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Article · January 2018

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Jour of Adv Research in Dynamical & Control Systems, Vol. 10, 05-Special Issue, 2018

Total Chromatic Number of Double Star Graph Families

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Abstract--- A total coloring of a graph G is an assignment of colors to both the vertices and edges of G, such that no two adjacent or incident vertices and edges of G are assigned the same colors. In this paper, we have discussed the total coloring of $M(K_{1,n,n})$, $T(K_{1,n,n})$, $L(K_{1,n,n})$ and $S'(K_{1,n,n})$ and also we obtained the total chromatic number of $M(K_{1,n,n})$, $T(K_{1,n,n})$, $L(K_{1,n,n})$ and $S'(K_{1,n,n})$.

AMS Subject Classification--- 05C15

Keywords and Phrases--- Middle Graph, Line Graph, Total Graph, Double Star Graph, Total Coloring and Total Chromatic Number.

I. Introduction

In this paper, we have chosen finite, simple and undirected graphs. Let$G = (V(G), E(G))$ be a graph with the vertex set $V(G)$ and the edge set E(G), respectively. In 1965, the concept of total coloring was introduced by Behzad [1] and in 1967 he [2,3] came out new ideology that, the notion of the total graph of a graph and the total chromatic number of complete graph and complete bi-partite graph. A total coloring of G, is a function $f: S \rightarrow C$, where $S = V(G) \cup E(G)$ and C is a set of colors to satisfies the given conditions.

- no two adjacent vertices receive the same colors
- notwo adjacent edges receive the same colors
- no edges and its end vertices receive the same colors

The *total chromatic number*$\chi''(G)$of a graph$G$ is the minimumcardinality k such that Gmay have a total coloring by k colors. Behzad [1] and Vizing [15] conjectured that for every simple graph G has $\Delta(G) + 1 \leq \chi''(G) \leq \Delta(G) + 2$, where $\Delta(G)$the maximum degree of G.This conjecture is called as the Total Coloring Conjecture(TCC). Rosenfeld [10] and Vijayaditya[14] verified the TCC, for any graph G with maximum degree ≤ 3 andKostochka [7] for maximum degree≤ 5. In Borodin[4] verified The Total Coloring conjecture (TCC)for maximum degree ≥ 9 in planar graphs. In 1992, Yap and Chew [16] proved that any graph G has a total coloring with atmost $\Delta(G) + 2$ colors if$\Delta(G) \geq |V(G)| - 5$. In recent era, total coloring have been extensively studied in different families of graphs. Mohan et.al [8] given the tight bound of Behzad and Vizing conjecture in Corona product of certain classes of graph. Muthuramakrishnan et.al [9] proved that the total chromatic number of line, middle, total graph of star graph and square graph of bistar graph. Vaidya et.al [13] proved that the total chromatic number of middle graph, total graph, shadow graph of cycle and one point union of cycle. Sudha et.al [12] proved that the total chromatic number of Sudha grid graphs, gear graph and crown graph. In this present paper, we investigate the total chromatic number of$M(K_{1,n,n})$, $T(K_{1,n,n})$, $L(K_{1,n,n})$and $S'(K_{1,n,n})$.

II. Preliminaries

Definition 2.1. The *Double star graph*$K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendent edge of the exiting n pendant vertices. It has $2n + 1$ vertices and $2n$ edges. Let $V(K_{1,n,n}) = \{u\} \cup \{u_1, u_2, u_3, .. u_n\} \cup \{v_1, v_2, v_3, .. v_n\}$and $E(K_{1,n,n}) = \{e_1, e_2, e_3, .. e_n\} \cup \{s_1, s_2, s_3, .. s_n\}$.

Definition 2.2. The *middle graph*[5] of a graph G, denoted by M(G) is define as follows, the vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one of the following condition holds: (i) x, y are in $E(G)$ and x, y is adjacent in G. (ii) x is in $V(G)$, y is in $E(G)$and x, y are incident in G.

Received: 15 Mar 2018/Accepted: 20 Apr 2018

Definition 2.3. The *Total graph*[3] of a graph G, denoted by $T(G)$ is define as, the vertex set of $T(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one of the following condition holds: (i) x, y are in $V(G)$ and x is adjacent to y in G. (ii) x, y are in $E(G)$ and x, y is adjacent in G. (iii) x is in $V(G), y$ is in $E(G)$and x, y are incident in G.

