$ with lucky number 2 is shown in fig 3, i.e., $\eta\big(J(6,6)\big) = 2$.

Corollary 1

The lucky labeling of jelly fish graph is one less than minimum degree i.e.,$\eta\big(J(m,n)\big) = \delta\big(J(m,n)\big) - 1$

Corollary 2

The lucky labeling of jelly fish graph is n subtracted from maximum degree i.e.,$\eta\big(J(m,n)\big) = \Delta\big(J(m,n)\big) - n$

Theorem 2

The cocktail party graph CP_k is lucky graph with lucky number $\eta(CP_k) = 2$.

Proof

Let $f(CP_k) \to \{1,2\}$ for cocktail party graph CP_k, where k is the $2n$ vertices i.e., $V(CP_k) = \{u_i;\ 1 \le i \le n\} \cup \{v_i;\ 1 \le i \le n\}$ and $n + n^2$ edges i.e., $E(CP_k) = \{u_i u_j;\ 1 \le i \le n, 1 \le j \le n, i \ne j\} \cup \{(v_i v_j;\ 1 \le i \le n, 1 \le j \le n, i \ne j)\} \cup \{u_i v_i; i \ne j\}$ be defined by

Case(i) for odd n

$$f(u_i) = f(v_i) = \begin{cases} 1 & odd\ i \\ 2 & even\ i \end{cases}$$

We obtain the sum of neighbor vertices as

$$s(u_i) = s(v_i) = \begin{cases} 3n - 3 & odd\ i \\ 3(n-1) - 2 & even\ i \end{cases}$$

Here a total of adjoining vertices isn't equal.. So cocktail party graph CP_k is a lucky graph with lucky number $\eta(CP_k) = 2$ for odd n.

Case(ii) for even n

$$f(u_i) = f(v_i) = \begin{cases} 1 & odd\ i \\ 2 & even\ i \end{cases}$$

We obtain the sum of neighbor vertices as

$$s(u_i) = s(v_i) = \begin{cases} 3n - 2 & odd\ i \\ 3n - 4 & even\ i \end{cases}$$

Here a total of adjoining vertices isn't equal.. So cocktail party graph CP_k is a lucky graph with lucky number $\eta(CP_k) = 2$ for even n.

Hence from case(i) and (ii), it is obvious the cocktail party graph CP_k is a lucky graph with lucky number $\eta(CP_k) = 2$

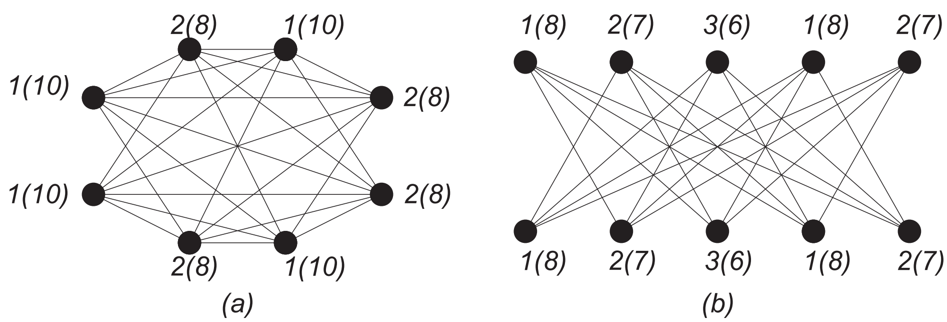

Figure 4: (a) Lucky cocktail party graph CP_8; (b) lucky crown graph C_5^*

Illustration 2

The lucky cocktail party graph CP_8 with lucky number 2 is shown in fig 4(a), i.e., $\eta(CP_8) = 2$.

Corollary 3

The lucky labeling of cocktail graph is minimum degree divide by $n - 1$ i.e.,$\eta(CP_k) = \frac{\delta(CP_k)}{n-1}$

Corollary 4

The lucky labeling of cocktail graph is maximum degree divide by $n - 1$ i.e.,$\eta(CP_k) = \frac{\Delta(CP_k)}{n-1}$

Theorem 3

The crown graph C_n^* is lucky graph with lucky number $\eta(C_n^*) = 2$when $n = 2$.

Proof

Let $f(C_n^*) \rightarrow \{1,2\}$ for crown graph C_n^* for $n = 2$ is obtained from $2n$ vertices i.e., $V(C_n^*) = \{u_i;\ 1 \leq i \leq n\} \cup \{v_i;\ 1 \leq i \leq n\}$ and $n(n-1)$ edges ie.,$EC_n^* = \{u_i, v_j; 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$ be defines as

$$f(u_i) = f(v_i) = \begin{cases} 1 & i = 1 \\ 2 & i = 2 \end{cases}$$

The sum of neighbor vertices as

$$s(u_i) = s(v_i) = \begin{cases} 1 & i = 2 \\ 2 & i = 1 \end{cases}$$

Here a total of adjoining vertices isn't equal.. So crown graph C_n^* is a lucky graph with lucky number $\eta(C_n^*) = 2$when $n = 2$.

Corollary 5

The lucky labeling of crown graph is 3 more than minimum degree minus n when $n = 2$ i.e.,$\eta(C_n^*) = 3 + \delta(C_n^*) - n$

Corollary 6

The lucky labeling of crown graph is 3 more than maximum degree minus $nn = 2$ i.e.,$\eta(C_n^*) = 3 + \Delta(C_n^*) - n$

Theorem 4

The crown graph C_n^* is lucky graph with lucky number $\eta(CP_k) = 3$when $n > 2$.

Proof

Let $f(C_n^*) \rightarrow \{1,2\}$ for crown graph C_n^* when $n > 2$ is obtained from $2n$ vertices i.e., $V(C_n^*) = \{u_i;\ 1 \leq i \leq n\} \cup \{v_i;\ 1 \leq i \leq n\}$ and $n(n-1)$ edges ie.,$EC_n^* = \{u_i, v_j; 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$ be defines as

$$f(u_i) = f(v_i) = \begin{cases} 1 & i = 3m - 2 \\ 2 & i = 3m - 1 \\ 3 & i = 3m \end{cases}$$

The sum of neighbor vertices as

Case(i) for $n = 3k$

$$s(u_i) = s(v_i) = \begin{cases} 2n - 1 & i = 3m - 2 \\ 2n - 2 & i = 3m - 1 \\ 2n - 3 & i = 3m \end{cases}$$

Case(ii) for $n \neq 3k$

$$s(u_i) = s(v_i) = \begin{cases} 2n - 2 & i = 3m - 2 \\ 2n - 3 & i = 3m - 1 \\ 2n - 4 & i = 3m \end{cases}$$

From case(i) and (ii) a total of adjoining vertices isn't equal.. So crown graph C_n^* is a lucky graph with lucky number $\eta(C_n^*) = 3$when $n > 2$.

Illustration 3

The lucky crown graph C_5^* with lucky number 2 is shown in fig 4(b), i.e., $\eta(C_5^*) = 3$.

Corollary 7

The lucky labeling of crown graph is 4 more than minimum degree minus n when $n > 2$ i.e.,$\eta(C_n^*) = 4 + \delta(C_n^*) - n$

Corollary 8

The lucky labeling of crown graph is 4 more than maximum degree minus $nn > 2$ i.e.,$\eta(C_n^*) = 4 + \Delta(C_n^*) - n$

CONCLUSION

The lucky number of the Jelly fish graph $J(m,n)$, cocktail party graph CP_k, and crown graph C_n^* is computed in this article. We also linked it to the minimum and maximum degree. We then extend this work to various graphs.

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