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Python based Approach for Modular Labeling in Switched Jewel Graphs

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***Abstract*— This research investigates the characteristics of Modular Multiplicative Divisor (MMD) labeling in a Jewel Graph under vertex switching when the number of jewels is odd. We formally establish that the transformed graph admits an MMD labeling under specific vertex assignments. By defining a structured labeling function that maps each vertex of the transformed graph to a sequential set of positive integers, we develop a systematic framework that ensures the total sum of edge labels is congruent to zero modulo the total number of vertices. The analysis is rigorously verified through algebraic formulations of edge sums and a Python pseudocode program, confirming the existence of MMD labeling in such cases. These findings contribute to a deeper understanding of MMD labeling in structured graph transformations, offering potential applications in network topology optimization and secure communication frameworks.**

Keywords— Jewel graph, Vertex switching graph, Graph labeling and Modular Multiplicative Divisor (MMD) labeling.

I. INTRODUCTION

The Seven Bridges of Königsberg problem, solved by Leonhard Euler in 1736, is considered the foundation of graph theory. This field plays a vital role in both theoretical and applied sciences, serving as a valuable tool for developing techniques in discrete mathematics. Mathematically, a graph is a structure consisting of a set of vertices and edges that connect specific couples of vertices, representing relationships involving them. Edges serve the function of depicting the relationships or connections between vertices. Bondy and Murty define a spanning tree as a minimal connected subgraph that includes all vertices of a graph without forming cycles. The significance of spanning trees is emphasized in applications such as network optimization and reliability analysis. While primarily focused on classical graph properties, the study acknowledges that graph labeling plays a role in combinatorial mathematics and structural analysis [1]. In authors [2], Modular Multiplicative Divisor (MMD) graphs are characterized, focusing on path, cycle, star, and wheel graphs. Conditions for MMD labeling are established, and the impact of structural modifications, like graph joining and edge alterations, on these properties is analyzed. The study includes proofs and examples, contributing to the broader understanding of labeled graphs. In [3], authors explored Super Vertex Graceful graphs and their behavior under certain graph operations. Conditions for graphs to admit SVG labeling are analyzed, and the impact of operations like graph union, join, and subdivision on these properties is investigated. The authors in [4] investigated five new cordial graphs, proving that the Shadow graph, splitting graph, and Degree Splitting graph of the star graph are cordial. Further, it is established that the Jewel graph and Jellyfish graph also admit cordial labeling. Each theorem in the paper rigorously defines and proves the cordiality of these graphs, supported by illustrations to enhance understanding of the labeling patterns. The findings contribute to the study of graph labeling and cordial graphs. The authors [5] established that the Jewel graph (for both odd and even values) and the Even Arbitrary Super Subdivision (EASS) graph of the Jewel graph admit Modular Multiplicative Divisor (MMD) labeling. MMD labeling is defined as a bijection from vertices to $\{1, 2,\ldots,n\}$ with an induced edge function satisfying divisibility conditions. The findings confirm that these graphs satisfy the necessary conditions for MMD labeling, contributing to graph labeling theory. Additionally, open problems related to this work are presented for further exploration. The authors [6] investigate Modular Multiplicative Divisor (MMD) labeling for various graph classes. It is proved that the triangular book with n-pages, triangular snake, and the graph obtained by duplicating every edge with a vertex in a cycle admit MMD labeling. Additionally, it is established that graphs formed by switching end vertices in a path and switching a vertex in a cycle also satisfy MMD labeling, with some exceptions. This work expands the study of MMD labeling by analyzing new graph structures and their labeling properties. The concept of graph labeling was first began by Rosa in 1967 [7]. The authors proved that modifications of the Jewel graph, including vertex switching, path union, and cycle formation, admit cordial labeling. The study extended the application of cordial labeling to these transformations, contributing to the classification of cordial graphs [8]. The authors [9] explored prime labeling in graphs obtained by switching a vertex and examined whether certain standard graphs retain their prime labeling properties. The authors proved that the graphs resulting from vertex switching in path graph and star graph admit prime labeling. The author analyzed the wheel graph and its prime labeling behavior under vertex switching, highlighting challenges due to the scattered nature of prime numbers. A conjecture related to prime labeling in such transformations is proposed. Additionally, it is proved that the Lilly graph admits Prime Cordial Labeling (PCL), extending to its variants under vertex switching, duplication, degree splitting, and barycentric subdivision [10]. Recent studies have applied Modular Multiplicative Divisor (MMD) labeling to diverse graph structures with promising outcomes. Kalarani and Revathi [14] applied MMD labeling techniques for efficient Minimum Spanning Tree (MST) computation in Jellyfish graphs using Kruskal's algorithm, demonstrating its role in network optimization. Kalarani et al. [15] examined the use of MMD labeling to expand various graph families, highlighting its versatility in structural graph analysis. Revathi et al. [16] focused on the EASS of the Cartesian product of two graphs, showing that MMD labeling can be effectively extended to product graphs with complex topology.