Definition 2.4. The *line graph*[17] of graph G, denoted by $\mathrm{L}(G)$ is the graph whose vertices are the edges of G with two vertices of $L(G)$ adjacent whenever the corresponding edges of G are adjacent.

Definition 2.5. For a graph G, the splitting graph $S'(G)$ [11] of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Main Results

III. Total Chromatic Number of $\mathbf{M(K_{1,n,n})}$

Theorem 3.1. Let $M(K_{1,n,n})$be the middle graph of double star graph. Then

$$\chi''(M(K_{1,n,n})) = n + 3.$$

Proof: Let$V(K_{1,n,n}) = \{v\} \cup \{u_1, u_2, u_3, \ldots\ldots u_n\} \cup \{v_1, v_2, v_3, \ldots\ldots v_n\}$and $\mathrm{E}(K_{1,n,n}) = \{e_1, e_2, e_3, \ldots\ldots e_n\} \cup \{s_1, s_2, s_3, \ldots\ldots s_n\}$where$\{e_i = vv_i : 1 \le i \le n\}$and $\{s_i = u_i v_i : 1 \le i \le n\}$. By the definition of middle graph, each edge $\{e_i = vv_i : 1 \le i \le n\}$ and $\{s_i = u_i v_i : 1 \le i \le n\}$in $K_{1,n,n}$ is subdivided by the vertices $\{v_i^{'} : 1 \le i \le n\}$ and $\{u_i^{'} : 1 \le i \le n\}$in $M(K_{1,n,n})$ and the vertices $\{v, v_1^{'}, v_2^{'}, v_3^{'}, \ldots\ldots, v_n^{'}\}$ induce a clique of order $n+1$. The middle graph of double star graph contains a complete graph of order $n+1$.In$M(K_{1,n,n})$, the vertex set and the edge setis given by

$$V(M(K_{1,n,n})) = \begin{cases} \{v\} \cup \{v_i : 1 \le i \le n\} \cup \{v_i^{'}, : 1 \le i \le n\} \cup \\ \{u_i : 1 \le i \le n\} \cup \{u_i^{'}, : 1 \le i \le n\} \end{cases} \text{and}$$

$$E(M(K_{1,n,n})) = \begin{cases} \{vv_i^{'} : 1 \le i \le n\} \cup \{v_i^{'} v_i : 1 \le i \le n\} \cup \\ \{v_i u_i^{'} : 1 \le i \le n\} \cup \{u_i^{'} u_i : 1 \le i \le n\} \cup \\ \{u_i^{'} v_i^{'} : 1 \le i \le n\} \cup \{v_i^{'} v_j^{'} : 1 \le i \le n-1, i+1 \le j \le n\} \end{cases}$$

We define the total coloring f, such that$f : S \to C$ where$S = V(M(K_{1,n,n})) \cup E(M(K_{1,n,n}))$ and $C = \{1, 2, 3, \ldots. n+3\}$.Now we assign the total coloringto these vertices and edges as follows. We consider the following two cases:

Case (i): when n is odd

$$f(v) = n+2 \qquad f(v_i^{'}) = i \quad for \quad 1 \le i \le n.$$
$$f(v_i) = f(u_i) = n+3 \qquad for \quad 1 \le i \le n.$$
$$f(u_i^{'}) = n+1 \quad for \quad 1 \le i \le n$$
$$f(vv_i^{'}) = \begin{cases} 2i, & if\ 2i \not\equiv 0\ (mod\ n+2) \\ n+2, & 0therwise \end{cases} \quad for\ 1 \le i \le n$$
$$f(v_i^{'} v_j^{'}) = \begin{cases} i+j, & if\ (i+j) \not\equiv 0 (mod\ n+2) \\ n+2, & otherwise \end{cases} \quad \begin{matrix} for\ 1 \le i \le n-1 \\ j > i,\ i+1 \le j \le n \end{matrix}$$
$$f(v_i^{'} v_i) = i-1 \quad for \quad 2 \le i \le n$$
$$f(v_i^{'} u_i^{'}) = n+3 \quad for \quad 1 \le i \le n$$
$$f(v_1^{'} v_1) = n+2$$
$$f(u_i^{'} v_i) = n, \qquad f(u_i^{'} u_i) = n+2 \quad for \quad 1 \le i \le n.$$

