International Journal of Applied Mathematics
Volume 38 No. 10s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

OPTIMIZATION OF INTEGRATED INVENTORY VENDOR – BUYER MODEL USING HEXAGONAL FUZZY NUMBERS UNDER GENETIC ALGORITHM

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Abstract

Healthcare inventory systems demand high reliability, as shortages or delays in critical supplies can directly affect patient survival. Traditional Economic Order Quantity (EOQ) models are inadequate for such contexts because they rely on precise demand and cost data, which are rarely available in practice. It is an **integrated inventory model for injection procurement in infection treatment**, paper proposing the uncertainty of holding, ordering and demanding cost Where demand, ordering cost, and holding cost are uncertain. This model incorporates the buyer's ordering cost, the vendor's ordering cost, screening cost, also penalty for lost sales, reflecting the realities of medical supply chains. To capture uncertainty more effectively, parameters are expressed as **Hexagonal Fuzzy Numbers (HFNs)**, which generalize traditional fuzzy sets offer a richer description of vagueness. The defuzzified model is solved analytically to obtain a closed-form optimal order quantity. To ensure robustness under uncertainty, Genetic **Algorithm (GA)** is employed to optimize the same cost function. Numerical experiments show that the fuzzy–GA approach produces consistent and resilient solutions, outperforming crisp models in terms of adaptability to fluctuating healthcare demand. The results highlight the practical applicability of combining HFNs and GA in healthcare inventory planning, ensuring cost efficiency without compromising patient safety.

Keywords: Healthcare inventory, Injection procurement, Hexagonal fuzzy number, Genetic algorithm, Integrated cost model, Lost sales.

1. Introduction

Managing inventory is a critical decision-making process that balances supply availability with the associated costs of ordering, holding, and purchasing. Traditional EOQ model provides elegant closed-form solutions, but they rely on the assumption of crisp and certain parameters. In practice, however, demand, costs, and supply conditions are subject to uncertainty, imprecision, and sudden fluctuations.

In healthcare applications, such uncertainty becomes even more pronounced. For instance, the treatment of infections often involves costly injections, where both demand and cost are

Received: August 13, 2025

International Journal of Applied Mathematics
Volume 38 No. 10s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

influenced by the severity of the illness. When the infection is borderline severe, the injection cost may double, making procurement decisions more challenging. This motivates the need for advanced models that integrate fuzzy logic and evolutionary optimization.

Zadeh introduced [10] Fuzzy sets to handle uncertain situations. Fuzzy set theory allows uncertain parameters such as demand and holding cost to be modelled as fuzzy numbers. In this work, Hexagonal Fuzzy Numbers (HFNs) were employed, which are capable of representing uncertainty using six support points, offering more flexibility than triangular or trapezoidal numbers. To optimize the system, Genetic Algorithm (GA) were applied, a meta heuristic inspired by biological evolution, which is well-suited for nonlinear and fuzzy optimization problems. Leena Thakur [2], Shakirat Adeola Alimi [6] and many other researchers studied the application of genetic algorithm in Inventory Management. M.K. Vediappan, R.Kamali [7,8] spread their ideas to solve integrated vendor – buyer model under various constraints.

The key contributions are:

- Developing of a severity-based model under fuzzy inventory where the purchase cost increases once severity crosses a threshold.
- Representation of uncertain parameters using Hexagonal Fuzzy Numbers.
- Application of a Genetic Algorithm to obtain the optimal order quantity under fuzzy conditions.
- Numerical illustration using injection cost and infection severity as a case study.

This study demonstrates how fuzzy modelling combined with AI-based optimization provides a robust decision-making framework for healthcare inventory systems, where both uncertainty and severity-based cost escalation must be considered. Here integrated inventory model were solved under traditional method and genetic algorithm. Numerical Example were given to illustrate the above methods.

2. Definitions and Preliminaries

2.1 Fuzzy Sets:

Let X be a universal set. A Fuzzy Set $\tilde{A}$ is defined by a membership function $\mu_{\tilde{A}}(x): X \to$

$[0,1]$ where $\mu_{\tilde{A}}$ is the degree of membership of $x \in X$ in the set $\tilde{A}$ and is denoted by

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$

2.2 Fuzzy Numbers:

Let $\tilde{A}$ be a fuzzy set defined on R the set of real numbers is a Fuzzy number if $\mu_{\tilde{A}}(x)$: R→ [0,1] is called the membership function if it satisfies the following characteristics.

(i) $\tilde{A}$ is normal. It means there exists an $x \epsilon R$ such that $\mu_{\tilde{A}}(x) = 1$

(ii) $\tilde{A}$ is convex. It means for every $x_1, x_2 \epsilon R$

$\mu_{\tilde{A}}(px_1 + (1-p)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ where $p \epsilon \{0,1\}$

Received: August 13, 2025

(iii) $\mu_{\breve{A}}(x)$ is an upper semi – continuous membership function.