The authors [11] proved that the Triangular Book, Triangular Book with Bookmark, and Jewel Graph are k-cordial. The authors [12] discussed self-vertex switching and duplication self-vertex switching in self-centered graphs, defining conditions where vertex switching or duplication results in an isomorphic graph. The existence of such graphs with these properties is established. The authors J. Meena and T.N.M. Malini Mai focused on Roman and Double Roman Domination in graphs, using Python for computational analysis. In [13] examined Roman domination in complete binary trees, Their studies provide a strong foundation for algorithmic approaches like Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jewel graphs for network security.

Section 2 provides definitions of all key terms along with relevant examples. Section 3 focuses on the MMD labeling of the Jewel Graph under vertex switching when the number of jewels is odd. The study applies MMD labeling to the vertex-switched Jewel graph, with the primary objective of exploring and presenting its properties in the even case. Figure 1 shows the graphical representation of the proposed strategy.

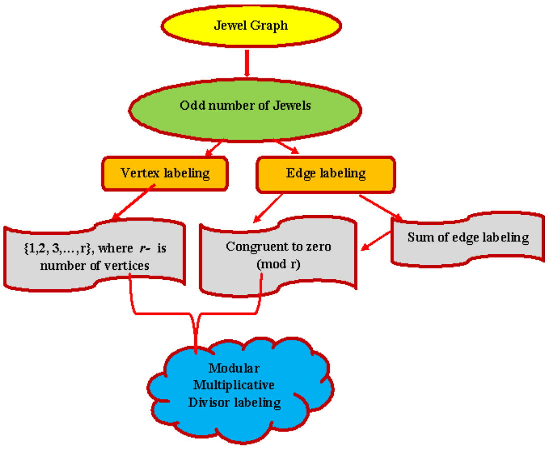

Fig. 1. Graphical representation of the proposed strategy

A. *Novelty of this study*

The paper introduces a novel approach to Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jewel graphs, a topic that has not been extensively studied. It establishes a structured bijection-based vertex labeling function that ensures the edge labels satisfy MMD conditions, contributing to the theoretical advancement of graph labeling techniques. A key finding is that the sum of edge labels remains congruent to zero modulo the total vertex count, which is mathematically proven and verified through numerical examples. The study further extends the applicability of MMD labeling to new graph transformations, specifically under vertex switching, providing fresh insights into structured graph modifications. Additionally, the research highlights practical applications in network security, cryptography, and combinatorial optimization, demonstrating its significance in both theoretical and applied domains.

II. *PRELIMINARIES*

A. *Jewel graph*

A Jewel graph J_η contains of a set of nodes $V(J_\eta) = \{\alpha, \beta, \gamma, \delta, \delta_i : 1 \le i \le \eta\}$ and a corresponding set of edges $E(J_\eta) = \{\alpha\delta, \gamma\delta, \alpha\delta_i, \gamma\delta_i : 1 \le i \le \eta\}$ that connect them. Each node in the graph is adjacent to specific other nodes, ensuring that every jewel node has a degree of 2. The prime edge in a J_η refers to the edge that connects two vertices β and δ. The term jewel in a Jewel graph denotes a vertex with a degree of two, intending it is connected to accurately two edges. The structure of the J_η is illustrated in diagram 2.