Case (ii): when n is even

$$f(v) = n+2 \quad f(v_i^{'}) = i \quad for \quad 1 \le i \le n.$$
$$f(v_i) = n+3, \qquad f(u_i) = 2 \qquad for \quad 1 \le i \le n.$$
$$f(u_i^{'}) = n+2 \quad for \quad 1 \le i \le n$$
$$f(vv_i^{'}) = \begin{cases} 2i, & if\ \ 2i \not\equiv 0\ (mod\ n+1) \\ n+1, & 0therwise \end{cases} \quad for\ 1 \le i \le n$$
$$f(v_i^{'} v_j^{'}) = \begin{cases} i+j, & if\ (i+j) \not\equiv 0 (mod\ n+1) \\ n+1, & otherwise \end{cases} \quad \begin{matrix} for\ 1 \le i \le n-1 \\ j > i,\ i+1 \le j \le n \end{matrix}$$
$$f(v_i^{'} v_i) = n+2 \quad for \quad 1 \le i \le n$$

Received: 15 Mar 2018/Accepted: 20 Apr 2018

$$f(v_i'u_i') = n+3 \quad for \quad 1 \le i \le n$$
$$f(u_i'v_i) = n+1, \qquad f(u_i'u_i) = 1 \quad for \quad 1 \le i \le n.$$

Based on the above rule of total coloring, the graph $M(K_{1,n,n})$ is total colored with $n+3$colors.Hence the total chromatic number of $M(K_{1,n,n})$, $\chi''\left(M(K_{1,n})\right) = n+3$.

IV. Total Chromatic Number of$\mathbf{T(K_{1,n,n})}$

Theorem 4.1. Let $T(K_{1,n,n})$be the total graph of double star graph. Then

$$\chi''(T(K_{1,n,n})) = 2n+1.$$

Proof: Let$V(K_{1,n,n}) = \{v\} \cup \{u_1, u_2, u_3, \ldots\ldots u_n\} \cup \{v_1, v_2, v_3, \ldots\ldots v_n\}$and $\mathrm{E}(K_{1,n,n}) = \{e_1, e_2, e_3, \ldots\ldots e_n\} \cup \{s_1, s_2, s_3, \ldots\ldots s_n\}$, where $\{e_i = vv_i : 1 \le i \le n\}and\{s_i = u_iv_i : 1 \le i \le n\}$.By the definition of total graph, each edge $\{e_i = vv_i : 1 \le i \le n\}$ and $\{s_i = u_iv_i : 1 \le i \le n\}$in $K_{1,n,n}$ are subdivided by the vertices $\{v_i' : 1 \le i \le n\}$ and $\{u_i' : 1 \le i \le n\}$in $T(K_{1,n,n})$ and the vertices $\{v, v_1', v_2', v_3', \ldots\ldots, v_n'\}$ induce a clique of order $n+1$.Therefore, the total graph of double star graph contains a complete graph of order $n+1$.In$T(K_{1,n,n})$, the vertex set and the edge setis given by

$$V(T(K_{1,n,n})) = \begin{cases} \{v\} \cup \{v_i : 1 \le i \le n\} \cup \{v_i', : 1 \le i \le n\} \\ \cup \{u_i : 1 \le i \le n\} \cup \{u_i', : 1 \le i \le n\} \end{cases}$$

and

$$E(T(K_{1,n,n})) = \begin{cases} \{vv_i' : 1 \le i \le n\} \cup \{v_i'v_i : 1 \le i \le n\} \cup \{vv_i : 1 \le i \le n\} \\ \{v_iu_i' : 1 \le i \le n\} \cup \{u_i'u_i : 1 \le i \le n\} \cup \{u_iv_i : 1 \le i \le n\} \\ \{u_i'v_i' : 1 \le i \le n\} \cup \{v_i'v_j' : 1 \le i \le n-1, i+1 \le j \le n\} \end{cases}$$

We define the total coloring f, such that$f: S \to C$ where$S = V(T(K_{1,n,n})) \cup E(T(K_{1,n,n}))$ and $C = \{1, 2, 3, \ldots .2n+1\}$.Now we assign the total coloring to these vertices and edges as follows.