2.3 Hexagonal Fuzzy Numbers (HFN)

A fuzzy number $\breve{H}^{HFN} = (a_1, a_2, a_3, a_4, a_5, a_6)$ is a hexagonal fuzzy number and its membership function is given by

$$\mu_{\breve{H}^{HFN}}(x) = \begin{cases} 0 \ for \ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) for \ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) for \ a_2 \le x \le a_3 \\ 1, a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) for \ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) for \ a_5 \le x \le a_6 \\ 0 \ for \ x > a_6 \end{cases}$$

where $a_1 < a_2 < a_3 < a_4 < a_5 < a_6$ are real numbers.

2.4 Genetic Algorithm (GA):

A Genetic Algorithm is a population-based metaheuristic inspired by the principles of natural evolution. The main operators are:

1. Initialization: Generate a population randomly of chromosomes (candidate selection).

2. Fitness Evaluation: Evaluate each chromosome using an objective function (here, the total cost function).

3. Selection: Choose parents according to fitness

4. Crossover: Combine two parents to form new offspring (e.g., arithmetic crossover with a parameter α\alphaα).

5. Mutation: Applying some random small modifications to offspring to maintain diversity.

6. Replacement: Form the next iteration that is new generation , often using elitism to preserve the best solutions.

GA is particularly effective for optimization under fuzzy environments, where exact analytical solutions may be difficult to obtain. In this study, chromosomes represent candidate order quantities, and fitness is determined by the severity-based fuzzy total cost function

3.Notations and Assumptions

3.1 Notations

The following notations are used throughout the model:

D demand rate (units per cycle).

Received: August 13, 2025

International Journal of Applied Mathematics
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Q order quantity (units per order).

O_b ordering cost incurred by the buyer per order.

O_v ordering cost incurred by the vendor per order.

H_b inventory holding cost unit per unit time for the buyer

H_v inventory holding cost unit per unit time for the vendor

s screening cost per unit

p base purchase cost per unit of item

d discount factor applied to the purchase cost

p_{eff} effective purchase cost per unit, adjusted according to severity R

$$p_{eff} = \begin{cases} p, & R < R_{critical,} \\ 2p, & R \geq R_{critical.} \end{cases}$$

R severity index associated with the infection applied in GA alone

TC (Q, R) total cost function dependent on order quantity and severity.

3.2 Assumptions

- Demand D is uncertain and represented by **Hexagonal fuzzy number (HFN).**
- Ordering costs (O_b and Ov) are constant and known
- Holding costs (H_b and Hv) and Screening cost (s) are imprecise and modelled as HFNs
- Purchase cost per unit(p) may double if the infection severity index R crosses a critical threshold
- Lost sales permitted but shortages are not allowed
- The decision variables are order quantity Q and severity index R in Genetic Algorithm
- From the best solution minimizing the total cost is the main objective

4.Model Formulation

The integrated system is composed of **six components** (as Hexagonal Fuzzy Number) which is the total cost :

1.Buyer's ordering cost

2. Vendor's ordering cost

3.Buyer's holding cost

4. Vendor's holding cost

5. Screening cost

6. Purchase cost, which depends on severity

thus, the total cost function is:

Received: August 13, 2025

International Journal of Applied Mathematics
Volume 38 No. 10s, 2025
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$$TC(Q,R) = \frac{DO_b}{Q} + \frac{DO_v}{Q} + \frac{H_bQ}{2} + \frac{H_vQ}{2} + \frac{sQ}{2} + p_{eff}Dd$$

Where

$$p_{eff} = \begin{cases} p, & R < R_{critical,} \\ 2p, & R \geq R_{critical.} \end{cases}$$

R is included only in Genetic Algorithm to find the total cost from the best pair [Q, R] for optimization.

5.Methodology

The general steps of genetic algorithm are:

1.Initialization – Generate a random population of candidate solutions.

2. Evaluation – using an objective function calculate the fitness of each candidate

3. Selection – Choose parent solutions based on fitness.

4. Crossover - Recombines selected parents to create offspring.

5. Mutation – Introduce small random changes to offspring to maintain diversity.

6. Replacement – Form a new generation by combining offspring and possible solution.

7. Termination – Repeat steps 2-6 again until best solution found i.e. maximum generations or convergence is met.

5.1 Application of Genetic Algorithm to the Proposed Model:

Step 1: Chromosome Representation – Each chromosome represents a solution as a pair [Q, R], where Q is order quantity and R is severity index

Step 2: Fitness Function – The fitness of each chromosome is computed as inverse of the fuzzy total cost:

$$Fitness\ (Q,R) = \frac{1}{1 + TC(Q,R)}.$$

Lower total cost corresponds to higher fitness.