B. *Vertex switching of the graph*

The vertex switching $\boldsymbol{R_v}$ of a graph R is defined as the transformation of R into a new graph by modifying the adjacent relationships of a selected vertex v. Specifically, this process involves removing all edges directly connected to v in R and then connecting v to every vertex that was previously not adjacent to it in R. The result is a graph where the neighborhood of v is complemented, while the rest of the graph remains unchanged. Figure2 represents graph $\boldsymbol{R_v}$ and figure3 denotes the switching of the vertex v_4 of Graph R_v

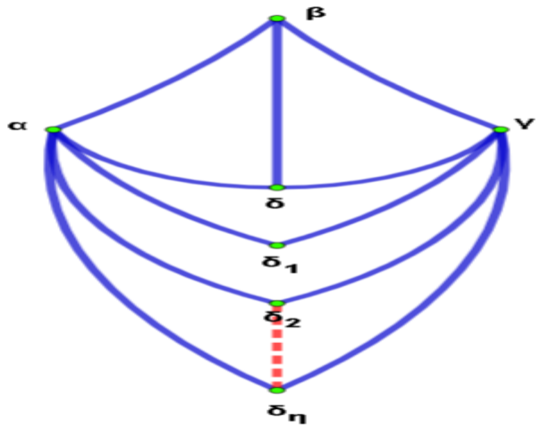

Fig.2. Jewel graph

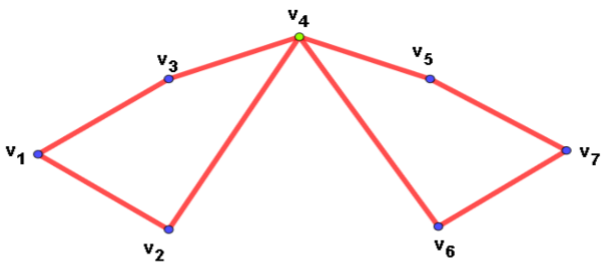

Fig.3. Graph R_v

C. *Graph labeling*

Assigning integers to points, lines, or both while adhering to predetermined guidelines in the graph is known as graph labeling. To provide a visual representation, Figure 3 displays the MMD assignment of labels to a graph with 7 points.

D. *MMD labeling*

The MMD graph R with C points. This graph involves a bijection Z from V(R) to the set $\{1,2,3,\ldots,C\}$, along with an induced function y assigned to the edges of R, taking values from $\{0,1,2,3,\ldots,C\text{-}1\}$. s Specifically, for any edge ab in R, the function *y(ab)* is determined by the equation y(ab) = Z (a) * Z (b) (mod C). It is important to note that the summation of the labels assigned to all the edges of this graph results in a value that is a multiple of C. The Figure 4&5 number obtained by the addition of all labels on edges of this labeling is 3+4+0+0+1+6+0+0 =14 ≡ 0 (mod 7).

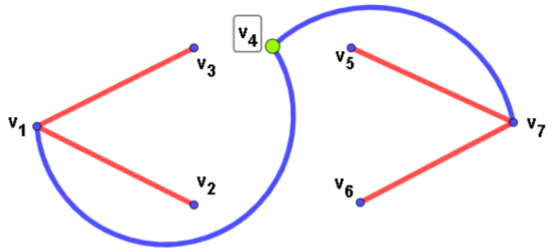

Fig.4. Switching of the vertex v_4 of Graph R_v

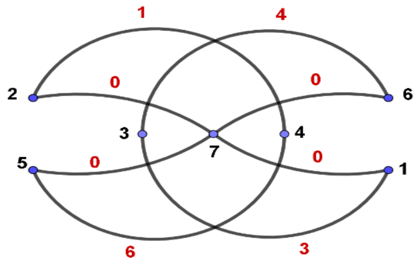

Fig.5. MMD labeling

III. Discussion of MMD Labeling for Jellyfish Graph

This study examines MMD labeling in vertex-switched Jewel graphs j'_η with an odd number of jewels (η). It establishes a structured bijection-based vertex labeling to maintain MMD conditions. The research confirms that the sum of edge labels remains congruent to zero modulo the total vertex count. A systematic approach is used to validate MMD properties under vertex switching transformations. The algebraic proof and numerical verification confirm the correctness of the proposed method. The findings highlight how graph modifications preserve modular arithmetic properties. The study also establishes a relationship between jewel count, vertex set, and edge labels. These results extend graph labeling techniques to new structured transformations. The approach offers potential applications in network security, cryptography, and combinatorial optimization. Overall, the research strengthens the theoretical foundation of graph labeling under structural modifications.

A. Effect of Vertex Switching on MMD Labeling

Vertex switching in Jewel graphs with an odd number of jewels preserves MMD labeling by assigning a bijective vertex labeling from 1 to σ^V, where σ^V is the number of vertices. The edge labels, calculated as the product of vertex labels modulo σ^V, continue to satisfy the MMD condition. As established in Theorem 3.1, the sum of all edge labels remains congruent to zero modulo σ^V even after switching. This transformation retains the essential labeling properties and is verified through both algebraic formulation and Python-based computation.