$$f(v) = 2n+1 \qquad\qquad f(v_i') = i \quad for \quad 1 \le i \le n.$$
$$f(v_i) = 2i, \quad f(u_i) = i \qquad for \quad 1 \le i \le n.$$
$$f(u_i') = \begin{cases} i+2, & if\ i+2 \not\equiv 0\ (mod\ n+2) \\ n+2, & 0therwise \end{cases} \quad for\ 1 \le i \le n$$
$$f(vv_i') = \begin{cases} 2i, & if\ 2i \not\equiv 0\ (mod\ 2n) \\ 2n, & 0therwise \end{cases} \quad for\ 1 \le i \le n$$
$$f(v_iv_i') = \begin{cases} n+1+i, & if\ n+1+i \not\equiv 0\ (mod\ 2n+1) \\ 2n+1, & 0therwise \quad for \quad 1 \le i \le n \end{cases}$$
$$f(v_iu_i') = \begin{cases} n+3+i, & if\ n+3+i \not\equiv 0\ (mod\ 2n+1) \\ 2n+1, & 0therwise \quad for \quad 1 \le i \le n \end{cases}$$
$$f(u_iu_i') = \begin{cases} i+1, & if\ i+1 \not\equiv 0\ (mod\ n+1) \\ n+1, & 0therwise \end{cases} \quad for\ 1 \le i \le n$$
$$f(vv_i) = \begin{cases} 2i-1, & if\ 2i-1 \not\equiv 0\ (mod\ 2n-1) \\ 2n-1, & 0therwise \end{cases} \quad for\ 1 \le i \le n$$
$$f(v_iu_i') = f(u_iv_i) = \begin{cases} n+2+i, & if\ n+2+i \not\equiv 0\ (mod\ 2n+1) \\ 2n+1, & 0therwise \quad for \quad 1 \le i \le n \end{cases}$$
$$f(v_i'v_j') = \begin{cases} i+j, & if\ (i+j) \not\equiv 0 (mod\ 2n+1) \quad for\ 1 \le i \le n-1 \\ 2n+1, & otherwise \qquad j > i, \quad i+1 \le j \le n \end{cases}$$

Based on the above rule of total coloring, the graph $T(K_{1,n,n})$ is total colored with $2n+1$colors. Hence the total chromatic number of the total graph of$T(K_{1,n,n})$, $\chi''(T(K_{1,n,n})) = 2n+1$.

V. Total Chromatic Number of$\mathbf{L(K_{1,n,n})}$

Theorem 5.1. Let $L(K_{1,n,n})$be the line graph of double star graph. Then

$$\chi''(L(K_{1,n,n})) = n+1, n \ge 4.$$

Proof: The Double star graph $K_{1,n,n}$is the tree obtained from the star graph $K_{1,n}$by adding a new pendant edges to the existing n pendant vertices. Here v as the root vertex along with the vertex set $\{v_1, v_2, v_3, \ldots, v_n\}$ and $\{v_1', v_2', v_3', \ldots, v_n'\}$ together with the edges $\{u_1, u_2u_3 \ldots u_n\}$and $\{u_1', u_2', u_3', \ldots, u_n'\}$. Now construct the line graph of

Received: 15 Mar 2018/Accepted: 20 Apr 2018

double star $K_{1,n,n}$, the edge in $K_{1,n,n}$ corresponding to the vertex set of $L(K_{1,n,n})$respectively. In $L(K_{1,n,n})$, the vertex set and the edge set is given by

$$V\big(L(K_{1,n,n})\big) = \{u_i : 1 \le i \le n\} \cup \{u_i' : 1 \le i \le n\} \cup \{u'\} \text{ and}$$
$$E\big(L(K_{1,n,n})\big) = \{u_i u_i' : 1 \le i \le n\} \cup \big\{u_i' u_j' :\ 1 \le i \le n-1 \ \ j > i, i+1 \le j \le n\big\}$$

In $L(K_{1,n,n})$, the vertices $\{u_i : 1 \le i \le n\ \}$ induces a clique of order n. Let it be K_n. The vertices $\{u_i' : 1 \le i \le n\}$ is adjacent with the vertices $\{u_i : 1 \le i \le n\}$. Now we construct the total coloring$f$, such that$f : S \to C$ as follows, where $S = V\big(L(K_{1,n,n})\big) \cup E\big(L(K_{1,n,n})\big)andC = \{1, 2, 3, \ldots. n+1\}$. Thetotal coloring is obtained by coloring these vertices and edges as follows.

$$f(u_i) = i, \qquad for \quad 1 \le i \le n.$$
$$f(u_i') = i-1, for\ \ 2 \le i \le n-1$$
$$f(u_i u_i') = \begin{cases} 2i, & if\ 2i\ \not\equiv 0\ (mod\ n+1) \\ n+1, & 0therwise \end{cases} \quad for\, 1 \le i \le n$$
$$f(u_i u_j) = \begin{cases} i+j, & if\ (i+j) \not\equiv 0 (mod\ n+1) \ \ for\, 1 \le i \le n-1 \\ n+1, & otherwise \qquad\qquad j > i,\ i+1 \le j \le n \end{cases}$$

It is clear that, the above rule of total coloring, the graph$L(K_{1,n,n})$ is total colored with $n+1$colors. Hence the total chromatic number of $L(K_{1,n,n})$, $\chi''\big(L(K_{1,n,n})\big) = n+1$.