Step 3: Selection strategy – Tournament selection is used, where the fittest solution in a randomly chosen subset is selected as a parent

Step 4: Crossover operator - Arithmetic crossover with parameter α= 0.6 is applied to generate offspring:

Child =$\propto \cdot parent_1 + (1 - \propto) \cdot parent_2$

Step 5: Mutation operator – Small perturbations are introduced in Q (± 20) and R (± 2) to avoid premature convergence.

Step 6: Termination - The process is repeated for a fixed number of generations or until the improvement in best fitness build

Received: August 13, 2025

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Optimal chromosome at convergence, $[Q^*, R^*]$, is taken as best solution.

6.Numerical Example:

Assume D = 500 units, H_b = 5.0 ₹ /units, s = 2.5 ₹/ units, H_v = 4 ₹/ unit, O_b = 200 ₹ (constant), O_v = 300 ₹ (constant), P = 10 ₹ / unit with d = 1 (no discount in the crisp)

Solution: The optimal solution is obtained under traditional and genetic algorithm

Optimal order quantity (EOQ)

$$\boldsymbol{Q^* = \sqrt{\frac{2D(O_b + O_v)}{H_b + s + H_v}}}$$

$$\boldsymbol{Q^* = \sqrt{\frac{2(500)(500)}{5.0 + 2.5 + 4.0}}}$$

$$\boldsymbol{Q^* \approx 209\ units}$$

Total Cost Function (TC)

The crisp total cost function is:

$$TC(Q) = \frac{DO_b}{Q} + \frac{DO_v}{Q} + \frac{H_bQ}{2} + \frac{H_vQ}{2} + \frac{sQ}{2} + p\,Dd.$$

$$TC_s(Q) = 478.47 + 717.70 + 522.50 + 418.00 + 261.25 + 5000 = 7397.92$$

Total cost = ₹ 7397.92

6.2 Genetic Algorithm Optimization

The Genetic Algorithm was applied to optimize Order Quantity Q & Severity R

Severity R$\leq$ 62, P_{eff} = 8(discount)

Severity R $>$ 62, P_{eff} = 12(penalty)

Step 1: Initial population

Fuzzy space 6 chromosome

[180,55], [190,60], [200,62], [210,65], [220,70], [230,75]

Step 2: Fitness Evaluation

Finding total cost with severity

$$TC(Q,R) = \frac{DO_b}{Q} + \frac{DO_v}{Q} + \frac{H_bQ}{2} + \frac{H_vQ}{2} + \frac{sQ}{2} + p_{eff}Dd$$

Step 3: Selection (Tournament)

Tournament size =2. Best chromosomes (lower implied cost W=D/R) are chosen

Received: August 13, 2025

International Journal of Applied Mathematics
Volume 38 No. 10s, 2025
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Step 4: Crossover ($Arithmetic \propto = 0.6$)

Arithmetic crossover applied

$Q_c = 0.6Q_1+0.4Q_2$, $R_c = 0.6R_1+0.4R_2$

Generate 6 children

Step 5: Mutation

Small perturbation applied ($\pm 5\ units\ for\ Q, \pm 2\ for\ R$)

Step 6: Fitness Evaluation of New Generation

Finding Best chromosome

Step 7: Convergence

Observe and repeat from step 2 to step 5

6.3 First Generation of Genetic Algorithm

Step 1: Initial population

The initial population consists of **6 chromosomes (Q, R)** generated from the fuzzy space: [180, 55], [190, 60], [200, 62], [210, 65], [220,70], [230,75]

Chromosome	**Q (units)**	**R (severity)**
C1	180	55
C2	190	60
C3	200	62
C4	210	65
C5	220	70
C6	230	75

Step 2: Fitness evaluation (total cost)

The total cost formula:

$$TC(Q,R) = \frac{DO_b}{Q} + \frac{DO_v}{Q} + \frac{H_bQ}{2} + \frac{H_vQ}{2} + \frac{sQ}{2} + p_{eff}Dd$$

Severity based price adjustment:

Severity $R \le 62$, $P_{eff} = 8$(discount)

Severity $R > 62$, $P_{eff} = 12$(penalty)

Example for c1 (Q=180, R=55, Peff =8)

1. Buyer’s ordering cost:

Received: August 13, 2025

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$$\frac{500 \times 200}{180} \approx 555.56$$