Theorem :3.1

A Jewel Graph with an odd number of jewels admits Modular Multiplicative Divisor (MMD) labeling under vertex switching. There exists a bijective function on the vertex set that induces valid edge labels satisfying MMD conditions. Additionally, the total sum of edge labels remains a multiple of the vertex count after switching.

Proof:

Let J_η be the jewel graph with the node set $V(J_\eta) = \{\alpha, \beta, \gamma, \delta, \delta_i : 1 \le i \le \eta\}$ and $E(J_\eta) = \{\alpha\delta, \gamma\delta, \alpha\delta_i, \gamma\delta_i : i = 1 \ to \ \eta\}$. In this case, α and γ are adjacent, while each δ_i has a degree two. The edge β & δ is referred to as the prime edge of the J_η . The parameter η denote the number of jewels in J_η and the number of vertices is given by $n[v(J_\eta)] = \eta + 4$. Let us consider β as a switching vertex in the J_η . Remove all the edges $\alpha\beta, \beta\gamma, \beta\delta$ which are incident with β and make β to be adjacent with all the vertices which are not initially adjacent to it. The resultant graph termed as j'_η , the vertex switching of jewel graph. The vertex set of j'_η, is denoted as $V(j'_\eta) = \{\alpha, \beta, \gamma, \delta, \delta_i : 1 \le i \le \eta\}$ and $E(j'_\eta) = \{\alpha\delta, \gamma\delta, \alpha\delta_i, \gamma\delta_i : 1 \le i \le \eta\}$ is the edge set of j'_η,which is described in figure 6. The number of vertices in j'_η is denoted as $\sigma^V = \eta + 4$ and the number of edges in j'_η is denoted as $\sigma^E = 3\eta + 2$. The labeling function for the vertices is defined as : $V(j'_\eta) \to \{1,2,3, \dots, \eta + 4\}$, with the following assignments: are $\mu(\alpha) = 1, \mu(\beta) = \sigma^V, \mu(\gamma) = \mu(\beta) - \mu(\alpha) = \sigma^V - 1, \mu(\delta) = \mu(\alpha) + 1 = 2$. The derivation for the even number of jewels even case is as follows: the labels for the vertices are $\mu(\delta_{2i-1})= \mu(\gamma) - i; 1 \le i \le \frac{\eta+1}{2}$, $\mu(\delta_{2i})= \mu(\delta) + i; 1 \le i \le \frac{\eta-1}{2}$, where η represents an odd number of jewels. The number of edges is $\sigma^E = |E\,[j'_\eta]| = 3\eta + 2$. The edge labeling function is defined as $\mu^*(u'v') = \mu(u')\mu(v')(mod\sigma)$ for every line $e = (u'v') \in E[j'_\eta]$.

The total of the line labels μ_1 is expressed as:

$$\mu_1 = \mu(\alpha)\left[\sum_{i=1}^{\eta}\mu(\delta_i)\right] + \mu(\beta)\left[\sum_{i=1}^{\eta}\mu(\delta_i)\right] + \mu(\gamma)\left[\sum_{i=1}^{\eta}\mu(\delta_i)\right] + \mu(\alpha)\mu(\delta) + \mu(\gamma)\mu(\delta)$$

$$\mu_1 = [\mu(\alpha) + \mu(\beta) + \mu(\gamma)]\left[\sum_{i=1}^{\eta}\mu(\delta_i)\right] + \mu(\alpha)\mu(\delta) + \mu(\gamma)\mu(\delta)$$

$$\mu_1 = [1 + \sigma^V + \sigma^V - 1]\left[\sum_{i=1}^{\eta}\mu(\delta_i)\right] + 1.2 + (\sigma^V - 1)2$$

$$\mu_1 = [2\sigma^V]\left[\sum_{i=1}^{\eta}\mu(\delta_i)\right] + 2 + (2\sigma^V - 2)$$

From the vertex labeling function

$$\mu(\delta_{2i-1}) = \mu(\gamma) - i = (\sigma^V - 1) - i \text{ for } 1 \le i \le \frac{\eta+1}{2}$$

$$\mu(\delta_{2i}) = \mu(\delta) + i = 2 + i \text{ for } 1 \le i \le \frac{\eta-1}{2}$$

Thus, summing over all η values

$$\mu_1 = \left[2\sigma^V\left\{\left[\frac{\eta+1}{2}X\sigma^V\right] - 2\right\} + 2\sigma^V\right]$$

Since j'_η follows MMD labeling and verify that

$$\mu_1 \equiv 0(mod\sigma^V) \quad (1)$$

By using (1), it is shown that the total sum of edge labels μ_1 is congruent to zero modulo the number of vertices σ^V, confirming that the MMD labeling condition is satisfied.