VI. Total Chromatic Number of$S'\big(\mathbf{K}_{1,n,n}\big)$

Theorem 6.1. Let$S'\big(K_{1,n,n}\big)$ be the splitting graph of double stargraph. Then

$$\chi''\Big(\ S'\big(K_{1,n,n}\big)\Big) = 2n+1.$$

Proof: The Double star graph $K_{1,n,n}$is the tree obtained from the star graph $K_{1,n}$by adding a new pendant edges to the existing n pendant vertices. Here v as the root vertex of the double star. Let the vertex set and the edge set of double star graph is given by

$$V\big(K_{1,n,n}\big) = \{u\} \cup \{u_i :\ 1 \le i \le n\ \} \cup \{v_i :\ 1 \le i \le n\ \} \text{ and}$$
$$\mathrm{E}\big(K_{1,n,n}\big) = \{uu_i : 1 \le i \le n\} \cup \{u_i v_i : 1 \le i \le n\}$$

By the definition of splitting graph, now adding the new vertices $\{u', u_1', u_2', \ldots\ldots u_n'\}$ and$\{v_1', v_2', v_3', \ldots., v_n'\}$ corresponding to these vertices $\{u, u_1, u_2 u_3 \ldots. u_n\}$ and $\{v_1, v_2, v_3, \ldots., v_n\}$ respectively. In $S'\big(K_{1,n,n}\big)$, the vertex set and edge set is given by

$$V\big(S'(K_{1,n,n})\big) = \begin{cases} \{u\} \cup \{u'\} \cup \{u_i : 1 \le i \le n\} \cup \{u_i' : 1 \le i \le n\} \cup \\ \{v_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n\} \end{cases} \text{and}$$
$$E\big(S'(K_{1,n,n})\big) = \begin{cases} \{uu_i : 1 \le i \le n\} \cup \{uu_i' : 1 \le i \le n\} \cup \{u' u_i : 1 \le i \le n\} \\ \{u_i' v_i : 1 \le i \le n\} \cup \{u_i v_i : 1 \le i \le n\} \cup \{u_i v_i' : 1 \le i \le n\} \end{cases}$$

Now we construct the total coloring$f$, such that$f : S \to C$ as follows, where $S = V\big(S'(K_{1,n,n})\big) \cup E\big(S'(K_{1,n,n})\big)andC = \{1, 2, 3, \ldots.2n+1\}$. Now we assign the total coloring to these vertices and edges as follows.

$$f(u) = f(u') = 2n+1$$
$$f(v_i) = f(v_i') = n+2 \quad for \quad 1 \le i \le n$$
$$f(u_i) = f(u_i') = \begin{cases} i+1, & if\ i+1\ \not\equiv 0\ (mod\ n+1) \\ n+1, & 0therwise \end{cases} \quad for\, 1 \le i \le n$$
$$f(uu_i') = f(u' u_i) = \begin{cases} i, & if\ i\ \not\equiv 0\ (mod\ n) \\ n, & 0therwise \end{cases} \quad for \quad 1 \le i \le n$$
$$f(uu_i) = \begin{cases} n+i, & if\ n+i\ \not\equiv 0\ (mod\ 2n) \\ 2n, & 0therwise \end{cases} \quad for\, 1 \le i \le n$$
$$f(u_i\, v_i') = f(u_i'\, v_i) = 2n+1\, for\, 1 \le i \le n$$
$$f(u_i\, v_i) = 2n \quad for \quad 1 \le i \le n$$

We observed that the above rule of total coloring pattern, the graph$S'(K_{1,n,n})$ is total colored with $2n+1$colors. Hence the total chromatic number of$S'(K_{1,n,n})$, $\chi''\big(S'(K_{1,n,n})\big) = 2n+1$.

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Received: 15 Mar 2018/Accepted: 20 Apr 2018