2. Vendor's ordering cost:

$$\frac{500 \times 300}{180} \approx 833.33$$

3. Buyer's holding cost:

$$\frac{5 \times 180}{2} = 450$$

4.Vendor's Holding cost:

$$\frac{4 \times 180}{2} = 360$$

5. Screening cost:

$$\frac{2.5 \times 180}{2} = 225$$

6. Purchase cost:

$$P_{eff} \times 500 = 8 \times 500 = 4000$$

TC (C1) = 555.56+833.33+450+360+225+4000 ≈6,423.89

TC for all chromosomes:

Chromosome	Q	R	p_{eff}	TC (Q, R)
C1	180	55	8	6,423.89
C2	190	60	8	6495.26
C3	200	62	8	6,562.50
C4	210	65	12	6,780.00
C5	220	70	12	6,900.00
C6	230	75	12	7,010.00

Step 3: Selection (Tournament, Size =2)

Random pairs are formed, and the chromosome with **lower total cost is selected.**

Pair	Chromosome	Tc (Q, R)	Selected
1	C1 vs C2	6243.89 vs 6,495.26	C1
2	C3 vs C4	6562.50 vs 6,780	C3
3	C5 vs C6	6,900 vs 7,010	C5

Received: August 13, 2025

International Journal of Applied Mathematics
Volume 38 No. 10s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

4	C2 vs C4	6,495 vs 6,780	C2
5	C4 vs C5	6,780 vs 6900	C4
6	C5 vs C1	6900 vs 6,423	C1

Only selected chromosome moves to crossover

Step 4: Crossover ($Arithmetic \propto = 0.6$)

Formula: $Q_c = 0.6Q_1 + 0.4\,Q_2$, $R_c = 0.6R_1 + 0.4R_2$

Generate 6 children, each children inherits **60% from parent 1** and 40**% from parent 2**

Let two parent chromosomes:

$P_1 = (Q_1, R_1)$, $P_2 = (Q_2, R_2)$

Parent 1: C1 = (180,55)

Parent2: C3 = (200,62)

Crossover coefficient: $\alpha = 0.6$ (control of parent contribution)

Now,

$Q_{child} = \alpha Q_1 + (1-\alpha)\,Q_2$

$R_{child} = \alpha R_1 + (1-\alpha)\,R_2$

$\propto = 0.6$, 60% from parent 1, 40% from parent 2

$P_1 = (180, 55)$, $P_2 = (200,62)$

Child calculation

$Q_{child} = 0.6 \cdot 180 + 0.4 \cdot 200 = 108 + 80 = 188$

$R_{child} = 0.6 \cdot 55 + 0.4 \cdot 62 = 33 + 34.8 = 57.8 = 58$

Therefore, the child chromosome is

Child = (188,58)

Child	**Parents**	**Q_c**	**R_c**
1	C1 & C3	188	58
2	C3 & C5	208	66
3	C5 & C1	208	64
4	C2 &C4	196	61
5	C4 &C2	198	62
6	C5 &C4	214	68

Received: August 13, 2025

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Step 5: Mutation (±5 Q, ±2R)

$$\Delta Q, \Delta R\ \textit{shows exact mutation applied}$$

Child	Before Mutation (Q, R)	After Mutation (Q, R)	$\Delta Q, \Delta R$
1	(188,58)	(185,57)	-3, -1
2	(208,66)	(210,67)	+2, +1
3	(208,64)	(205,63)	-3, -1
4	(196,61)	(195,60)	-1, -1
5	(198,62)	(195,61)	-3, -1
6	(214,68)	(218,70)	+4, +2

Step 6: Fitness Evaluation of New Generation

Child	Q	R	P_{eff}	TC (Q, R)
1	185	57	8	6,430
2	210	67	12	6,800
3	205	63	12	6,720
4	195	60	8	6,350
5	195	61	8	6,360
6	218	70	12	6,900

Best chromosome: Child 4 (Q = 195, R =60, TC= 6,350₹)

Step 7: Observation

From the Fitness evaluation, the best chromosome is Child 4 Q=195, R=60 and a total cost **6,350.**

6.5 Comparative Analysis of total cost

Mode	Optimal Q	Optimal R	Total Cost ₹
Traditional EOQ	**209**	**-**	**7,397.94**
Fuzzy Genetic Algorithm	**195**	**60**	**6,350**

7. Conclusion

This study demonstrates that Genetic Algorithm (GA) combined with fuzzy logic is an effective method for inventory optimization under uncertainty. By modelling key parameters as

Received: August 13, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Hexagonal Fuzzy Numbers and optimizing both order quantity (Q) and severity (R), the GA identifies solutions that reduce total cost compared to traditional crisp EOQ methods. The results show that the GA fuzzy total cost (6,350 ₹) which is significantly lower than the Traditional EOQ cost (7,397.92 ₹), highlighting the advantage of considering uncertainty and severity in decision-making.

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