B. Pseudocode for MMD Labeling in Vertex-Switched Jewel Graphs

Establishing Theorem 3.1 Proof Using Pseudocode for the given Python Program.

Algorithm1: MMD labeling verification of Jewel Graphs

Input: An odd number η of jewels

Output: Verification of Modular Multiplicative Divisor (MMD) labeling condition

1. If η is even:

Output "Error: Jewel count must be odd" and terminate.

2. Compute the number of vertices: $\sigma^V \leftarrow \eta + 4$

3. Computing the number of Edges: $\sigma^E \leftarrow 3\eta + 2$

4. Compute the sum of Edge labels:

$$\mu_1 \leftarrow \left[2\sigma^V\left\{\left[\frac{\eta+1}{2}X\sigma^V\right]-2\right\}+2\sigma^V\right]$$

5. Verify the MMD condition:

MMD Verified $\leftarrow (\mu_1 mod\sigma^V = 0)$

6. Return results: jewel count η, σ^V, : σ^E, μ_1, and MMD condition status.

The given pseudocode systematically verifies the MMD labeling condition for a vertex-switched Jewel Graph with an odd number of jewels. It first ensures that the jewel count (η) is odd, rejecting even values to maintain validity. The program then computes key graph properties, including the number of vertices (σ^V) and number of edges (σ^E) using predefined formulas. The sum of edge labels (μ_1) is calculated based on the given mathematical expression. To confirm the MMD labeling condition, the program checks whether μ_1 is divisible by σ^V. If the condition holds, the labeling is considered valid. The results are returned in a structured format, showing all computed values. Additionally, test cases for different odd jewel counts are executed to verify correctness. This pseudocode is beneficial for manuscripts as it provides a clear algorithmic structure for theoretical proofs and computational verification. It ensures mathematical rigor while being easy to implement in programming languages. The approach enhances understanding and reproducibility of results in research.

This confirms that graph with an odd number of jewels j'_η and a vertex switch known as J_η , admits Modular Multiplicative Divisor (MMD) labeling, ensuring that the total of its edge labels remain congruent to 0 modulo σ^V. Figure 6 shows generalized MMD labeling of vertex switching of jewel graph. This table I. presents numerical values for different jewel counts (η) in a vertex-switched Jewel graph, including the number of vertices (σ^V), number of edges (σ^E), sum of edge labels (μ_1), and verification of the MMD labeling condition ($\mu_1 \equiv 0($ mod σ^V). The results presented in Table I confirm that the Modular Multiplicative Divisor (MMD) labeling condition is satisfied for all tested values of η (jewels count) up to 50. Specifically, the sum of edge labels μ_1 remains divisible by the number of vertices (σ^V) in every case, ensuring that $\mu_1 \equiv 0(mod\sigma^V)$ holds true. This consistency across different values of η validates theoretical proof that vertex-switched Jewel graphs with an even number of jewels always admit MMD labeling. The findings further strengthen the structural properties of MMD-labeled graphs under vertex switching and highlight their potential applications in areas such as network topology, cryptography, and combinatorial mathematics.

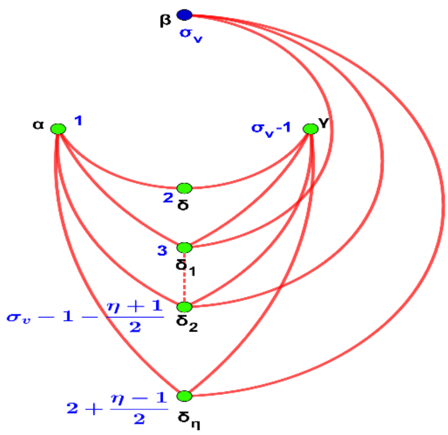

Fig.6.Generalised MMD labeling of vertex switching jewel graph

C. Computational Load in MMD Verification Under Vertex Permutations

Verifying MMD labeling in large Jewel graphs under arbitrary vertex permutations involves calculating edge labels by multiplying vertex labels and reducing modulo the total number of vertices. This requires checking all edges and summing their labels to confirm the MMD condition. Since each vertex permutation alters the labeling pattern, every verification step involves multiple arithmetic operations over the entire edge set. As the graph grows, the number of calculations increases significantly, especially for graphs with a high number of jewels, making the process computationally intensive.

The figure7 illustrates the relationship between the jewel count and the number of vertices and edges in the vertex switched Jewel graph. The x-axis represents the jewel count, while the y-axis shows the corresponding number of vertices and edges. As the jewel count increases, the difference between the number of edges and vertices becomes more pronounced, indicating a more complex graph structure. This trend highlights how the connectivity of the Jewel graph expands significantly as more jewels are added.

Figure 8 illustrates the total edge labeling in j'_η as the quantity number of vertices rises. The horizontal axis represents the number of jewels, whereas the vertical axis displays the related total of vertex labels. The pattern exhibits a nonlinear upward trend, signifying a swift increase in labeling totals as the jewel quantity grows. Each data point is emphasized for better visibility.

D. *Validation of Results*

The results of the study are validated through a combination of mathematical proofs, algorithmic implementation, and extensive numerical testing. First, a formal algebraic framework is established to prove that the Modular Multiplicative Divisor (MMD) labeling condition is satisfied in vertex-switched Jewel graphs with an odd number of jewels. This condition requires that the sum of all edge labels be congruent to zero modulo the total number of vertices. To support the theoretical findings, a Python-based pseudocode is developed and executed for various values of jewel counts (η), ensuring reproducibility and computational accuracy. The results for each tested case are presented in Table I, where the calculated edge label sums consistently satisfy the MMD condition. Additionally, Figures 7 and 8 graphically illustrate the trends in the number of vertices, edges, and total edge labels, reinforcing the correctness of the approach. This multi-layered validation strategy combining algebraic, algorithmic, and numerical methods confirms the soundness and reliability of the results.

TABLE I. NUMERICAL EXAMPLES OF MMD LABELING IN VERTEX-SWITCHED JEWEL GRAPHS

Jewels *Count* η	Number of Vertices σ^V	Number of Edges τ	Sum of Edge Labels μ_1	MMD Labeling Condition $\mu_1 \equiv 0(mod\sigma^V)$
1	5	5	40	40 ≡ 0 (mod 5) ✓
3	7	11	182	182 ≡ 0 (mod 7) ✓
5	9	17	468	468 ≡ 0 (mod 9) ✓
7	11	23	946	946 ≡ 0 (mod 11) ✓
9	13	29	1664	1664 ≡ 0 (mod 13) ✓
11	15	35	2670	2670 ≡ 0 (mod 15) ✓
13	17	41	4012	4012 ≡ 0 (mod 17) ✓
15	19	47	5738	5738 ≡ 0 (mod 19) ✓
17	21	53	7896	7896 ≡ 0 (mod 21) ✓
19	23	59	10534	10534 ≡ 0 (mod 23) ✓
21	25	65	13700	13700 ≡ 0 (mod 25) ✓
23	27	71	17442	17442 ≡ 0 (mod 27) ✓
25	29	77	21808	21808 ≡ 0 (mod 29) ✓
27	31	83	26846	26846 ≡ 0 (mod 31) ✓
29	33	89	32604	32604 ≡ 0 (mod 33) ✓
31	35	95	39130	39130 ≡ 0 (mod 35) ✓
33	37	101	46472	46472 ≡ 0 (mod 37) ✓
35	39	107	54678	54678 ≡ 0 (mod 39) ✓
37	41	113	63796	63796 ≡ 0 (mod 41) ✓
39	43	119	73874	73874 ≡ 0 (mod 43) ✓
41	45	125	84960	84960 ≡ 0 (mod 45) ✓
43	47	131	97102	97102 ≡ 0 (mod 47) ✓
45	49	137	110348	110348 ≡ 0 (mod 49) ✓
47	51	143	124746	124746 ≡ 0 (mod 51) ✓
49	53	149	140344	140344 ≡ 0 (mod 53) ✓

IV. COMPARISON OF EXISTING SURVEY AND PRESENT WORK

Table II. presents a comparative analysis between this research and earlier studies. In [14], the authors confirmed that the vertex switching of the Jewel Graph, as well as the path union of its vertex switching, exhibit Cordial labeling. In contrast, our work establishes the MMD labeling for the vertex switching of the J_η in the even case. The table provides a comparative overview of past findings and our present research.

A. *Techniques to Enhance Accuracy in MMD Labeling*

Enhancing the accuracy of Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jewel graphs involves several key techniques. Firstly, establishing a clear bijective function that maps vertices to a specific set of integers is crucial. This precise mapping ensures that each vertex receives a unique label, which is fundamental for accurate edge labeling. Secondly, the induced edge labeling function should be meticulously defined to assign labels based on the product of the labels of adjacent vertices, taken modulo the total number of vertices. This approach maintains consistency and adheres to MMD labeling rules. Additionally, rigorous verification processes, such as algebraic proofs and computational checks, are essential to confirm that the sum of all edge labels is congruent to zero modulo the number of vertices. These verification steps help in identifying and rectifying any discrepancies in the labeling process, thereby enhancing overall accuracy.

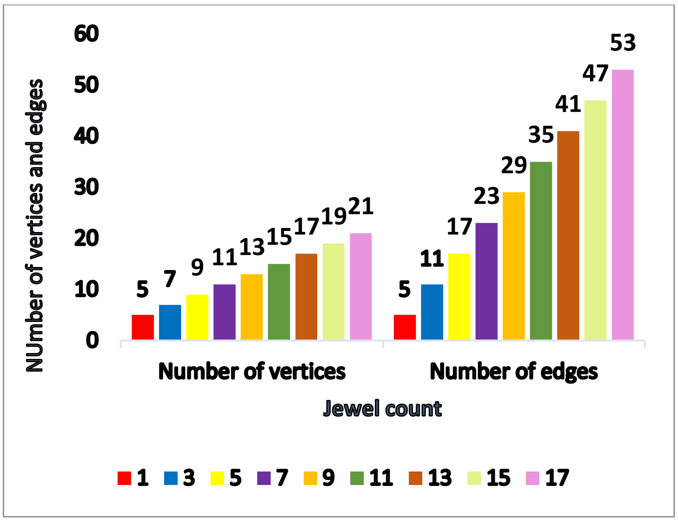

Fig.7 Variation of the number of vertices and edges in the Jewel graph with increasing jewel count

B. *Reliability of the Proposed Labeling Method*

To ensure the reliability of the Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jewel graphs, the proposed study employs both mathematical rigor and computational validation. The labeling method is first verified algebraically, ensuring that the sum of edge labels is always congruent to zero modulo the number of vertices, a necessary condition for MMD labeling. This theoretical foundation is further reinforced through Python-based pseudocode, which systematically computes and confirms the labeling conditions across multiple jewel counts. Numerical validation is conducted up to 49 jewel values, with all results presented in

Table I, consistently satisfying the MMD condition. Additionally, graphical trends and edge label sums are illustrated to support the correctness and consistency of the method. By combining formal proof, algorithmic computation, and repeated testing across a wide input range, the study establishes a high degree of reliability for the proposed labeling technique.

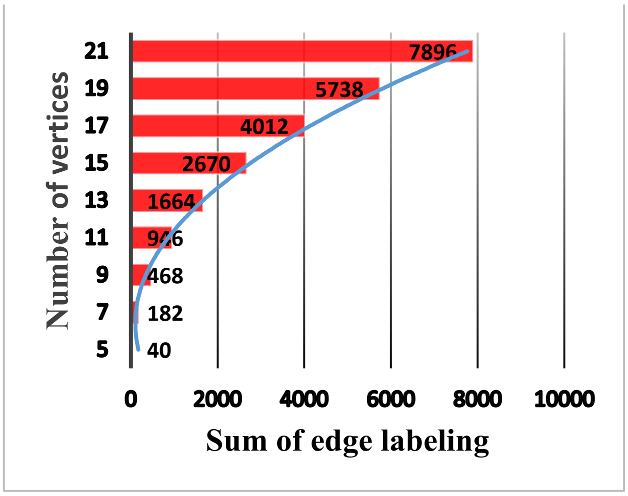

Fig.8 Sum of edge labeling in Jewel graph for varying number of vertices

TABLE II.
OMPARISON OF PRESENT AND PREVIOUS RESULTS ON LABELING TECHNIQUES FOR VERTEX SWITCHING OF THE JEWE

Aspect	Present result	Previous result
Title	Python Based Approach for Modular Labeling in Switched Jewel Graphs	Cordial Labeling on the Vertex Switching of Jewel Graph
Graph Types Investigated	Vertex switching of Jewel graph (Jewels count is odd)	vertex switching of the jewel graph
Labeling Technique	MMD labeling	Cordial Labeling
Objective	Explored the novel application of MMD labeling in the vertex switching of a Jewel Graph.	Proved that the vertex switching of jewel is Cordial labeling
Labeling Function	The function f maps the vertices of the graph L to the set 1 to n, while the function f* maps the edges of the graph L to the set 0 to n – 1.	f: V(G) → {0, 1}
Main Results	- MMD labeling method is developed for vertex switching in the Jewel graph. graph with MMD labeling demonstrated.	- Vertex switching of the jewel graph is Cordial labeling

C. Strategies for Enhancing MMD Labeling Performance

To improve the performance of Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jewel graphs, the paper adopts several algorithmic and structural strategies. A key enhancement involves the implementation of automated Python-based algorithms that systematically assign labels to vertices and edges, significantly reducing manual effort and increasing computational efficiency. Additionally, the modular decomposition of the graph into smaller subgraphs allows for localized labeling, which can later be recombined to preserve the overall MMD labeling properties. The use of computational tools also facilitates the processing of large and complex graphs, enabling accurate and faster validation. Furthermore, the labeling process is rigorously verified through both algebraic proofs and programmatic checks, ensuring that the sum of edge labels remains congruent to zero modulo the number of vertices. These combined approaches enhance the accuracy, scalability, and robustness of the proposed MMD labeling technique.

D. Future Scope

In future work, we aim to switch the remaining vertices that admit MMD labeling and analyze their structural impact. We will explore the effects of these transformations on various graph properties. Additionally, we plan to investigate the existence of other labeling schemes applicable to Jewel graphs. The study can be extended to different classes of graphs to examine similar switching behaviors. Furthermore, algorithmic approaches for efficiently identifying and transforming such graphs will be developed.

E. Impact of Vertex Switching on MMD Labeling Preservation

Vertex switching in Jewel graphs with an odd number of jewels results in a transformation that preserves the Modular Multiplicative Divisor (MMD) labeling properties. Specifically, the operation modifies the adjacent relations of a selected vertex (e.g., vertex β), leading to a new but isomorphic structure where the original topology is altered, yet the graph retains a bijective labeling function. The induced edge labels still satisfy the MMD condition, i.e., the total sum of edge labels remains congruent to zero modulo the number of vertices. This transformation demonstrates that MMD labeling is robust under structural modifications like vertex switching. The result extends to a broader class of isomorphic graphs derived from the original Jewel graph, confirming that the MMD labeling property is invariant under such isomorphic transformations when carefully constructed. This reinforces the structural stability and applicability of MMD labeling in dynamic network environments.

F. Congruency in Recursive Expansions and Graph Products

The congruency condition on edge label sums in Jewel graphs remains valid under vertex switching due to structured labeling and modular arithmetic. For recursive expansions or graph product operations, the condition can still hold if vertex labels are reassigned properly and the labeling rule (product modulo total vertices) is maintained. However, since these operations alter the graph's structure, the labeling must be recalculated and verified for each case. Algorithmic validation is essential to ensure the MMD condition continues to be satisfied.

G. Cryptographic Applications of MMD Labeling

MMD labeling can support cryptographic protocols by using the modular edge sum invariance for key generation and hash functions. In vertex-switched Jewel graphs, the structured vertex labeling ensures that the total edge label sum remains congruent to zero modulo the number of vertices. This predictable property can act as a verification tool in key exchange or as part of a hash function sensitive to label changes. The preserved modular structure under vertex switching ensures robustness, making it suitable for secure communication frameworks.

V Conclusion

In this study, we established the MMD labeling for the vertex switching of the J_η when the η is odd. By defining a structured labeling function, we demonstrated that the addition of the edge labels remains congruent to zero modulo the total number of vertices. This work extends the applicability of MMD labeling to new graph j'_η structures, contributing to the broader study of graph labeling techniques. The findings offer insights into structured graph transformations and their mathematical properties, providing a foundation for further research in this domain.

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